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Optimal environment design and revenue allocation: Under cap-and-trade policy in the cooperation supply chain

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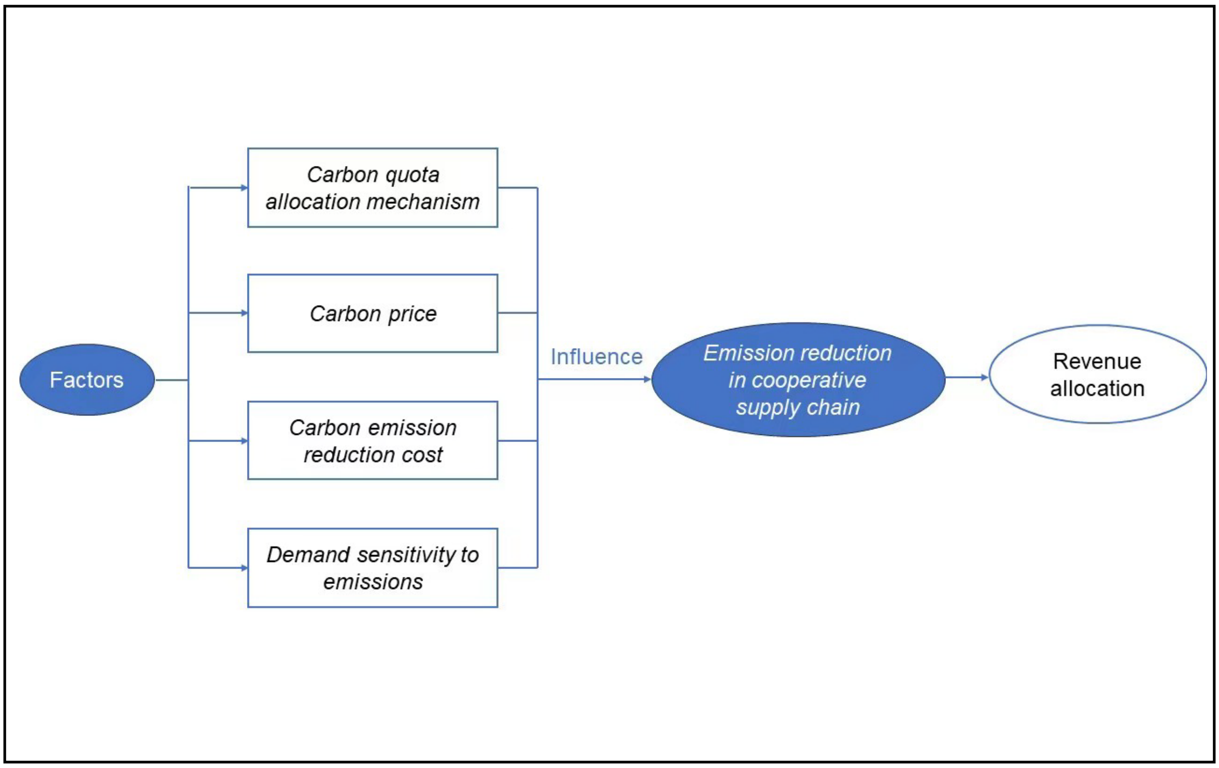
Graphical abstract



Analyzing the influence of some environmental factors on carbon emission reduction and optimizing revenue allocation in cooperative supply chain.

Public summary

■ From the perspective of a supply chain manager, we analyze how to guide the carbon reduction decision of the whole supply chain is a question worth studying.

■ We analyze the external environment, such as how carbon trading price, unit carbon emission reduction cost, the impact of efforts on emissions per unit of product, and the sensitivity of demand to unit emissions of the product affect the co-operative supply chain emission reduction in a carbon trading environment.

■ Combined with Shapley value, we introduce the concept of a "core" to propose a fair and stable revenue allocation mechanism to stabilize the cooperation supply chain stable.

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Abstract: Since the supply chains of the world's 2500 largest companies alone emit more than 20% of global greenhouse gases, how to achieve optimal cooperative supply chain emission reduction effects in supply chain optimal emission reduction efforts and effectively distribute revenue in cooperative supply chains is a difficult complex problem. In this paper, a green supply chain model of joint production is constructed based on the framework of the Stackelberg model and with carbon trading under three quota methods being taken into account. First, from the perspective of a supply chain leader, we obtain the optimal efforts to reduce emissions, the optimal price, and the yield of the products. Then, from the perspective of carbon market regulators, we obtain the environment that is most conducive to reducing emissions in the supply chain. Finally, we offer a profit distribution method based on the modified Shapley value, which maximizes fairness and stability. The data calculation example analysis further verifies the results of the theoretical analysis.

Keywords: green cooperation supply chain; cap-and-trade policy; carbon emission reduction; optimal environment; revenue allocation

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1 Introduction

Given the growing urgency of the climate issue, countries worldwide have paid much attention to the issue of greenhouse gas emissions. The 1992 United Nations Framework Convention on Climate Change aims to keep atmospheric concentrations of greenhouse gases at a stable level. The Kyoto Protocol, adopted in 1991 to support the United Nations Framework Convention on Climate Change, came into force in 2005. This is the first time in human history that greenhouse gas emissions have been regulated by regulations. It adopts a "top-down" model, assigning developed countries uniform targets for reducing greenhouse gas emissions by an average of 5% from 1990 levels between 2008 and 2012. Then, the Paris Agreement reached in 2015 lays out arrangements for post-2020 global action on climate change and defines a "bottom-up" model of relatively flexible emission reduction with nationally determined contributions at the core.

Carbon dioxide (CO_2) is an essential component of greenhouse gases. To restrain the greenhouse effect, reducing carbon dioxide emissions is very important. A carbon trading mechanism is a market mechanism aiming to promote global greenhouse gas emission reduction and reduce global carbon dioxide emissions. The first additional agreement of the Kyoto Protocol takes the market mechanism as a new way to solve the problem of greenhouse gas emission reduction, represented by carbon dioxide. Carbon credit, a generic term for any tradable certificate or permit representing the right to emit a set amount of carbon dioxide or the equivalent amount of different greenhouse gases, is regarded as a commodity, thus forming carbon trading. The carbon trading process is as follows: first, according to specific regulations, determine the total amount of emissions and emission quotas, and then allocate these quotas to key emission enterprises or units. Second, firms can carry out cap-and-trade during the implementation period. Finally, firms need to pay their full emissions quota at the end of the period or face a penalty for breach of contract.

There are generally three-carbon quota allocation rules under the current carbon cap-and-trade mechanism. However, researchers have not consensus on the optimal carbon quota allocation rules, which will affect the final emission reduction results. China's national carbon trading market, officially launched in July 2021, has been using a mix of carbon quota allocation rules, for example, combining auction rules with two free allocation rules. The flourishing European Union carbon market is experiencing a gradual reduction in the proportion of free quotas and increased the auctioned part. As far as the free quota mechanism is concerned, it has also experienced a transformation from "historical law" to "industry benchmark law". That is, the carbon quota allocation rules are still changing. Therefore, the influence of different carbon quota mechanisms on emission reduction has attracted research interest. The emissions from the supply chains of the 2500 largest global corporations account for more than 20% of global greenhouse gas emissions[1]. The Centre for Global Environmental Information research (CDP) calculated that a company's supply chain often produces 5.5 times more carbon emissions than its operational process. However, only 4% of companies set supply chain targets. Therefore, carbon emission reduction in the supply chain is essential and urgent. Considering the extensive literature, we define a "green" supply chain[2–5]: A green supply chain is a supply chain that balances environmental and operational performance through effective implementation of appropriate value cocreation and collaboration. It manages the flow of material, information, capital, and other necessary resources along with the nodes and links of a supply chain network while integrating triple bottom-line objectives from internal and external stakeholders.

On the one hand, superior supply chain management (SCM) is recognized as a contributor to firm performance, and the role of supply chain leaders is playing a role in an increasing number of supply chains. However, few studies take their role into account and examine carbon reduction as a whole in the context of the supply chain as a decision-making body. Specifically, most existing studies have modeled the game between firms in the supply chain or supplemented it with government regulation. For example, Yu et al.[6] model the equilibrium abatement and pricing strategy with manufacturers as leaders determining wholesale prices and abatement levels, and retailers as followers determining retail prices, deriving a cost and benefit sharing contract. From the perspective of a supply chain manager, how to guide the carbon reduction decision of the whole supply chain is a question worth studying.

In addition, the external environment, such as how carbon trading price, unit carbon emission reduction cost, the impact of efforts on emissions per unit of product, and the sensitivity of demand to unit emissions of the product affect the cooperative supply chain emission reduction in a carbon trading environment, is also a question worth studying. Moreover, most of the existing studies have focuse on non-cooperative supply chains. The few studies that examin the impact of the external environment on cooperative supply chains also involve fewer factors, e.g., Cheng et al.[7] find that on the network state consumers' increased environmental awareness can increase product transactions for low-carbon manufacturers; Kou et al.[8] find that when consumers have strong green preferences, cooperative emission reduction is of greater value.

On the other hand, how cooperative supply chains can effectively allocate their carbon reduction profits also needs to be discussed. Cooperative supply chain management aims to reduce production costs and increase profit through information sharing. In the cooperation supply chain, if the revenue allocation is not reasonable, the coalition will be out of balance, resulting in conflict. Therefore, an effective allocation of supply chain revenue is crucial to maintaining the cooperative relationship among firms in the supply chain. In conclusion, this paper takes a green cooperation supply chain under the cap-and-trade policy as the research object, establishes a model to minimize the total emissions of the supply chain, studies carbon emission reduction under various external environmental factors, and obtains the external environmental conditions conducive to supply chain emission reduction. In addition, combined with the Shapley value, we introduce the concept of a "core" to propose a fair and stable income distribution mechanism to keep the cooperation supply chain stable.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 establishes the green supply chain model and carries out the analysis. Section 4 provides a revenue distribution mechanism based on the Shapley value. Section 5 discusses the stability of cooperation and its solutions. Numerical examples are given in Section 6. Section 7 concludes this paper.

2 Literature review

Our study is related to two streams of literature: carbon emission reduction and revenue allocation.

In carbon emission reduction, many researchers study the drivers of changes in carbon emissions. Wang et al.[9] identify economic development, energy mix, and low energy efficiency as the three main factors contributing to the increase in carbon dioxide emissions. Jiang et al.[10] find that the contribution of different factors to global carbon emissions changes with time. Influencing factors of carbon emissions for different countries are diverse. They only attribute the driving factors of carbon emission reduction to carbon emission reduction cost and the impact of efforts on each emissions unit. However, in our study, we consider more factors, such as the carbon quota allocation mechanism, carbon emission reduction cost, demand sensitivity to emissions, and carbon price.

Several scholars address the mechanism and method of emission reduction. İşlegen and Reichelstein[11] find that carbon capture and storage technology can offer the potential to reduce carbon emissions from fossil fuel power plants significantly. Toptal et al.[12] analyze the joint decision of retailer inventory replenishment and carbon emission reduction investment under three carbon emission regulatory policies and shows that the opportunities of investing in carbon emission reduction can simultaneously reduce costs while reducing carbon emissions. Chen et al.[13] look at how to design a carbon tax scheme to reduce emissions without hindering long-term economic development. Lovelace and Bironneau[14] look for solutions to reduce the carbon emissions associated with the operations. Wang et al.[15] optimize the emission reduction path for "lagging regions" in China that do not meet the 2030 industrial carbon reduction target, based on the two-dimensional perspective of carbon emission efficiency and emission reduction cost.

In addition, Mirzaee et al.[16] develops a stochastic game theoretical model consisting of a manufacturer, a third-party carbon emission verifier, and the government to study the effect of the interaction of external and internal factors on the effectiveness of the strategy; Zhang et al.[17] find that under a consignment inventory policy, increasing the price of carbon trading reduces carbon emissions but increases the economic cost of the supply chain; Li et al.[18] study a single manufacturer under government regulation and find that if technological upgrading is necessary, it is the strategy-based promotion of increasing the level of consumers' low-carbon preferences that is an essential factor in achieving total social. All of the above studies are for individual firms or noncooperative

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supply chains, while our study explores the impact of external factors on cooperative supply chains. Our work differs from the above in several dimensions, as China's current rules for allocating carbon quotas are still in their infancy. The literature fails to study the influence mechanism of environmental factors on cooperative supply chain emission reduction under various allocation rules. Therefore, supply chain enterprises covered by carbon trading cannot predict these changes positive or negative impact on product prices. More importantly, the existing literature primarily focuses on non-cooperative supply chains, and very few studies have explored the influence of external factors on cooperative supply chains.There are fewer factors studied, which inspires the research in this paper.

For revenue allocation, the literature is mainly based on the concept of "core" to find a steady allocation of profits[19, 20]. On this basis, Tang et al.[21] describe the relationship between core and Shapley values. Furthermore, several researchers show that their Shapley value is at the core, resulting in fair and stable revenue allocation[22–24]. If Shapley values are not in the core, Nguyen et al.[25] introduce the "fairest core". We propose a fair and stable allocation scheme using Shapley values and cores and introduce the concept of nucleolus to study the fairest solution when Shapley values are not included in cores.

The Shapley value method is an allocation method based on contributions in the literature. Zhang et al.[26] considered cost and proposed the Shapley value with cost correction. Zhang et al.[27] modify the Shapley value with cost and risk correction. This paper considers the difference between cost input and risk-sharing among enterprises and introduces innovation modification. Moreover, in the subsequent determination of the weight of correction factors, this paper combines the analytic hierarchy process and entropy method, making the weight of qualitative and quantitative factors more scientific. In brief, considering cap-and-trade policies under three carbon trading allocation rules, we adopt a green cooperation model of the cooperation supply chain to analyze the optimal carbon-reduction environment and formulate rules for revenue allocation.

3 Green cooperation model

We discuss a green supply chain composed of multirole firms that jointly produce a product by participating in part of the process through green means in a carbon trading environment. Our joint production model is based on Ref. [1]. Let $i \in N = \{1, \cdots, n\}$ represent firm, $j \in M = \{1, \cdots, m\}$ represent process, N and M represent firm set and process set respectively. A firm can participate in multiple processes, and each process can involve multiple firms simultaneously. We use 0-1 matrix $B = (b_{i,j})$ to represent the firm's participation in each process in the supply chain, that is, if and only if firm i participates in j process, $b_{i,j} = 1$. In multiplayer cooperative games, any subset S of N is called a coalition. If at least one firm in coalition S participates in j process, $b_{S,j} = 1$. That is, for some $i \in S$, if $b_{i,j} = 1$, then $b_{S,j} = 1$.

3.1 Definitions and assumptions

Under a cap-and-trade scheme, the government sets limits or "caps" on the emissions allowed by specific industries. It issues a limited number of annual credits allowing firms to emit a certain amount of carbon dioxide that contributes to global warming. The total cap is divided into quotas, with the government setting a cap on the total amount of carbon emitted. Each quota allows firms to emit one ton of emissions. The government allocates subsidies to firms for free or through auctions. Nevertheless, the government reduces the number of credits each year, lowering the overall emissions cap. If a firm reduces its emissions, it can keep excess credits to meet future demand or sell them to another firm with an inadequate quota. The overall environmental objectives of a carbon emission trading system depend on the total amount of carbon emission quotas set by the system (i.e., the total amount) Q^c. Carbon quotas are allocated under Q^c to each unit that participates in cap-and-trade, and regulators typically assign carbon quotas on a business basis. The current carbon quota allocation methods of countries worldwide can be divided into three types: historical carbon intensity reduction, benchmarking, and auctioning.

a) Historical carbon intensity reduction rule

The historical carbon intensity reduction rule determines the quota according to control unit history emissions (on average) in recent years, which applies to the product characteristics of the complex production process industry. Moreover, its advantage lies in the more straightforward calculation and the minor requirement of data quantity. Nevertheless, its shortcomings are obvious, such as the unfair rewards for past emissions to the reduction action against the earlier firm, which not only did not take into account the recent economic development of firms but also did not use the historical emission data of new firms as a reference. Firms receive free quotas based on their historical emissions during a specified period. The carbon quota q_i allocated to each unit free of charge is[27]

$$q_i = \beta_i \cdot Q^c, \tag{1}$$

where β_i is the ratio of the average CO_2 emission of unit i in previous years to the average total CO_2 emission of the whole country in previous years.

b) Benchmarking rule

The benchmarking rule takes carbon emission intensity as the industry benchmark value. Moreover, industry carbon emissions represent the carbon emission level of unit activity under a certain production level, which is used as a reference index for the initial quota allocation of carbon trading. It is suitable for industries with simple production processes, a single product style, and a good data base. The number of free allowances allocated to a company is a measure of its performance relative to the emissions intensity of its products or sectors. The carbon quota q_i' allocated to each unit free of charge is[28]

$$q_i' = \beta_i' \cdot Q^c, \tag{2}$$

where β_i' is the ratio of the average annual output of i firm to the average total output of all firms in the industry over the

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years.

Both carbon quotas as above are obtained free of charge.

A supply chain cooperative relationship is a cooperative relationship in which all supply chain firms share information, risks, and profits for a certain period. Therefore, it is no longer reasonable for firms to conduct carbon trading as independent entities. We consider a contractual relationship: carbon quota sharing between cooperative enterprises in the supply chain. Under this contractual relationship, the total carbon quota of the cooperative supply chain is the sum of the carbon quota of each firm. In the carbon trading system, the whole supply chain as a trading party carries out carbon trading with the outside world. Therefore, the carbon quota of the whole supply chain this year is as follows:

$$q_N = \sum_{i\in N} q_i. \tag{3}$$

We assume that in the case of free allocation, the carbon quota θ of the whole supply chain in the carbon market in that year is either bought (>0) or sold (<0) :

$$\theta = f_N - q_N, \tag{4}$$

where f_N is the CO_2 emissions of the whole supply chain in that year, and $f_N = \sum_{i\in N} f_i$, q_N', q_i', θ' under the benchmarking rule are the same as those in Eqs. (3) and (4).

The revenue π° of the whole supply chain in the carbon market in that year is

$$\pi^\circ = -p^c \cdot \theta, \tag{5}$$

where p^c is the carbon trading price of the year. $\pi^{\circ\prime}$ under the benchmarking rule is the same.

c) Auctioning rule

Under the auctioning rule, participating units need to purchase quotas from designated auctioneers. The actual carbon emission of the supply chain is the quantity of carbon quota θ'' purchase:

$$f_N = \theta''. \tag{6}$$

The revenue $\pi^{\circ\prime\prime}$ of the whole supply chain in the carbon market in that year is

$$\pi^{\circ\prime\prime} = -p^c \cdot \theta'', \tag{7}$$

where p^c is the auction price of unit carbon credit. We assume that the emission reduction effort of firm i in process j is $e_{i,j}$. In unit time, firm i's emission reduction efforts remain unchanged, $e_{i,j}\begin{cases} \geqslant 0, & \text{when } b_{i,j}=1; \\ =0, & \text{when } b_{i,j}=0; \end{cases}$ obviously.

That is when firm i participates in the process of j, firm i can choose whether to pay and how much emission reduction efforts to pay. When firm i does not participate in the process j, firm i cannot make efforts to achieve carbon emission reduction in the process j. The corresponding cost of emission reduction effort $e_{i,j}$ is $k_{i,j}e_{i,j}$, where $k_{i,j}$ is the carbon emission reduction cost factor of firm i in process j. Let $f_{i,j}$ be the carbon emission generated by firm i in process j for producing each unit of product, $f_j = \sum_{i\in N} f_{i,j}$; f_N is the carbon emission per unit of product produced for the entire supply chain, $f_N = \sum_{j\in m} f_j$. The carbon footprint of the entire supply chain is random and influenced by emission reduction efforts $e_{i,j}$ as follows:

$$f_N = \alpha - \sum_{i,j} \alpha_{i,j} e_{i,j} + \varepsilon, \tag{8}$$

where α is the expected emission per unit of product in the absence of carbon emission reduction efforts, $\alpha_{i,j}$ is the marginal effect of effort on emissions per unit of product, and ε is a random variable with $E(\varepsilon)$ equal to 0. Although the cost of emission reduction is usually a quadratic function of emission reduction effort, for ease of subsequent analysis and calculations, we assume a linear relationship referring to Ref. [29].

To prevent a negative carbon footprint, we make the following assumption.

Assumption 1. $\alpha - \sum_{i,j} \alpha_{i,j} e_{i,j} + \varepsilon \geqslant 0$, so that $E(f_N) = \alpha - \sum_{i,j} \alpha_{i,j} e_{i,j} \geqslant 0$.

We assume that market demand is sensitive to both product price and carbon emission levels, referring to Ref. [30]. We assume that demand information is common sense, and the demand function per unit of time is

$$D = a - p - \lambda f_N, \tag{9}$$

where a is the base market size and λ is the emission impact factor, that is, the sensitivity of demand to unit emissions of a product. We assume that each process's carbon footprint $f = (f_1, \cdots, f_m)$ of each process is measurable, while the emission reduction effort $e_{i,j}$ is obviously not.

From Eq. (7),

$$D = a - p - \lambda\left(\alpha - \sum_{i,j} \alpha_{i,j} e_{i,j} + \varepsilon\right). \tag{10}$$

That is, when one firm increases its emission reduction efforts, all the other firms benefit from the eventual reduction in emissions of their products because reducing the total carbon emissions of the supply chain increases demand. This is the positive externality of mitigation efforts, and the greater the λ, the greater the positive externality.

Note that at equilibrium, supply chain production $Q = E(D)$, i.e.,

$$Q = a - p - \lambda\left(\alpha - \sum_{i,j} \alpha_{i,j} e_{i,j}\right). \tag{11}$$

The total emissions of the supply chain are $F_N = E(Q \cdot f_N)$.

The profit of the whole supply chain is composed of sales, production cost (basic production cost and carbon emission reduction effort cost), and carbon trading income.

$$\pi = p \cdot Q - \left(c + \sum_{i,j} k_{i,j} e_{i,j}\right) \cdot Q - p^c \theta, \tag{12}$$

where p^c is the carbon price, p is the product price, c is the unit base production cost (not affected by emission reduction

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efforts), and $p > c$.

3.2 Solution and analysis

In this section, the model we consider is the response of the cooperative supply chain under the supervision of the supply chain leader to exogenous variables, such as the unit price of external carbon trading under three carbon quota allocation methods, the allocation method of the carbon quota, and the analysis. First, assuming that external environmental factors, including carbon prices, are exogenous, we find the optimal response strategy from the perspective of supply chain leaders. It can be interpreted as a two-stage model with the following sequence of decisions in each stage:

Stage 1. In regard to making decisions about carbon reduction efforts, the firm optimizes its market response by solving $\pi_e(p) = \max_{p\geqslant 0}\pi_e(p)$ to obtain the optimal price p_e^*.

Stage 2. At price p_e^*, the firm optimizes its emission reduction effort $e^* = \operatorname{argmin}_{e\in[0,\bar{e}]} F_N(e)$.

Therefore, the decision of a firm can be expressed as $\{e^*, p_{e^*}^*\}$, and the optimal production quantity $Q_{e^*}^* = E[D(p_{e^*}^*)]$, it can be regarded as the response function of the firm to the carbon price p^c in the Stackelberg game.

Second, based on the decision-making of supply chain leaders, we stand in the position of market regulators such as the government, take external environmental factors as endogenous variables, and analyze what kind of environment is more conducive to the impact of supply chain emission reduction. For the sake of convenience, let $\sum_{i,j} k_{i,j}e_{i,j} = \widetilde{k_{i,j}}\cdot\overline{e_{i,j}}$, $\sum_{i,j}\alpha_{i,j}e_{i,j} = \widetilde{\alpha_{i,j}}\cdot\overline{e_{i,j}}$ is average emission reduction efforts, and $\widetilde{k_{i,j}}$, and $\widetilde{\alpha_{i,j}}$ are the comprehensive carbon emission reduction cost factor and the impact factor of comprehensive emission reduction efforts on emissions per unit of product respectively. Given Eqs. (10)–(12), for a given carbon emission effort e and carbon price p^c, we obtain the following results.

Lemma 1. The optimal price and quantity are given by

$$p_e^*(p^c) = \frac{a + c + \widetilde{k_{i,j}}\cdot\overline{e_{i,j}} + (p^c - \lambda)(\alpha - \widetilde{\alpha_{i,j}}\cdot\overline{e_{i,j}})}{2}, \tag{13}$$

$$Q_e^*(p^c) = \frac{a - c - \widetilde{k_{i,j}}\cdot\overline{e_{i,j}} - (p^c + \lambda)(\alpha - \widetilde{\alpha_{i,j}}\cdot\overline{e_{i,j}})}{2}, \tag{14}$$

respectively.

We make the following assumptions.

Assumption 2.

(ⅰ) $p_e^*(p^c) = \dfrac{a + c + \widetilde{k_{i,j}}\cdot\overline{e_{i,j}} + (p^c - \lambda)(\alpha - \widetilde{\alpha_{i,j}}\cdot\overline{e_{i,j}})}{2} \geqslant 0$;
$Q_e^*(p^c) = \dfrac{a - c - \widetilde{k_{i,j}}\cdot\overline{e_{i,j}} - (p^c + \lambda)(\alpha - \widetilde{\alpha_{i,j}}\cdot\overline{e_{i,j}})}{2} \geqslant 0$.

(ⅱ) $p^c \in \left[0, \min\left\{\dfrac{\widetilde{k_{i,j}}}{\widetilde{\alpha_{i,j}}} - \lambda, \dfrac{a-c}{\alpha} - \lambda\right\}\right.$.

(ⅲ) $\dfrac{\widetilde{k_{i,j}}}{\widetilde{\alpha_{i,j}}} - \lambda \geqslant 0$, $\dfrac{a-c}{\alpha} - \lambda \geqslant 0$.

The first assumption holds w.l.o.g. and helps us to avoid trivial cases: If $p_e^*(p^c)$, $Q_e^*(p^c) < 0$, then there is some kind of emissions-reduction effort that is never chosen, and can therefore be removed from consideration. The second assumption is solved by the first and is considered in Stage 2. If Assumption 2(ⅱ) is not satisfied, $\overline{e^*} = +\infty$. That is, under the condition $p^c \in \left[0, \min\left\{\dfrac{\widetilde{k_{i,j}}}{\widetilde{\alpha_{i,j}}} - \lambda, \dfrac{a-c}{\alpha} - \lambda\right\}\right]$, firms will not excessively invest in carbon emission reduction efforts. Otherwise, the problem will become a trivial case with infinite demand. Therefore, we require $p^c \in \left[0, \min\left\{\dfrac{\widetilde{k_{i,j}}}{\widetilde{\alpha_{i,j}}} - \lambda, \dfrac{a-c}{\alpha} - \lambda\right\}\right]$ throughout the paper to guarantee that all firms do not excessively invest in carbon emission reduction efforts.

The third assumption is made to ensure Assumption 2(ⅱ).

Based on Assumption 2, we can get $\overline{e_{i,j}} \in \left[0, \overline{e_{i,j}}^{\lim}\right]$ and $\overline{e_{i,j}}^{\lim} = \dfrac{(p^c + \lambda)\alpha - (a - c)}{(p^c + \lambda)\widetilde{\alpha_{i,j}} - \widetilde{k_{i,j}}}$.

Lemma 2.

(ⅰ) Under the historical and benchmarking rule, $\pi_e(p^c)$ is a convex function of p^c on $\left[0, \min\left\{\dfrac{\widetilde{k_{i,j}}}{\widetilde{\alpha_{i,j}}} - \lambda, \dfrac{a-c}{\alpha} - \lambda\right\}\right]$, and $\pi_e(0) > \pi_e\left(\min\left\{\dfrac{\widetilde{k_{i,j}}}{\widetilde{\alpha_{i,j}}} - \lambda, \dfrac{a-c}{\alpha} - \lambda\right\}\right)$.

(ⅱ) Under the auditioning rule, $\pi_e(p^c)$ is a decreasing function of p^c on $\left[0, \min\left\{\dfrac{\widetilde{k_{i,j}}}{\widetilde{\alpha_{i,j}}} - \lambda, \dfrac{a-c}{\alpha} - \lambda\right\}\right]$.

This implies that firms can obtain high profits when p^c is either too large or too small and obtains the optimal profit when $p^c = 0$ under the historical and benchmarking rules. In other words, firms prefer p^c to be in its largest or smallest range. In addition, it would be better if there were no carbon trading prices. However, under the auditioning rule, the higher the carbon price, the lower the profit of the supply chain.

For convenience, in the cooperative supply chain, we denote

$$h_{i,j} = \frac{\alpha_{i,j}}{k_{i,j}}, \tag{15}$$

$$\widetilde{h_{i,j}} = \frac{\widetilde{\alpha_{i,j}}}{\widetilde{k_{i,j}}}. \tag{16}$$

$h_{i,j}$ can be interpreted in two ways. First, the higher $h_{i,j}$ is, the higher the impact factor $\alpha_{i,j}$ or the lower the carbon emission reduction cost factor $k_{i,j}$ will be. And the higher the emission reduction effort efficiency (call it "emission reduction efficiency" for short) will be. Therefore, we call $h_{i,j}$ the emission reduction efficiency factor of enterprise i in process j. Second, $h_{i,j}$ captures the ratio between the degree of positive externalities generated by the emission reduction efforts of firm i to process j and the cost of providing such mitigation efforts. Therefore, the higher $h_{i,j}$ is, the more favorable the emission reduction will be.

$\widetilde{h_{i,j}}$ has a similar interpretation to $h_{i,j}$. $\widetilde{h_{i,j}}$ is the emission reduction efficiency factor of the whole supply chain. $\widetilde{h_{i,j}}$ represents the ratio between the degree of positive externalities generated by the mitigation efforts of all firms in the supply chain and the cost of providing such mitigation efforts. Obviously, the higher $\widetilde{h_{i,j}}$ is, the more beneficial it is to reduce the

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whole supply chain.

Lemma 3. Under three carbon quota methods:

(i) When $\widetilde{h_{i,j}} \geqslant \dfrac{\alpha}{a-c}$, $p^c \in \left[0, \dfrac{2(a-c)}{\alpha} - \dfrac{1}{\widetilde{h_{i,j}}} - \lambda\right]$, $F_N(p^c)$ is a decreasing function of $\overline{e_{i,j}}$ on $[0, \overline{e_{i,j}}^{\lim}]$ with a unique minimum at $\overline{e_{i,j1}}^* = \dfrac{\alpha}{2\widetilde{a_{i,j}}}$ and the corresponding $p^*_{e^*(p^c)}(p^c)_1 = \dfrac{a+c+\dfrac{\alpha}{2}(\dfrac{1}{\widetilde{h_{i,j}}} + p^c - \lambda)}{2}$.

(ii) $\widetilde{h_{i,j}} \geqslant \dfrac{\alpha}{a-c}$, $p^c \in \left[\dfrac{2(a-c)}{\alpha} - \dfrac{1}{\widetilde{h_{i,j}}} - \lambda, \dfrac{1}{\widetilde{h_{i,j}}} - \lambda\right]$, $F_N(p^c)$ is a decreasing function of $\overline{e_{i,j}}$ on $[0, \overline{e_{i,j}}^{\lim}]$ with a unique minimum at $\overline{e_{i,j2}}^* = \overline{e_{i,j}}^{\lim}$ and the corresponding $p^*_{e^*(p^c)}(p^c)_2 = \dfrac{a+c+\widetilde{k_{i,j}}\overline{e_{i,j}}^{\lim} + (p^c-\lambda)(\alpha - \widetilde{a_{i,j}}\overline{e_{i,j}}^{\lim})}{2}$.

(iii) $\widetilde{h_{i,j}} \leqslant \dfrac{\alpha}{a-c}$, $p^c \in \left[0, \dfrac{a-c}{\alpha} - \lambda\right]$, $F_N(p^c)$ is a convex function of $\overline{e_{i,j}}$ on $[0, \overline{e_{i,j}}^{\lim}]$ with a unique minimum at $\overline{e_{i,j3}}^* = \overline{e_{i,j1}}^* + \dfrac{1}{2}\overline{e_{i,j2}}^*$ and the corresponding $p^*_{e^*(p^c)}(p^c)_3 = \dfrac{1}{2}p^*(p^c)_2 + \dfrac{a+c+\dfrac{\alpha}{2}\dfrac{1}{\widetilde{h_{i,j}}}}{2}$.

At this point, we obtain the supply chain's decision $(e^*(p^c), p^*_{e^*(p^c)}(p^c))$ as above.

According to Lemma 3, when $\widetilde{h_{i,j}} \geqslant \dfrac{\alpha}{a-c}$, the effort is not the bigger the better. And when $\widetilde{h_{i,j}} \leqslant \dfrac{\alpha}{a-c}$, the effort is not the bigger the better. It does not accord with our usual cognitive cognition, mainly because the increase of efforts in this situation makes the equilibrium price increase, while the increasing demand the unit emission made may not be enough to offset the impact of the price, so more extraordinary efforts may increase supply chain emissions instead.

From Lemma 3, we can establish the following proposition.

Proposition 1.

(i) The carbon price p^c in Lemma 3(ii) is higher than that in Lemma 3(i), but $\overline{e_{i,j2}}^* \leqslant \overline{e_{i,j1}}^*$.

(ii) In Lemma 3(i), regardless of the value of p^c is, $e^*(p^c)$ remains the same.

We can conclude that it is not that the higher p^c is, the more emission reduction efforts the firms should make. This indicates that the optimal emission reduction efforts are non-monotone concerning carbon prices: Increasing the carbon price level does not necessarily favor emission reduction efforts.

Proposition 2. $\overline{e_{i,j}}^*$ in the case of $\widetilde{h_{i,j}} \leqslant \dfrac{\alpha}{a-c}$ is greater than that in the case of $\widetilde{h_{i,j}} \geqslant \dfrac{\alpha}{a-c}$.

The combination of Lemma 3 and Proposition 2 implies that although in the case of $\widetilde{h_{i,j}} \leqslant \dfrac{\alpha}{a-c}$, $\overline{e_{i,j}}^*$ is greater than that in the other case. However, it is inversely proportional to F_N. Therefore, in the condition of $\widetilde{h_{i,j}} \geqslant \dfrac{\alpha}{a-c}$, the supply chain is more conducive to exerting optimal emission reduction efforts to generate optimal emissions.

Next, we calculate the supply chain profit under this decision.

Proposition 3. Under the three carbon quota methods, $\pi_e(p^c)$ is always a decreasing function of $\overline{e_{i,j}}$ on $[0, \overline{e_{i,j}}^{\lim}]$.

Considering that firms are profit-oriented in making decisions, they always tend to make the most profitable decisions. Although we assume that supply chain leaders can make decisions for firms, practical risks still exist. Furthermore, Proposition 3 indicates that firms will try to make fewer emission reduction efforts decisions to achieve higher profits. Therefore, firms always have no incentives to invest more in emission reduction efforts.

Beyond that, we do not have a better way to better meet the needs of supply chain profitability in terms of carbon price regulation.

We suppose that $\pi_1{}^*$, $\pi_2{}^*$, and $\pi_3{}^*$ correspond to the optimal profit of the condition of $\widetilde{h_{i,j}} \geqslant \dfrac{\alpha}{a-c}$, $p^c \in \left[0, \dfrac{2(a-c)}{\alpha} - \dfrac{1}{\widetilde{h_{i,j}}} - \lambda\right]$; $\widetilde{h_{i,j}} \geqslant \dfrac{\alpha}{a-c}$, $p^c \in \left[\dfrac{2(a-c)}{\alpha} - \dfrac{1}{\widetilde{h_{i,j}}} - \lambda, \dfrac{1}{\widetilde{h_{i,j}}} - \lambda\right]$, and $\widetilde{h_{i,j}} \leqslant \dfrac{\alpha}{a-c}$, $p^c \in \left[0, \dfrac{a-c}{\alpha} - \lambda\right]$, respectively.

Proposition 4. Under the three carbon quota methods, $\pi_1{}^*$ and $\pi_3{}^*$ are always positively correlated with $\widetilde{h_{i,j}}$, but there is no connection between $\pi_2{}^*$ and $\widetilde{h_{i,j}}$.

This indicates that under the condition of $\widetilde{h_{i,j}} \leqslant \dfrac{\alpha}{a-c}$, as long as p^c is in the domain, the higher emission reduction efficiency $\widetilde{h_{i,j}}$ will lead to the higher optimal profit π^*. However, in the case of $\widetilde{h_{i,j}} \geqslant \dfrac{\alpha}{a-c}$, $p^c \in \left[\dfrac{2(a-c)}{\alpha} - \dfrac{1}{\widetilde{h_{i,j}}} - \lambda, \dfrac{1}{\widetilde{h_{i,j}}} - \lambda\right]$, π^* does not change with a change in $\widetilde{h_{i,j}}$. In other words, when both $\widetilde{h_{i,j}}$ and p^c are large enough, the change in $\widetilde{h_{i,j}}$ has no effect on π^*.

4 Revenue allocation model

4.1 The Shapley allocation

Cooperation inevitably involves benefit allocation, and only a reasonable allocation of benefits can achieve a long-term cooperation model. To distribute the benefits more fairly and reasonably in the process of multiperson cooperation, Shapley and Shubik[31] proposed the Shapley value of distributing the benefits according to each member's contribution degree to the alliance's overall goal. The benefit shared by a member is equal to the average of the marginal benefits created by the member for the alliance he/she participates in. This section builds a revenue distribution model based on the Shapley value.

Assuming that enterprise i participates in j process, product utility $u_{i,j}$ and carbon emissions $f_{i,j}$ are generated, here we define that $u_i = \sum_i u_{i,j}$; $u_j = \sum_j u_{i,j}$; f_i, f_j is similar. We assume that emissions f_j from all processes in the supply chain are measurable.

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We discuss this joint production issue under the carbon cap-and-trade policy. Assuming that the motivation of the supply chain leader is to achieve collaborative emission reduction to a great extent, it is committed to developing a revenue distribution scheme ϕ to allocate the total profit π to lay the foundation and provide incentives for carbon emission reduction. The assignment scheme is set as $\phi = \phi(B, f)$ and defined on firm set N, process set M and responsibility matrix B, the distributed income of firm i is $\phi(i)$, $\sum_{i=1}^{n}\phi(i) = \pi$.

The cooperative game in the supply chain is represented by a binary array $G = \langle N, v\rangle$ composed of the firm set N and characteristic function v in the supply chain, where the characteristic function $v(S)$ is a function of any coalition, $S \in 2^N$, v is the mapping that $v: 2^N \to R$, and refers to the overall utility of the coalition obtained by each member through cooperation. We generally only consider the case of complete information, where N and v are common knowledge. In the current green environment of energy conservation and emission reduction, the value of supply chain results lies in the actual output and the emission reduction situation. Accordingly, we establish the utility characteristic function as follows.

$$v(S) = \sum_{j\in M}(u_j - \rho f_j)b_{S,j}. \tag{17}$$

The characteristic function $v(\{i\})$ of a single member coalition is often abbreviated as $v(i)$.

$$v(i) = \sum_{j\in M}(u_j - \rho f_j)b_{i,j}, \tag{18}$$

where ρ is the constant coefficient.

The revenue that must be distributed to all firms in the supply chain is given by the profit π of joint production per unit of time, and the unit of time in this article is one year.

The Shapley value φ of income distribution model (N, v) conforms to the following characteristics:

1) Symmetry

The sequencing or labeling of stakeholders does not affect the results of revenue allocation. In other words, equal pay for equal work.

For any permutation τ of N, $\varphi(\tau(i))(\tau v) = \varphi(i)(v)$.

2) Efficiency

The total value of the stakeholder coalition is the sum of all Shapley values, namely the characteristic function value.

$$\sum_{i\in N}\varphi(i)(v) = v(N).$$

3) Law of aggregation

When there are many kinds of cooperation, the way of revenue allocation of each cooperation has nothing to do with other cooperation results, and the total allocation is the sum of two terms. For any game v and w on N, there is $\varphi(i)(v+w) = \varphi(i)(v) + \varphi(i)(w)$.

4) Null player

If a member does not contribute to any of the cooperative coalitions in which he/she participates, he/she should not benefit from collective cooperation. If $v(S\backslash\{i\}) = v(S)$ for all subsets S containing member i, $\varphi_{(i)}(v) = 0$, where $S\{i\}$ is set S after removing element i.

Thus, there exists a unique assignment $\varphi(v) = (\varphi_{(1)}(v), \varphi_{(2)}(v), \cdots, \varphi_{(n)}(v)) = (\varphi(1), \varphi(2), \cdots, \varphi(n))$ for a particular set of players N, $\varphi(i) \in R$ simultaneously satisfies the above four postulates and can be calculated by the following formula.

$$\varphi(i) = \sum_{\substack{S\subseteq N,\\ i\in S}} w[v(S) - v(S\backslash\{i\})], \tag{19}$$

$$w = (|S|-1)!(n-|S|)!\frac{1}{n!}, \tag{20}$$

where $|S|$ is the number of participants in union S, w is the coalition's weight coefficient, $v(S\backslash\{i\})$ denotes all the members of the league except i formed by the union of total utility. That is, a player's Shapley value is the weighted average of his/her marginal contribution in all possible coalitions.

4.2 Modified Shapley value

Although the Shapley value avoids egalitarianism and makes allocation more reasonable and fair, it ignores the heterogeneity of different players in forming a particular coalition, leading to players without individual characteristics. In collaborative supply chain practice, the positions of each enterprise in the system are different due to different investment levels, risk-sharing, and innovation results. At the same time, the Shapley value method essentially assumes that the listed indexes are equal for all alliance members. Thus, this paper revises the Shapley value model according to participating firms' capital investment, risk-taking level, and innovation research efforts. To make the correction factors closer to reality, we further differentiate the subordinate indexes as a comprehensive reference based on the above factors and construct a comprehensive correction algorithm based on the Shapley value method.

4.2.1 Cost of investment

We use C to represent the cost factor as the first-level indicator with a weight of λ. Capital input can be divided into:

a) Direct production cost, C_1

It points to the cost that relates directly to the product's production process, such as raw material, primary material, purchase of a semifinished product, wages of a production worker, and depreciation of machine equipment.

b) Information sharing cost, C_2

To achieve coordinated emission reduction, supply chain enterprises often need to share emission reduction information and even some noncore technologies with each other. This information is of value, which is collected or developed by the enterprise and should be converted into the investment cost of the enterprise.

c) Human resource cost, C_3

It consists of ① original cost of human resources, which is the cost of acquiring and developing human resources, and ② the replacement cost of human resources, which is the cost of replacing an employee.

d) Internal transaction costs between cooperative members, C_4

In the process of collaborative supply chain emission reduction, the flow of material resources often occurs between enterprises, and users need to pay part of the cost to the provider. Thus, it will form an internal transaction.

The cost of the above specific types is represented by C_1, C_2, C_3, C_4 as the secondary index, the total cost of enterprise i is expressed as C_i, and the cost of each secondary index is C_{i1}, C_{i2}, C_{i3}, C_{i4} in sequence. The weights are λ_1, λ_2, λ_3, λ_4 in that order, that is $C_{i,\mathrm{sum}} = \lambda_1 C_{i1} + \lambda_2 C_{i2} + \lambda_3 C_{i3} + \lambda_4 C_{i4}$. And $\overline{C_{i,\mathrm{sum}}}$ is the result of normalizing $C_{i,\mathrm{sum}}$ with respect to $\sum_{i\in N} C_{i,\mathrm{sum}}$.

In other words, under the cost correction factor, the revenue allocation obtained by firm i is

$$\varphi(i)_1 = \sum_{\substack{S\subseteq N,\\ i\in S}} w\left[1! + \sum_{i\in S}\left(\overline{C_{i,\mathrm{sum}}} - \frac{1}{n}\right)\right]v(S) - \left[1 + \sum_{S n\{i\}}\left(\overline{C_{i,\mathrm{sum}}} - \frac{1}{n}\right)\right]v(S\setminus\{i\}). \tag{21}$$

4.2.2 Degree of risk-bearing

We use R to represent the risk factor, whose weight is μ. Risk-taking levels can be divided into:

a) Market risk, R_1: It refers to the influence and change of a variety of factors, resulting in increased business risk to not achieving the expected emission reduction effect of the possibility. For example, ① changes in the policy environment. Carbon trading systems are proliferating around the world. However, to date, there has not been a unified global carbon trading market, and different countries and regional markets have different management rules for trading. The European Union's emissions trading scheme, although relatively advanced, is still in the stage of gradual improvement. Therefore, both in time and space, relevant policies are in a high degree of change. ② Competitive risk. In market competition, there are many uncertain factors. Although every competitor expects to achieve its expected profit target, they cannot all succeed.

b) Cooperation risk, R_2: This refers to the risks brought to the manufacturers in the core supply chain by the noncooperation or inability of the partners to cooperate.

c) Technical risk, R_3: It is mainly derived from two aspects of hardware and software. Moreover, the main types include technology deficiency risk, development risk, technology protection risk, technology use risk, and technology acquisition and transfer risk.

The weights are μ_1, μ_2, and μ_3 in order, that is, $R_{i,\mathrm{sum}} = \mu_1 R_{i1} + \mu_2 R_{i2} + \mu_3 R_{i3}$. And $\overline{R_{i,\mathrm{sum}}}$ is the result of normalizing $R_{i,\mathrm{sum}}$ with respect to $\sum_{i\in N} R_{i,\mathrm{sum}}$.

Then we can obtain

$$\varphi(i)_2 = \sum_{\substack{S\subseteq N,\\ i\in S}} w\left[1 + \sum_{i\in S}\left(\overline{C_{i,\mathrm{sum}}} - \frac{1}{n}\right)\right]\left[1 + \sum_{i\in S}\left(\overline{R_{i,\mathrm{sum}}} - \frac{1}{n}\right)\right]v(S) - \left[1 + \sum_{S n\{i\}}\left(\overline{C_{i,\mathrm{sum}}} - \frac{1}{n}\right)\right]\left[1 + \sum_{S n\{i\}}\left(\overline{R_{i,\mathrm{sum}}} - \frac{1}{n}\right)\right]v(S\setminus\{i\}). \tag{22}$$

4.2.3 Innovation contribution

The innovation factor is denoted by I with a weight of γ. Innovative research efforts can be divided into:

a) Innovation contribution, I_1: System innovation is the premise of innovation of other kinds. A perfect enterprise system innovation mechanism can ensure the effective implementation of technological and management innovation.

b) Technical innovation, I_2: Design products and packaging using cleaner materials and product technologies. Changing environmental trends and regulations complicate the design of green products, minimizing their impact on the environment during their usable life cycle and at the end of life.

c) Cultural innovation, I_3: Technological innovation encompasses the development and application of new or existing technology, drawing from scientific knowledge and resources. It is a crucial source of competitive advantage for businesses and a key factor in sustainable development. Science forms the foundation of technology, which in turn drives industry. Technological innovation stems from the discovery of scientific principles, and industrial innovation is primarily built upon technological advances. A thorough understanding of the essence, characteristics, and principles of technological innovation is essential for effective management in this area.

d) Business innovation, I_4: To expand the market and pursue profit maximization, business development has become the goal of many managers. However, it is challenging to predict the effects of business development direction and strategy, so significant risks exist.

The above specific types of innovation of firm i are represented by I_{i1}, I_{i2}, I_{i3}, I_{i4}. Their weights are γ_1, γ_2, γ_3, γ_4 in order, that is $I_{i,\mathrm{sum}} = I_{i1} + I_{i2} + I_{i3} + I_{i4}$. And $\overline{I_{i,\mathrm{sum}}}$ is the result of normalizing $I_{i,\mathrm{sum}}$ with respect to $\sum_{i\in N} I_{i,\mathrm{sum}}$. Here, all secondary indicators are normalized, i.e., $\sum_{l=1}^{4}\lambda_l + \sum_{l=1}^{3}\mu_l + \sum_{l=1}^{4}\gamma_l = 1$.

Then we can obtain

$$\varphi(i)_3 = \sum_{\substack{S\subseteq N,\\ i\in S}} w\left[1 + \sum_{i\in S}\left(\overline{C_{i,\mathrm{sum}}} - \frac{1}{n}\right)\right]\cdot \left[1 + \sum_{i\in S}\left(\overline{R_{i,\mathrm{sum}}} - \frac{1}{n}\right)\right]\left[1 + \sum_{i\in S}\left(\overline{I_{i,\mathrm{sum}}} - \frac{1}{n}\right)\right]v(S) - \left[1 + \sum_{S n\{i\}}\left(\overline{C_{i,\mathrm{sum}}} - \frac{1}{n}\right)\right]\left[1 + \sum_{S n\{i\}}\left(\overline{R_{i,\mathrm{sum}}} - \frac{1}{n}\right)\right]\cdot \left[1 + \sum_{S n\{i\}}\left(\overline{I_{i,\mathrm{sum}}} - \frac{1}{n}\right)\right]v(S\setminus\{i\}). \tag{23}$$

5 Stability of the allocation

Considering the difference between firms, we establish a modified revenue allocation model based on the Shapley value, making the distribution highly fair. However, the firms in the coalition may be attracted by external economic interests and deviate from the system, which means that the cooperation under the allocation may not be stable. In the following section, we combine the core guarantee mechanism in a cooperative game to achieve stability and fairness through the "fairest core".

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5.1 Definitions and properties

The solution of a cooperative game is the distribution of the benefits obtained by all the partners. Much research on cooperative game solutions is divided into the following three types: core, least core/nucleolus, and Shapley value.

5.1.1 Core

The core is the "no one will complain" set of allocation schemes, that is, the set of all allocation schemes that can stabilize overall cooperation. We define the core as in an n-person cooperative game (N,v); all non-superior distribution sets become the core, denoted as $C(v)$. In this case, no principal coalition can improve the utility of its members by reallocating the initial resources of its members among them, and all cooperative enterprises in the collection reach a stable equilibrium state. In other words, any firm will not deviate from the core due to the change of strategy because the deviation of any party will cause the firm to suffer losses, expressed as $x_i \geqslant v(i)$, $\forall i \in N$. As a result, the parties reached binding agreements to form a stable state of cooperation when the core elements are used as allocation schemes.

Theorem 1. Let $x = (x_1, x_2, \cdots, x_n)$, x_i is the payoff distributed to player i, x is assigned to core $C(v)$ if and only if

$$\sum_{i=1}^{n} x_i = v(N), \tag{24}$$

$$\sum_{i \in S} x_i \geqslant v(S), \forall S \subseteq N. \tag{25}$$

For each solution x, we assume that $e(S,x) = v(S)x(S)$ is the excess allocation of set S, that is, the difference between the value of coalition S and the value allocated by the coalition, which can be understood as the dissatisfaction of the players in coalition S with the proposed solution x. The core of the game is the set of solution x for $\forall S \subset N$, making $e(S,x) \geqslant 0$.

However, the core solution requirement is too strong, and a cooperative game does not always have a core, which can guarantee stability but cannot guarantee existence and fairness, and is not unique.

5.1.2 Nucleolus

In some games, the core may not exist. At this point, we define nucleolus as a similar solution. Its essence is a collection of allocation methods to minimize the maximum dissatisfaction of coalitions in cooperative games. The ε-core of the game is the set of solutions x for $\forall S \subset N$, making $e(S,x) \leqslant \varepsilon$, $\varepsilon \geqslant 0$.The core of the game is the nonempty ε-core at the lowest ε.

Like the core, it is stable, unfair, and not unique because it is a set, but it is always guaranteed to exist.

5.1.3 Shapley value

In contrast, the Shapley value has existence, uniqueness, and fairness but no stability.

Therefore, when looking for a unique allocation scheme for cooperative games, we ideally want the core to exist and the Shapley value to be in the core. According to Ref. [25] we introduce the concepts of the "fairest core" and "fairest nucleolus" in the following subsection.

5.2 Optimization model

Based on the previous Shapley values, we added finite constraints to the original distribution equation based on the "fairest core"[26]. Given that φ_i^* is the comprehensive modified Shapley value, $x = (x_1, x_2, \cdots, x_n)$ is the decision variable of allocation.

They are combined with the three necessary conditions in Section 5.1, the "fairest core" is the core closest to the Shapley value according to the only stable solution with fairness. We constructed the constrained optimization model as follows.

$$\begin{aligned} &\min \|x - \varphi_i^*\| \\ &\text{s.t.} \quad \sum_{i=1}^{n} x_i = v(N), \\ &\sum_{i \in S} x_i \geqslant v(S), \forall S \subseteq N, \\ &x_i \in R, i \in N. \end{aligned} \tag{26}$$

There may be three situations when solving the above model.

(i) The model has a feasible solution, and the objective function value is 0. The income distribution scheme is in the "core", equal to the modified Shapley value.

(ii) The model has feasible solutions, but the objective function value is not 0, and the income distribution scheme is in the "core" but not equal to the modified Shapley value.

(iii) The model has no running solution. In this case, the "core" of the cooperative game is an empty set, and there is no stable income distribution scheme.

In cases (i) and (ii), the "fairest core" is obtained, which is our ideal solution. For case (iii), we resolve it by extending it to the concept of "fairest nucleolus" in the following sections.

$$\begin{aligned} &\min \|x - \varphi_i^*\| \\ &\text{s.t.} \quad \sum_{i=1}^{n} x_i = v(N), \\ &\sum_{i \in S} x_i \geqslant v(S) - \varepsilon^*, \forall S \subseteq N, \\ &x_i \in R, i \in N. \end{aligned} \tag{27}$$

Among them, $\varepsilon^* \geqslant 0$ is the worst excess level of nucleolar solutions. It is always possible to find an approximation to ε^* so that the revenue allocation scheme for cooperative emission reduction is located closest to the core.

The constraints help us obtain the set of nucleoli or even cores, and the objective function makes it the closest solution to the Shapley value in the set. Based on the above, we obtain the solution to allocate revenue to both meet fairness and stability to the maximum extent.

6 Numerical example

To illustrate the applicability of the green cooperation model, we first present a simple example to instance the optimal market response and profit of the firms. Then, we continue this example to verify the rationality of the revenue allocation model.

6.1 Optimal model

According to the daily transaction data of the national carbon market released by the Shanghai Environmental Energy

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Exchange, the closing price of the carbon emission allowance (CEA) listing agreement in the national carbon market on 2022-07-25 was 57.30 CNY/t[32]. We consider the supply chain operating under a uniform carbon price of approximately 57 CNY/t. Suppose the total carbon quota of the cooperative supply chain q_N is 16 t per year, the expected emission per unit of product in the absence of carbon emission reduction efforts α is 50, the base market size a is 10000, the emission impact factor λ is 10. The unit base production cost is 10. In addition, the comprehensive carbon emission reduction cost factor $\widetilde{k_{i,j}}$ and the impact factor of comprehensive emission reduction efforts on emissions per unit of product $\widetilde{\alpha_{i,j}}$ are 500 and 5 respectively. Based on the above, we get $\frac{\widetilde{k_{i,j}}}{\widetilde{\alpha_{i,j}}}-\lambda=90$, $\frac{a-c}{\alpha}-\lambda=189.8$, $p^c\in[0,90]$ which satisfy Assumption 2 and the real data of p^c. And then, we obtain $p_e^*(p^c)=\frac{12360+265e}{2}$; $Q_e^*(p^c)=\frac{13340-835e}{2}$. According to Lemma 3, $\widetilde{h_{i,j}}=\frac{1}{100}\geqslant\frac{\alpha}{a-c}=\frac{5}{999}$, $p^c\in[0,\frac{1448}{5}]$, they satisfy Lemma 3(i). So that $\overline{e_{i,j1}^*}=\frac{\alpha}{2\widetilde{a_{i,j}}}=5$, $E(f_N)=25$, $p_e^*(p^c)=6842.5$, $Q_e^*(p^c)=9165$. Finally, the optimal revenue of the whole cooperative supply chain is $\pi^*\approx 3.97\times10^7$ CNY.

6.2 Allocation model

Suppose that there are three firms 1, 2, 3 in the supply chain. If these three firms choose to produce and reduce emissions, the revenue that can be best achieved is 5×10^6, 8×10^6, and 1×10^7 per year, respectively. Moreover, they will be much more profitable if they choose to cooperate. For example, if firms 1 and 2 choose to cooperate to reduce emissions, their total revenue can reach 2×10^7. The cooperation of emission reduction of firms1 and 3 can increase the total revenue to 2.5×10^7. The revenue of the cooperation of firms 2 and 3 can reach 3×10^7. According to Subsection 6.1, if all three firms choose to cooperate together, the revenue will be 3.97×10^7. Considering the factors of contribution, the level of investment cost, the condition of risk-bearing, and the achievements of innovation, we adopt the model developed in the above section to allocate the revenue of cooperative emission reduction.

6.2.1 Allocation model based on the Shapley value

The supply chain consisting of firms 1, 2 and 3 is denoted by $N=1,2,3$. The revenues obtained by the three firms without cooperation are represented as $\phi(1)=5\times10^6$, $\phi(2)=8\times10^6$, $\phi(3)=1\times10^7$. The set of all collaboration sets in which firm 1 participates is $S_1=1,1\cup2,1\cup3,1\cup2\cup3$, and the sets of firm 2 and firm 3 can be drawn by analogy. The revenue of all the cooperative sets are recorded as $\phi(1\cup2)=2\times10^7$, $\phi(1\cup3)=2.5\times10^7$, $\phi(2\cup3)=3\times10^7$, $\phi(1\cup2\cup3)=3.97\times10^7$. According to Eqs. (19) and (20), the Shapley value of the revenue of each firm can be calculated as $\varphi_1(v)$, $\varphi_2(v)$, $\varphi_3(v)$. The specific calculation process is shown in Tables 1, 2, and 3.

Then, we can obtain the Shapley value of each firm:

$$\varphi(1)=\frac{8\times10^6}{3}+\frac{7.5\times10^6}{3}+\frac{1\times10^7}{3}+\frac{1.47\times10^7}{3}=0.94\times10^7;$$

$$\varphi(2)=\frac{8\times10^6}{3}+\frac{7.5\times10^6}{3}+\frac{1\times10^7}{3}+\frac{1.47\times10^7}{3}=1.34\times10^7;$$

$$\varphi(3)=\frac{1\times10^7}{3}+\frac{1\times10^7}{3}+\frac{1.1\times10^7}{3}+\frac{1.97\times10^7}{3}=1.69\times10^7.$$

However, the Shapley value ignores the factors of contribution, the level of investment cost, the condition of risk-bearing and the achievements of innovation, thus, it needs to be further revised.

6.2.2 Allocation model based on the modified Shapley value

Suppose $C_{1,\text{sum}}=2.75\times10^3$, $C_{2,\text{sum}}=3.55\times10^3$, $C_{3,\text{sum}}=4.5\times10^3$, so that $\overline{C_{1,\text{sum}}}\approx0.2546$, $\overline{C_{2,\text{sum}}}\approx0.3287$, $\overline{C_{3,\text{sum}}}\approx0.4167$. In addition, $\overline{R_{1,\text{sum}}}=0.3$, $\overline{R_{2,\text{sum}}}=0.425$, $\overline{R_{3,\text{sum}}}=0.275$, $\overline{I_{1,\text{sum}}}=0.425$, $\overline{I_{2,\text{sum}}}=0.1875$, $\overline{I_{3,\text{sum}}}=0.3875$. The specific calculation process is shown in Tables 4, 5 and 6.

Based on Table 4, we obtain $\varphi(1)_3=1.62033\times10^6+1.92489\times10^6+2.45332\times10^6+3.10825\times10^6\approx9.10679\times10^6$.

Based on Table 5, we obtain $\varphi(2)_3=2.47507\times10^6+2.24835\times10^6+3.27015\times10^6+4.51985\times10^6\approx1.25134\times10^7$.

Based on Table 6, we obtain $\varphi(3)_3=3.58478\times10^6+3.54658\times10^6+3.82501\times10^6+7.11631\times10^6\approx1.80727\times10^7$.

6.2.3 Stability analysis of the allocation

To test whether the final allocation scheme based on the modified Shapley value is stable and can be implemented effectively over a longer period of time, this section introduces the concept of “core” and the “fairest core” in cooperative games for further analysis. If it is unstable, we optimize the model to determine the values that best satisfy both fairness and stability.

Based on Eq. (24), the sum of all firms’ revenue allocated

Table 1. The Shapley value calculation table of firm 1.

	S_1			
	1	$1\cup2$	$1\cup3$	N
$\phi(S_1)$	5×10^6	2×10^7	2.5×10^7	3.97×10^7
$\phi(S_1-1)$	0	8×10^6	1×10^7	3×10^7
$\phi(S_1-\phi(S_1-1))$	5×10^6	1.2×10^7	1.5×10^7	9.7×10^6
$\lvert S_1\rvert$	1	2	2	3
ω	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
$\omega[\phi(S_1)-\phi(S_1-1)]$	$\frac{5\times10^6}{3}$	$\frac{6\times10^6}{3}$	$\frac{7.5\times10^6}{3}$	$\frac{9.7\times10^6}{3}$

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Table 2. The Shapley value calculation table of firm 2.

	S_2			
	2	$1\cup 2$	$2\cup 3$	N
$\phi(S_2)$	8×10^6	2×10^7	3×10^7	3.97×10^7
$\phi(S_2-2)$	0	5×10^6	1×10^7	2.5×10^7
$\phi(S_2-\phi(S_2-2))$	8×10^6	1.5×10^7	2×10^7	1.47×10^7
$\lvert S_2\rvert$	1	2	2	3
ω	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
$\omega[\phi(S_2)-\phi(S_2-2)]$	$\frac{8\times 10^6}{3}$	$\frac{7.5\times 10^6}{3}$	$\frac{1\times 10^7}{3}$	$\frac{1.47\times 10^7}{3}$

Table 3. The Shapley value calculation table of firm 3.

	S_3			
	3	$1\cup 3$	$2\cup 3$	N
$\phi(S_3)$	1×10^7	2.5×10^7	3×10^7	3.97×10^7
$\phi(S_3-3)$	0	5×10^6	8×10^6	2×10^7
$\phi(S_3-\phi(S_3-3))$	1×10^7	2×10^7	2.2×10^7	1.97×10^7
$\lvert S_3\rvert$	1	2	2	3
ω	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
$\omega[\phi(S_3)-\phi(S_3-3)]$	$\frac{1\times 10^7}{3}$	$\frac{1\times 10^7}{3}$	$\frac{1.1\times 10^7}{3}$	$\frac{1.97\times 10^7}{3}$

Table 4. The modified Shapley value calculation table of firm 1.

	S_1			
	1	$1\cup 2$	$1\cup 3$	N
$\phi(S_1)$	5×10^6	2×10^7	2.5×10^7	3.97×10^7
$\left[1+\sum_{i\in S_1}\left(\overline{C_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_1}\left(\overline{R_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_1}\left(\overline{I_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]$	0.972196	0.917553	1.04562	1
$\phi(S_1-1)$	0	8×10^6	1×10^7	3×10^7
$\left[1+\sum_{i\in S_1-1}\left(\overline{C_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_1-1}\left(\overline{R_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_1-1}\left(\overline{I_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]$	0	0.850212	1.14205	1.01251
$\left[1+\sum_{i\in S_1}\left(\overline{C_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum_{i\in S_1}\left(\overline{R_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_1}\left(\overline{I_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\phi(S_1)-\left[1+\sum_{i\in S_1-1}\left(\overline{C_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_1-1}\left(\overline{R_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum_{i\in S_1-1}\left(\overline{I_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\phi(S_1-1)$	4.86098×10^6	1.15494×10^7	1.47199×10^7	9.32476×10^6
$\lvert S_1\rvert$	1	2	2	3
ω	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
$\omega\left[1+\sum_{i\in S_1}\left(\overline{C_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum_{i\in S_1}\left(\overline{R_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_1}\left(\overline{I_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\phi(S_1)\right]-\left[1+\sum_{i\in S_1-1}\left(\overline{C_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_1-1}\left(\overline{R_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum_{i\in S_1-1}\left(\overline{I_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\phi(S_1-1)$	1.62033×10^6	1.92489×10^6	2.45332×10^6	3.10825×10^6

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Table 5. The modified Shapley value calculation table of firm 2.

	S_2			
	2	$1\cup 2$	$2\cup 3$	N
$\phi(S_2)$	8×10^6	2×10^7	3×10^7	3.97×10^7
$\left[1+\sum\limits_{i\in S_2}\left(\overline{C_{i,\text{ sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum\limits_{i\in S_2}\left(\overline{R_{i,\text{ sum}}}-\frac{1}{n}\right)\right]\left[1+\sum\limits_{i\in S_2}\left(\overline{I_{i,\text{ sum}}}-\frac{1}{n}\right)\right]$	0.928148	0.917554	1.01251	1
$\phi(S_2-2)$	0	5×10^6	1×10^7	2.5×10^7
$\left[1+\sum\limits_{i\in S_2-2}\left(\overline{C_{i,\text{ sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum\limits_{i\in S_2-2}\left(\overline{R_{i,\text{ sum}}}-\frac{1}{n}\right)\right]\left[1+\sum\limits_{i\in S_2-2}\left(\overline{I_{i,\text{ sum}}}-\frac{1}{n}\right)\right]$	0	0.972196	1.07543	1.04562
$\left[1+\sum\limits_{i\in S_2}\left(\overline{C_{i,\text{sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum\limits_{i\in S_2}\left(\overline{R_{i,\text{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum\limits_{i\in S_2}\left(\overline{I_{i,\text{sum}}}-\frac{1}{n}\right)\right]\phi(S_2)-\left[1+\sum\limits_{i\in S_2-2}\left(\overline{C_{i,\text{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum\limits_{i\in S_2-2}\left(\overline{R_{i,\text{sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum\limits_{i\in S_2-2}\left(\overline{I_{i,\text{sum}}}-\frac{1}{n}\right)\right]\phi(S_2-2)$	7.42520×10^6	1.34901×10^7	1.96209×10^7	1.35596×10^7
$\lvert S_2\rvert$	1	2	2	3
ω	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
$\omega\left[1+\sum\limits_{i\in S_2}\left(\overline{C_{i,\text{ sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum\limits_{i\in S_2}\left(\overline{R_{i,\text{ sum}}}-\frac{1}{n}\right)\right]\left[1+\sum\limits_{i\in S_2}\left(\overline{I_{i,\text{ sum}}}-\frac{1}{n}\right)\phi(S_2)\right]-\left[1+\sum\limits_{i\in S_2-2}\left(\overline{C_{i,\text{ sum}}}-\frac{1}{n}\right)\right]\left(1+\sum\limits_{i\in S_2-2}\left(\overline{R_{i,\text{ sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum\limits_{i\in S_2-2}\left(\overline{I_{i,\text{ sum}}}-\frac{1}{n}\right)\right]\phi(S_2-2)$	2.47507×10^6	2.24835×10^6	3.27015×10^6	4.51985×10^6

is equal to the overall revenue of the whole supply chain, which represents $\varphi(1)_3+\varphi(2)_3+\varphi(3)_3=3.969289\times 10^7\approx\pi=3.97\times 10^7$.

Based on Eq. (25), the current cooperative portfolio is the best of all the collaborative solutions. Here, $\varphi(1)_3=9.10679\times 10^6\geqslant\phi_1=5\times 10^6$; $\varphi(2)_3=1.25134\times 10^7\geqslant\phi_2=8\times 10^6$; $\varphi(3)_3=1.80727\times 10^7\geqslant\phi_3=1\times 10^7$; $\varphi(1)_3+\varphi(2)_3=2.162019\times 10^7\geqslant\phi(1\cup 2)$; $\varphi(1)_3+\varphi(3)_3=2.717949\times 10^7\geqslant\phi(1\cup 3)$; $\varphi(2)_3+\varphi(3)_3=3.50861\times 10^7\geqslant\phi(2\cup 3)$; $\varphi(1)_3+\varphi(3)_3=2.717949\times 10^7\geqslant\phi(1\cup 3)$; $\sum\limits_{i\in N}\varphi(i)_3=3.969289\times 10^7\geqslant\phi(1\cup 2\cup 3)$.

In summary, the modified Shaley value satisfies Theorem 1. That is, it is located in the "core" of the cooperative game. Here, the allocation value is fair and reasonable.

7 Conclusions

This paper establishes a game model with n firms as the main body, and explores the influence of the external environment of carbon emission reduction of cooperative supply chains on their decision-making under the cap-and-trade policy. First, we obtain the optimal price and the optimal yield of the product to harvest the highest possible profit. Second, we obtain the supply chain leader's decision on the most beneficial/practical carbon reduction efforts. In addition, we obtained some insights as follows.

(Ⅰ) Supply chain responses to carbon prices may not be monotonous, and higher carbon prices may lead to smaller rather than more significant emission-reduction efforts.

(Ⅱ) It is not that more extraordinary emission-reduction efforts always lead to less supply chain emissions. This is mainly because more extraordinary emission-reduction efforts tend to lead to more production, which is when the reduction in emissions per unit of product is not sufficient to offset the increase in production. We need to avoid both emission reduction efficiency and carbon prices being too small to avoid this situation. It can be considered that when the external environment pays too little attention to carbon emission reduction, the impact of efforts on product emissions is too

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Table 6. The modified Shapley value calculation table of firm 3.

	S_3			
	3	$1\cup 3$	$2\cup 3$	N
$\phi(S_3)$	1×10^7	2.5×10^7	3×10^7	3.97×10^7
$\left[1+\sum_{i\in S_3}\left(\overline{C_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum_{i\in S_3}\left(\overline{R_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_3}\left(\overline{I_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]$	1.07543	1.04562	1.01251	1
$\phi(S_3-3)$	0	5×10^6	8×10^6	2×10^7
$\left[1+\sum_{i\in S_3-3}\left(\overline{C_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum_{i\in S_3-3}\left(\overline{R_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_3-3}\left(\overline{I_{i,\,\mathrm{sum}}}-\frac{1}{n}\right)\right]$	0	0.972196	0.928148	0.917553
$\left[1+\sum_{i\in S_3}\left(\overline{C_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum_{i\in S_3}\left(\overline{R_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_3}\left(\overline{I_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\phi(S_3)-\left[1+\sum_{i\in S_3-3}\left(\overline{C_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_3-3}\left(\overline{R_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum_{i\in S_3-3}\left(\overline{I_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\phi(S_3-3)$	1.07543×10^7	2.12795×10^7	2.29501×10^7	2.13489×10^7
$\lvert S_3\rvert$	1	2	2	3
ω	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
$\omega\left[1+\sum_{i\in S_3}\left(\overline{C_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum_{i\in S_3}\left(\overline{R_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_3}\left(\overline{I_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\phi(S_3)-\left[1+\sum_{i\in S_3-3}\left(\overline{C_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\left[1+\sum_{i\in S_3-3}\left[\overline{R_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\cdot\left[1+\sum_{i\in S_3-3}\left(\overline{I_{i,\mathrm{sum}}}-\frac{1}{n}\right)\right]\phi(S_3-3)$	3.58478×10^6	3.54658×10^6	3.82501×10^6	7.11631×10^6

small, or the unit carbon emission reduction cost of enterprises is too high. If the external carbon price is too low, firms' more extraordinary emission reduction efforts will lead to higher emissions.

Therefore, we can confirm that creating a good environment helps to maximize the effectiveness of emissions efforts. To improve the effectiveness of national carbon markets, we need to develop technologies or take measures such as government subsidies to increase the change in emissions caused per unit of effort.

To consider the benefits of the supply chain, under the historical and benchmarking rule, we are supposed to keep the carbon price in its largest range. However, under the auctioning rule, the carbon price should be kept as small as possible without damaging the emission reduction effect.

In addition, we also analyze revenue allocation in the green supply chain. To optimize the profit distribution of emission reduction cooperation between enterprises, we adopt the Shapley value analysis method, which considers cost investment, risk undertaking, and innovation input to enrich the traditional contribution-only configuration model. Moreover, through the concept of "the fairest core", we guarantee that the final solution meets fairness and stability to the maximum extent.

We can extend our study to a more detailed environment in future research. Moreover, as regulators, governments should maintain an environment that is most conducive to carbon reduction. The extension can also include comparisons of the environments required by more complex players, such as firms under different or mixed carbon quota rules and more heterogeneous firms in perfectly competitive or oligopolistic markets. It may even be possible to analyze incentives for supply chain leaders to lead enterprises to reduce carbon emissions in the case of reputational risk.

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Conflict of interest

The authors declare that they have no conflict of interest.

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Appendix

A.1 Proof of Lemma 1

$$\frac{\partial\pi}{\partial p}=a-p-\lambda(\alpha-\sum_{i,j}\alpha_{i,j}e_{i,j}-p+c+\sum_{i,j}k_{i,j}e_{i,j}+p^c(\alpha-\sum_{i,j}\alpha_{i,j}e_{i,j})=a-2p+c+\sum_{i,j}k_{i,j}e_{i,j}+(p^c-\lambda)(\alpha-\sum_{i,j}\alpha_{i,j}e_{i,j}),$$

$$\frac{\partial^2\pi}{\partial p^2}=-2<0.$$

Let $\frac{\partial\pi}{\partial p}=0$, we can get $p_e^*(p^c)=\frac{a+c+\widetilde{k_{i,j}}\cdot\overline{e_{i,j}}+(p^c-\lambda)(\alpha-\widetilde{\alpha_{i,j}}\cdot\overline{e_{i,j}})}{2}$.

Plugging the above into Eq. (11), we can get $Q_e^*(p^c)=\frac{a-c-\widetilde{k_{i,j}}\cdot\overline{e_{i,j}}-(p^c+\lambda)(\alpha-\widetilde{\alpha_{i,j}}\cdot\overline{e_{i,j}})}{2}$.

A.2 Proof of Lemma 2

(i) Under the historical and benchmarking rules, the associated optimal profit is

$$\pi_e(p^c)=E\left[p^*Q^*-(c+\sum_{i,j}\frac{k_{i,j}e_{i,j}^2}{2})Q^*-p^c(Q^*f_N-\sum_i\beta_iQ^c)\right]=\frac{\left[a-c-\sum_{i,j}\frac{k_{i,j}e_{i,j}^2}{2}-(p^c+\lambda)(\alpha-\sum_{i,j}\alpha_{i,j}e_{i,j})\right]^2}{4}+p^c\sum_i\beta_iQ^c,$$

$$\frac{\partial\pi}{\partial p^c}=\frac{a-c-\sum_{i,j}\frac{k_{i,j}e_{i,j}^2}{2}-(p^c+\lambda)(\alpha-\sum_{i,j}\alpha_{i,j}e_{i,j})}{2}\left[-\frac{\alpha-\sum_{i,j}\alpha_{i,j}e_{i,j}}{2}\right]+\sum_i\beta_iQ^c,\quad \frac{\partial^2\pi}{\partial p^{c2}}=\frac{(\alpha-\sum_{i,j}\alpha_{i,j}e_{i,j})^2}{4}.$$

Obviously, $\frac{\partial^2\pi}{\partial p^{c2}}\geqslant 0$, thus, $\frac{\partial\pi}{\partial p^c}$ is an increasing function. Besides, when $p^c=0$, $\frac{\partial\pi}{\partial p^c}<0$, but on the condition of $p^c=\min\left\{\frac{\widetilde{k_{i,j}}}{\widetilde{\alpha_{i,j}}}-\lambda,\frac{a-c}{\alpha}-\lambda\right\}$, $\frac{\partial\pi}{\partial p^c}\geqslant 0$.

Therefore, $\pi_e(p^c)$ is a convex function of p^c.

(ii) Under the auditioning rule, the associated optimal profit is

$$\pi_e(p^c)=E\left[p^*Q^*-(c+\sum_{i,j}\frac{k_{i,j}e_{i,j}^2}{2})Q^*-p^c(Q^*f_N)\right]=\frac{1}{4}\left[a-c-\sum_{i,j}\frac{k_{i,j}e_{i,j}^2}{2}-(p^c+\lambda)(\alpha-\sum_{i,j}\alpha_{i,j}e_{i,j})\right]^2,$$

$$\frac{\partial\pi}{\partial p^c}=\frac{a-c-\sum_{i,j}\frac{k_{i,j}e_{i,j}^2}{2}-(p^c+\lambda)(\alpha-\sum_{i,j}\alpha_{i,j}e_{i,j})}{2}\left[-\frac{\alpha-\sum_{i,j}\alpha_{i,j}e_{i,j}}{2}\right],\quad \frac{\partial^2\pi}{\partial p^{c2}}=\frac{(\alpha-\sum_{i,j}\alpha_{i,j}e_{i,j})^2}{4}.$$

Obviously, $\frac{\partial^2\pi}{\partial p^{c2}}\geqslant 0$, thus, $\frac{\partial\pi}{\partial p^c}$ is an increasing function. Besides, $\frac{\partial\pi}{\partial p^c}\leqslant 0$. Furthermore, when $p^c=0$, $\frac{\partial\pi}{\partial p^c}<0$, $\pi_e(p^c)<0$, but on the condition of $p^c=\min\left\{\frac{\widetilde{k_{i,j}}}{\widetilde{\alpha_{i,j}}}-\lambda,\frac{a-c}{\alpha}-\lambda\right\}$, $\frac{\partial\pi}{\partial p^c}=0$, $\pi_e(p^c)=0$.

Therefore, $\pi_e(p^c)$ is a decreasing function of p^c.

A.3 Proof of Lemma 3

$$F_N=\frac{1}{2}\left[a-c-\widetilde{k_{i,j}}\overline{e_{i,j}}-(p^c+\lambda)(\alpha-\widetilde{\alpha_{i,j}}\widetilde{e_{i,j}})\right](\alpha-\widetilde{\alpha_{i,j}}\widetilde{e_{i,j}}),$$

$$\frac{\partial F_N}{\partial\widetilde{e_{i,j}}}=\frac{1}{2}\left[-\widetilde{k_{i,j}}\alpha+2\widetilde{k_{i,j}}\widetilde{\alpha_{i,j}}\widetilde{e_{i,j}}+2(p^c+\lambda)\alpha\widetilde{\alpha_{i,j}}-2(p^c+\lambda)\widetilde{\alpha_{i,j}}^2\widetilde{e_{i,j}}-(a-c)\widetilde{\alpha_{i,j}}\right],$$

$$\frac{\partial^2F_N}{\partial\widetilde{e_{i,j}}^2}=\left[\widetilde{k_{i,j}}-(p^c+\lambda)\widetilde{\alpha_{i,j}}\right]\widetilde{\alpha_{i,j}}.$$

(i) If $\overline{e_{i,j}}\leqslant\frac{\alpha}{2\widetilde{\alpha_{i,j}}}$ that $\frac{\partial F_N}{\partial\widetilde{e_{i,j}}}\leqslant 0$. Therefore, $F_N(p^c)$ is a decreasing function of $\overline{e_{i,j}}$. Thus, when $p^c\in\left[0,\frac{2(a-c)}{\alpha}-\frac{1}{\widetilde{h_{i,j}}}-\lambda\right]$, $\overline{e_{i,j}}_1^*=\frac{\alpha}{2\widetilde{a_{i,j}}}$. And it is easy to get that $p^*_{e^*(p^c)}(p^c)_1=\frac{a+c+\frac{\alpha}{2}(\frac{1}{\widetilde{h_{i,j}}}+p^c-\lambda)}{2}$.

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(ii) If $\overline{e_{i,j}} \leqslant \dfrac{\alpha}{2\widetilde{\alpha_{i,j}}}$ that $\dfrac{\partial F_N}{\partial \overline{e_{i,j}}} \leqslant 0$. Therefore, $F_N(p^c)$ is a decreasing function of $\overline{e_{i,j}}$. Thus, when $p^c \in \left[\dfrac{2(a-c)}{\alpha} - \dfrac{1}{\widetilde{h_{i,j}}} - \lambda, \dfrac{1}{\widetilde{h_{i,j}}} - \lambda\right]$, $\overline{e_{i,j_2}}^* = \overline{e_{i,j}}^{\lim}$. And it is easy to get that $p^*_{e^*(p^c)}(p^c)_2 = \dfrac{a+c+\widetilde{k_{i,j}}\overline{e_{i,j}}^{\lim} + (p^c-\lambda)(\alpha - \widetilde{a_{i,j}}\overline{e_{i,j}}^{\lim})}{2}$.

(iii) If $\dfrac{\alpha}{2\widetilde{\alpha_{i,j}}} \leqslant \overline{e_{i,j}} \leqslant \dfrac{\alpha}{\widetilde{\alpha_{i,j}}}$ that $\dfrac{\partial^2 F_N}{\partial \overline{e_{i,j}}^2} \geqslant 0$. Therefore, $F_N(p^c)$ is a convex function of $\overline{e_{i,j}}$.

Let $\dfrac{\partial F_N}{\partial}\overline{e_{i,j}} = 0$, we can get $\overline{e_{i,j_3}}^* = \overline{e_{i,j_1}}^* + \dfrac{1}{2}\overline{e_{i,j_2}}^*$. And it is easy to get that $p^*_{e^*(p^c)}(p^c)_3 = \dfrac{1}{2}p^*(p^c)_2 + \dfrac{a+c+\dfrac{\alpha}{2}\dfrac{1}{\widetilde{h_{i,j}}}}{2}$.

A.4 Proof of Lemma 4

a) Under the historical and benchmarking rules

Referring to $\pi_e(p^c) = \dfrac{\left[a-c-\sum\limits_{i,j}\dfrac{k_{i,j}e_{i,j}^2}{2} - (p^c+\lambda)(\alpha - \sum\limits_{i,j}\alpha_{i,j}e_{i,j})\right]^2}{4} + p^c\sum\limits_i \beta_i Q^c$ and the proof of Lemma 3 we can get:

(i) When $\widetilde{h_{i,j}} \geqslant \dfrac{\alpha}{a-c}$, $p^c \in \left[0, \dfrac{2(a-c)}{\alpha} - \dfrac{1}{\widetilde{h_{i,j}}} - \lambda\right]$, $\pi_1{}^* = \dfrac{\left[a-c-\dfrac{\alpha}{2}(\dfrac{1}{\widetilde{h_{i,j}}} + p^c + \lambda)\right]^2}{4} + p^c\sum\limits_i \beta_i Q^c$.

(ii) $\widetilde{h_{i,j}} \geqslant \dfrac{\alpha}{a-c}$, $p^c \in \left[\dfrac{2(a-c)}{\alpha} - \dfrac{1}{\widetilde{h_{i,j}}} - \lambda, \dfrac{1}{\widetilde{h_{i,j}}} - \lambda\right]$,

$$\pi_2{}^* = \frac{\left[a-c-\dfrac{(a-c)-(p^c+\lambda)\alpha}{1-(p^c+\lambda)\widetilde{h_{i,j}}} - (p^c+\lambda)(\alpha - \dfrac{(a-c)-(p^c+\lambda)\alpha}{\dfrac{1}{\widetilde{h_{i,j}}} - (p^c+\lambda)})\right]^2}{4} + p^c\sum_i \beta_i Q^c.$$

(iii) $\widetilde{h_{i,j}} \leqslant \dfrac{\alpha}{a-c}$, $p^c \in \left[0, \dfrac{a-c}{\alpha} - \lambda\right]$,

$$\pi_3{}^* = \frac{\left[a-c-\dfrac{\alpha}{2}\dfrac{1}{\widetilde{h_{i,j}}} - \dfrac{(a-c)-(p^c+\lambda)\alpha}{2\left[1-(p^c+\lambda)\widetilde{h_{i,j}}\right]} - (p^c+\lambda)\left(\dfrac{\alpha}{2} - \dfrac{(a-c)-(p^c+\lambda)\alpha}{2\left[\dfrac{1}{\widetilde{h_{i,j}}} - (p^c+\lambda)\right]}\right)\right]^2}{4} + p^c\sum_i \beta_i Q^c.$$

b) Under the auditioning rule

Referring to $\pi_e(p^c) = \dfrac{\left[a-c-\sum\limits_{i,j}\dfrac{k_{i,j}e_{i,j}^2}{2} - (p^c+\lambda)\left(\alpha - \sum\limits_{i,j}\alpha_{i,j}e_{i,j}\right)\right]^2}{4}$ and the proof of Lemma 3 we can get:

(i) When $\widetilde{h_{i,j}} \geqslant \dfrac{\alpha}{a-c}$, $p^c \in \left[0, \dfrac{2(a-c)}{\alpha} - \dfrac{1}{\widetilde{h_{i,j}}} - \lambda\right]$, $\pi_1{}^* = \dfrac{\left[a-c-\dfrac{\alpha}{2}\left(\dfrac{1}{\widetilde{h_{i,j}}} + p^c + \lambda\right)\right]^2}{4}$.

(ii) $\widetilde{h_{i,j}} \geqslant \dfrac{\alpha}{a-c}$, $p^c \in \left[\dfrac{2(a-c)}{\alpha} - \dfrac{1}{\widetilde{h_{i,j}}} - \lambda, \dfrac{1}{\widetilde{h_{i,j}}} - \lambda\right]$,

$$\pi_2{}^* = \frac{\left[a-c-\dfrac{(a-c)-(p^c+\lambda)\alpha}{1-(p^c+\lambda)\widetilde{h_{i,j}}} - (p^c+\lambda)\left(\alpha - \dfrac{(a-c)-(p^c+\lambda)\alpha}{\dfrac{1}{\widetilde{h_{i,j}}} - (p^c+\lambda)}\right)\right]^2}{4}$$

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(iii) $\widetilde{h_{i,j}} \leqslant \frac{\alpha}{a-c}$, $p^c \in \left[0, \frac{a-c}{\alpha} - \lambda\right]$,

$$\pi_3{}^* = \frac{\left[a - c - \frac{\alpha}{2}\frac{1}{\widetilde{h_{i,j}}} - \frac{(a-c)-(p^c+\lambda)\alpha}{2[1-(p^c+\lambda)\widetilde{h_{i,j}}]} - (p^c+\lambda)\left(\frac{\alpha}{2} - \frac{(a-c)-(p^c+\lambda)\alpha}{2\left[\frac{1}{\widetilde{h_{i,j}}} - (p^c+\lambda)\right]}\right)\right]^2}{4}.$$

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