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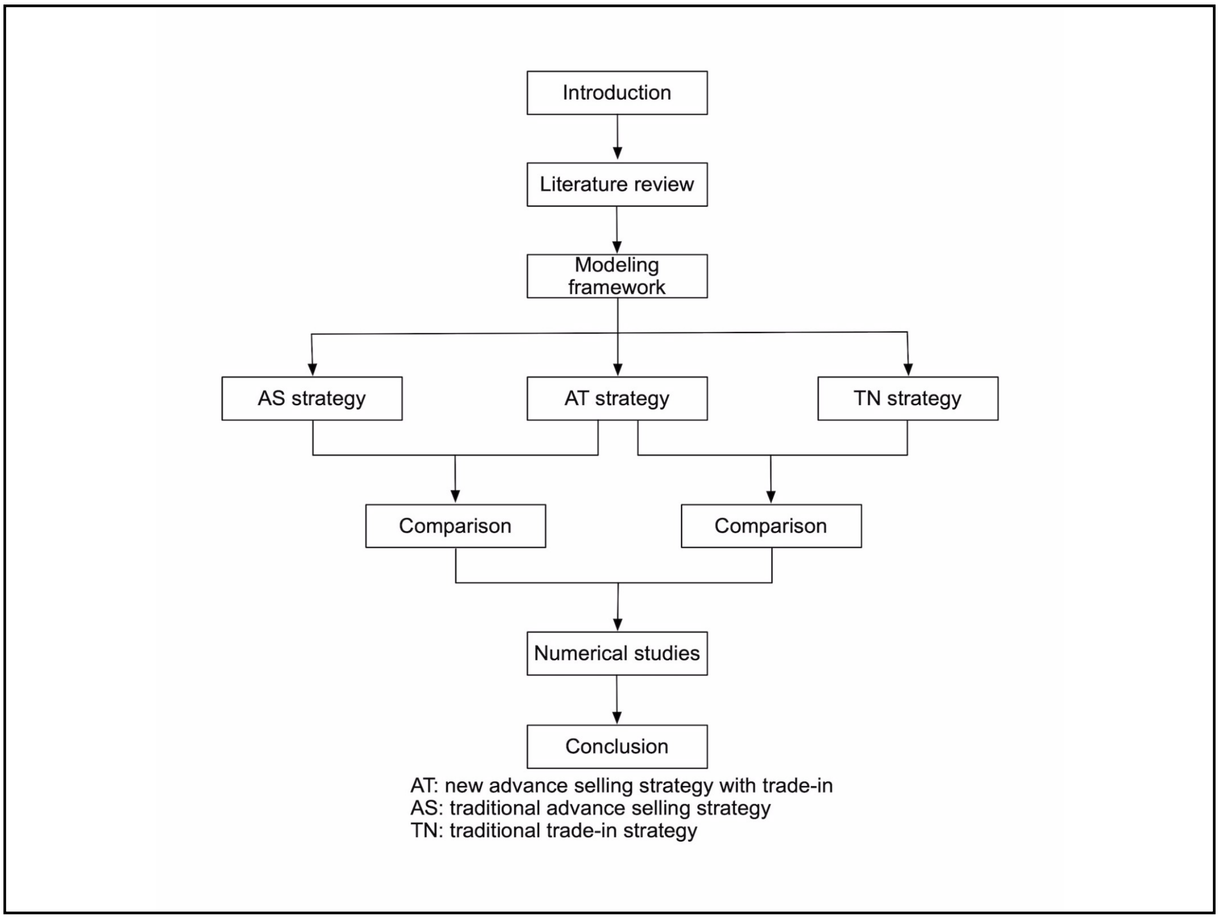
An advance selling strategy with a trade-in program

Jie Wu, Junjie Huang, Xiang Ji, and Zhixin Chen ✉

School of Management, University of Science and Technology of China, Hefei 230026, China

✉Correspondence: Zhixin Chen, E-mail: chenzhix@mail.ustc.edu.cn

Graphical abstract



Public summary

■ We study an emerging advance selling strategy that incorporates a trade-in program.

■ We explore a seller's optimal pricing decision and profit under different strategies.

■ This study examines the optimal strategy for the seller under different conditions.

■ We explore the factors that drive the sellers' preferences for the three strategies.

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Abstract: Recently, advance selling and trade-in programs have become increasingly popular in many industries. While previous studies have examined these two strategies individually, the combination of both strategies has not been studied. Inspired by business practices, we investigate an emerging advance selling strategy (hereinafter, the AT strategy) that utilizes the trade-in concept as a reward or discount for participating in the advance selling program. This study explores a seller's optimal pricing decision and profit when using the AT strategy, traditional advance selling (AS), and traditional trade-in (TN) strategies. We find that, compared to AT strategy, it is better for the seller to adopt the AS strategy when the salvage value of used products is sufficiently low and the product cost is not too small. In addition, compared to TN, when the salvage value and product cost are relatively low, the seller should adopt the AT strategy under most circumstances. Furthermore, we demonstrate that sellers can still gain profits when the product price is extremely high under the AT strategy. Even if consumer utility is negative during the advance selling period, consumers are willing to participate in AT programs because the total consumer surplus from buying two generations of products is positive.

Keywords: advance selling; trade-in; product durability; salvage value; strategic consumer

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1 Introduction

Advance selling occurs when buyers are allowed to purchase at a time preceding their consumption[1]. With the development of technology and innovation in business models, advance selling has become increasingly prevalent in various industries (e.g., travel and entertainment) as well as in many retail areas, such as books, ticketing, electronics, and media products. When sellers provide advance selling programs, customers are offered an opportunity to purchase a newly released product or service prior to the regular selling period. Advance selling can benefit customers as buying in advance guarantees that they will receive the product on time and avoid stock-out risk in the selling season. Additionally, advance selling can potentially benefit the seller. First, as consumers are often uncertain about the quality of the product and their own valuation of the product during the advance selling period, sellers can improve profits by utilizing the consumers' uncertainty[1–3]. Second, advance selling can reduce the sellers' demand uncertainty and inventory risk during the selling season[4–6]. Finally, as it enables dynamic pricing and facilitates demand learning, advance selling has been widely adopted.

As sellers frequently update products by releasing successive generations, several trade-in programs have been developed recently, under which buyers of an old-generation product can purchase the new-generation product at a discounted price by trading in the old product. Many sellers offer trade-in programs to promote the repeat purchases of successive product generations. For example, Apple and Huawei have launched trade-in programs that allow customers to replace their old devices through a discount on the purchase of new-generation smartphones. In addition to trading in old devices for new-generation products at an Apple store or Vmall, customers can also trade in their old mobile phones for cash payouts at some third-party recycling and reuse sites, including http://www.gazelle.com and https://huishou.jd.com.

While previous studies have examined the advance selling and trade-in programs individually and established their respective "bright sides"[1–6, 26–30], a combination of both strategies has not been analyzed. Thus, this study fills this gap in the extant literature by studying the emerging advance selling practice that utilizes the trade-in concept as a reward or discount for participating in the advance selling program, which we refer to as the AT strategy. Under the AT strategy, the consumer can buy a first-generation product in advance and obtain a prespecified trade-in value to be used for the purchase of the second-generation product in the third period. If the consumer delays a purchase until the spot-selling period, the consumer is not eligible to trade in the first-generation product to purchase the second-generation one. For example, Apple adopted this strategy in the Chinese market in 2020 on JD. com, and its poster is shown in Fig. 1. For convenience, we translated the poster into English.

The iPhone SE was launched in China in April, 2020. The retail platform JD.com adopted an advance selling strategy that incorporates a trade-in program. Consumers who buy iPhones during the advance selling period are eligible for a trade-in value equal to the original price of the iPhone to buy a new iPhone when the next-generation iPhone is released.

Fig. 1. iPhone SE trade-in plan on JD.com (http://tech.cnr.cn/techph/20200416/t20200416_525056130.shtml. Accessed April 16, 2020).

For example, if a customer purchases an iPhone SE at ¥ 3299 during the advance period, they will receive a trade-in value of ¥ 3299 when the new-generation iPhone is released one year later.

In addition to the AT strategy, we consider two benchmark strategies: the traditional advance selling (AS) and traditional trade-in (TN) strategies. This study aims to solve the sellers' optimal pricing decisions to maximize their profit when employing the AT, AS, and TN strategies. Additionally, it specifies the conditions under which AT strategy is more profitable than AS and TN strategies. Further, we explore the factors that influence the three strategies and explained how these factors drive sellers' preferences in the three strategies. We establish a stylized model to solve these questions and obtained the following interesting findings.

First, we reveal the optimal pricing decision and maximum profit when the seller adopts each of the three strategies. When product costs are high, the AT and AS strategies are not profitable for sellers. Under AS strategy, the price of the product and the advance selling discount are related to the product durability and cost. Under TN or AT strategies, the seller's price and profit depend on the combined effects of product cost, product durability, and salvage value of the used products. An AT program is profitable for strategic consumers even if the seller's product prices are extremely high. Therefore, sellers can set higher prices to squeeze out more consumer surplus, which justifies the extreme situation seen in Fig. 1 (i.e., the trade-in value equaling the original price of the iPhone).

Second, as the AT strategy incorporates the trade-in program, the sales volume and total profit are higher than those under AS strategy. Hence, under most circumstances, AT strategy is more profitable than AS strategy. Compared to AS strategy, it is more profitable for the seller to employ AT strategy when product durability is sufficiently low and the salvage value of the trade-in products to the seller is sufficiently high. Moreover, consumers are happy to purchase second-generation products when first-generation product durability is low. When the salvage value of used products is sufficiently high, there is more incentive for the seller to offer a trade-in program because it may benefit from the salvage option. Conversely, when the salvage value is sufficiently low and product cost is not too low, the seller should choose AS strategy instead of AT strategy to attract more customers.

Third, compared to TN strategy, if the salvage value is higher than product durability, the seller should adopt AT strategy when the salvage value product cost are relatively low. If the salvage value is lower than the product's durability, AT strategy is better because AT strategy provides a trade-in option and offers advance selling services. Advance selling can utilize the consumer valuation uncertainty to increase sales volume. When the product cost is high, demand for the product rapidly decreases, and the upside potential of the product price is reduced. At this point, it is fruitless to use advance selling as a promotion.

Finally, we numerically examine the impact of model parameters on the seller's pricing and profit performance. Marginal costs negatively influence the seller's profits under the three strategies. Under AT and TN strategies, the seller's total profit increases with the salvage value. Under TN strategy, when the salvage value is low, the price is not affected, and the total profit declines with the increase in product durability. However, the price and trade-in value decrease when the salvage value is sufficiently high, and both increase with the product's durability. Unlike TN strategy, the impact of product durability on the seller's profit is nonmonotonic under AT strategy.

The remainder of this paper is structured as follows. In the next section, we review the related literature. The proposed model is described in Section 3. We then analyze the equilibrium of the AT strategy in Section 4. The AS and TN strategies are modelled and discussed in Section 5. Section 6 compares the AT strategy with the AS and TN strategies. Next, we conduct numerical studies to further examine the effects of model parameters on the seller's optimal prices and profits. Finally, our conclusions are summarized in Section 8. All proofs are relegated to the Appendix.

2 Literature review

This study is closely related to two literature streams: advance selling and trade-in programs.

Numerous studies on advance selling have identified the benefits for sellers from offering advance selling and examined related issues, such as different consumer assumptions. Many studies have focused on the benefits of advance selling with respect to consumer valuation uncertainty. DeGraba[7] showed that a monopolist prefers selling to customers when they are uncertain about their valuations, and

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notes that selling fewer units than the number of customers may induce customers to purchase early. Gundepudi et al.[8] considered a seller that offers an information product in advance, assuming that consumer valuations are uncertain. Shugan and Xie[1−3, 9] considered that during advance selling, consumer valuation for a new to-be-released product is uncertain. They demonstrated that sellers can exploit consumer uncertainty to increase profits in either a monopoly or competitive setting. Xie and Shugan[10] further investigated the benefits of advance selling under different market and product conditions, and concluded that buyer uncertainty alone can allow advance selling to considerably increase profits. Fay and Xie[11] showed that both advance and probabilistic selling offer consumers a choice involving buyer uncertainty, and that these two conceptually different mechanisms can help the seller respond to unobservable buyer heterogeneity. In our study, valuation uncertainty is also identified as a key consumer characteristic.

Another benefit of advance selling is reducing the selling season's demand uncertainty and inventory risk for a seller. Tang et al.[4] showed that an advance selling strategy can reduce demand uncertainty and match supply with demand by utilizing advance booking data to update the demand forecast for the selling season. McCardle et al.[12] extended the work of Tang et al.[4] by incorporating the competitive nature of retailing while analyzing advance booking discount programs. Zhao and Stecke[5] showed that advance selling is a strategy for transferring inventory risk from a seller to consumers by jointly considering consumer valuation uncertainty and aggregate demand uncertainty when consumers are loss-averse. However, loss-averse consumers are not necessarily risk-averse. In subsequent work, Prasad et al.[6] examined the advance selling price and inventory decisions in a two-period setting when consumers are risk-averse. They shown that advance selling is not always an optimal strategy for sellers. Moreover, Gao et al.[13] showed that a weather-conditional rebate program allows the seller to price discriminate among a given customer's post-purchase states under different weather conditions and can increase the retailer's sales. Li and Zhang[14] showed that advance demand information may reduce the seller's profit.

In the advance selling literature, several studies have considered intertemporal pricing with strategic consumers. According to Aviv et al.[15, 16], substantial profit losses will ensue if the seller incorrectly assumes that strategic customers are myopic in their purchase decisions. Zhao et al.[17] shown that advance selling can reduce a seller's profit and supply chain performance in a decentralized supply chain when consumers are strategic. In the context of capacity decisions for advance selling, Yu et al.[18] explored the impact of the interdependence of customer valuation on both the behavior of strategic consumers and the seller's advance selling strategy. Wei and Zhang[19] studied the effectiveness of a new advance selling strategy (i.e., the preorder-contingent production strategy) in counteracting strategic consumer behavior. They shown that compared to other advance selling strategies, this strategy is more effective in mitigating strategic waiting behavior, and thus can significantly improve the seller's profit. Studying whether sellers should offer a price guarantee, and if so, how to set prices under the price guarantee, Pang and Xiao[20] shown that a preorder price guarantee is a new phenomenon related to both advance selling and price matching. In our model, consumers who participate in a TN program are myopic whereas consumers who participate in AT and AS programs are strategic.

Regarding the second stream of related literature, most previous studies on trade-in programs have focused on the pricing decisions of purchasing single or multigeneration products. For single-generation products, van Ackere and Reyniers[21] considered a setting in which a firm sells the same product in two periods, and a customer who purchased the product in the first period can trade in the old product for the new product (same but new) in the second period. Ray et al.[22] developed a single-period model to study the optimal pricing and trade-in strategies for durable, remanufacturable products while differentiating between two customer types: first-time buyers and customers who already own the product. Considering the used goods market or competition with other sellers, Rao et al.[23] modeled a trade-in policy as an intervention by the seller in the used goods market to reduce the inefficiencies arising from the lemons problem. They shown that trade-in programs are more valuable for products that are less reliable and deteriorate more slowly. Agrawal et al.[24] studied the decisions of an original equipment manufacturer (OEM) facing competition from third-party remanufacturers (the used goods market). However, in contrast to Rao et al.[23], they found that offering a trade-in program to compete with third-party remanufacturers can be detrimental to the OEM. Further, in a duopoly situation in which one firm implements a trade-in program and the other does not, Zhu et al.[25] found that adopting a trade-in strategy could generate a competitive advantage for the first firm in terms of market share and profit.

For multigeneration products, Fudenberg and Tirole[26] investigated the monopoly pricing of successive generations of a durable good with and without a secondhand market. Okada[27] adopted the framework of mental accounting and prospect theory to explain how both normative and psychological mechanisms drive replacement purchase decisions. He also explained why a replacement purchase decision is more sensitive to mental cost than the purchase price of the new product itself. Considering two successive generations of products, Yin and Tang[28] examined the effectiveness of different trade-in programs, including programs that require customers to pay an up-front fee. In a follow-up study, Yin et al.[29] developed a model to determine the optimal prices of two successive generations of a product in equilibrium and examine the conditions under which trade-in programs benefit the seller. Zhang et al.[30] explored how customer purchasing behavior and remanufacturing efficiency affect the economic and environmental values of such a business practice, and how a social planner should formulate a public policy to maximize social welfare. By analyzing the purchasing behavior of both myopic and strategic consumers, Hu et al.[31] investigated the optimal price of next-generation products and the optimal trade-in rebate in a trade-in program with a limited duration. Xiao and Zhou[32] demonstrated that a hybrid trade-in program can generate significantly more profit than either an upgrade-only or a cash-only trade-in program for a seller of a new-generation product. Our study also investigates the issue of two generations of products. However, we assume that the prices of the two generations of products are identical. For many electronic products (e.g., mobile phones), the price variation between the two generations is negligible relative to the total price.

Although the aforementioned advance selling and trade-in studies answered the question of whether a seller should sell in advance or offer a trade-in program in different scenarios, they did not explore a combined strategy of both programs. To fill this gap in the extant literature, our study focuses on the effectiveness of the new strategy.

3 Modeling framework

This section introduces our main model of this new advance selling strategy that incorporates a trade-in program: the AT strategy. We consider a monopolistic seller that offers two successive generations of a product to strategic customers over three periods. For the first-generation products, the first period is the advance selling period, while the second period is the spot selling period (i.e., regular selling period). The second-generation products are sold during the third period (the trade-in period). In our study, we focus on electronic or high-tech products, such as smartphones and computers, for which this model is very realistic. In the following, we describe the assumptions regarding the seller and consumer.

3.1 Seller setting

To obtain tractable results, this study follows the vast trade-in literature[21, 26, 28] in assuming that the seller has sufficient capacity to meet customer demand in each period without considering the issues of production planning and inventory control.

Smartphone sellers usually set identical prices for different generations of a product[32]. For example, the price of a new generation of iPads is the same as the release prices of its predecessors, and customers are "trained" to know such price information before the new generation is officially released into the market[28]. To facilitate calculations and simplify the model, we assume that the seller charges a constant price (p) per unit for successive generations of products. The unit costs for the first-generation (product 1) and second-generation products (product 2) are both c. Unlike the traditional AS strategy that includes a discount, the AT strategy replaces the discount with a trade-in program as a means of promotion. Owing to this condition, in the first period, the advance selling price (p_a) of the first-generation product is the same as its regular selling price (p_r), that is, $p_a = p_r = p$. Furthermore, we assume that the seller knows the distribution of consumer valuations[5, 6, 9, 19]. To capture the sizable profit associated with remanufacturing and reselling trade-in products, recycling materials, or harvesting parts, we assume that the seller may obtain a salvage value $s \geqslant 0$ for each trade-in unit, where s is exogenously given[28, 29] .

3.2 Consumer setting

In the AT and AS strategies, consumers are divided into two groups (i.e., informed and uninformed) depending on whether they know that advance selling is offered by the seller. Informed consumers become aware of advance selling and arrive in the advance period. Informed consumers must strategically decide whether to buy in the advance selling period or wait until the regular selling period. On the one hand, if the consumers buy in advance, they are not guaranteed to obtain the product and have the option to trade-in the first-generation product and receive a prespecified trade-in value t to be used for the purchase of the second-generation product in the third period. However, if the consumers delay the purchase until the regular selling period, they are not eligible to trade in the first-generation product or receive trade-in value t when they purchase the second-generation product. On the other hand, consumers arriving in advance know the valuation distribution but not their valuations. Therefore, informed consumers have a random valuation of the product in the first period. At the beginning of the second period, consumers realize their valuations of the product. We assume that the consumers' valuation, denoted by V, follows a uniform distribution between 0 and 1. As with the price assumption, we also assume that the consumer valuations of the two product generations are identical. Uninformed consumers are unaware of advance selling and arrive during the spot period. They consider whether to buy the product only in the second period after its release. There are N_1 informed and N_2 uninformed consumers in the market. Following Xie and Shugan[9] and Zhao et al.[17], N_1 and N_2 are deterministic and $N_1 = N_2 = N$, where N is a constant. The market size N is standardized to 1 in the propositions, as it has no effect on the analysis and conclusions of the model. Following convention, consumers buy when and only when they receive a nonnegative surplus.

In our model, those who participate in the AT and AS programs are strategic consumers, whereas those who participate in the TN program are myopic consumers. The reasons for this are as follows. First, in the TN program, the seller will only announce the product price p when it releases the first-generation product and the trade-in value t' when the second-generation product is released. When most consumers buy a product, they will only consider the current role and possible utility of the product but will not consider its salvage value and potential benefits in the future. We assume that when the seller offers a traditional trade-in program, consumers are myopic, that is, consumers will not deliberately consider the benefits of the trade-in program in the future. Second, when advance selling and the trade-in strategies are combined, the seller will announce both product price p and trade-in value t when the first-generation product is released, which will encourage consumers to think strategically about long-term benefits. Thereby, they will consider not only the benefits of current products but also the potential benefits of updating second-generation products. Therefore, we assume that when the seller provides an AT program, consumers will change from their normally myopic behavior to strategic behavior.

Following Yin and Tang[28], we assume that target customers in the primary market are only interested in buying new products and that the seller will not produce (or sell) refurbished products in the primary market. For any customer who

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purchased the first-generation product with valuation v in Period 1, we assume that this "holder" will value this old product in Period 2 at δv, where $\delta \in (0,1)$. van Ackere et al.[21] and Yin et al.[29] refered to δ as the durability parameter of the first-generation product, where δ denotes the rate at which the product loses its value over time.

4 New advance selling strategy with trade-in

4.1 Consumer surplus

The consumers' expected valuation is EV. If consumers choose to buy early, then their expected utility in the first period is $EV-p$. Consequently, considering the two choices that a holder has in the third period: either trading in the old product for product 2 at p, or keeping the old product and not purchasing product 2. The corresponding third-period surpluses associated with these two choices are $v-p+t$ and δv, respectively. In the third period, the consumer will trade in only if $v \geqslant \frac{p-t}{1-\delta}$, where $0 \leqslant t \leqslant p \leqslant 1$. The ex-ante expected total surplus (over three periods), denoted by $U_{\rm A}$, is given by

$$U_{\rm A} = EV - p + \int_{\frac{p-t}{1-\delta}}^{1} (v-p+t)\mathrm{d}v + \int_{0}^{\frac{p-t}{1-\delta}} \delta v \mathrm{d}v = 1-2p+t+\frac{(p-t)^2}{2(1-\delta)}.$$

If consumers do not buy in advance, they can decide whether to buy during the spot-selling season. If their realized valuation is greater than or equal to the market price p, consumers will buy and obtain a surplus of $v-p$. Otherwise, they will not buy, generating a surplus of 0. If consumers do not participate in advance selling, they will not receive the trade-in rebate t and can only buy second-generation products at the regular price. We assume that customers who do not have a trade-in rebate t will not update their products because high-tech products, such as mobile phones and computers, can be used for more than one year; however, the updating time of the two generations of products will generally not exceed one year. Additionally, we assume that without a trade-in rebate t, customers who have the first-generation product will not buy the second-generation product, that is, $p \geqslant 1-\delta$ or $v-p \leqslant \delta v$, where $v \in (0,1)$. Therefore, the corresponding third-period surplus is δv. Further, we assume that consumers who do not buy in the second period will not purchase in the third period because the consumer surplus $(v-p)$ of consumers who purchase products is negative.

$$U_{\rm W} = E(v-p)^{+} + \int_{p}^{1} \delta v \mathrm{d}v = (1-\delta)\frac{(1-p^2)}{2} - (1-p)p.$$

During the first period, to make their early purchase decision, informed consumers compare the expected utility from advance purchase ($U_{\rm A}$) and that from waiting to buy later ($U_{\rm W}$). They buy early if and only if $U_{\rm A} \geqslant U_{\rm W}$ and $U_{\rm A} \geqslant 0$.

4.2 Event sequence

The sequence of events is illustrated in Fig. 2. There are three periods for the two-generation products: advance selling, spot selling, and trade-in periods.

Period 1: A cohort of N_1 consumers who are uncertain about their individual valuations arrives in the advance period. The seller decides the product price p and trade-in value t. Then, consumers decide whether to buy the product in advance.

Period 2: During the spot selling period, another cohort of N_2 customers arrives. The product is available for consumption, and customers know both product price p and their individual valuations. All the remaining customers, including those who have not bought in advance and those who arrived during the spot-selling period, decide whether to buy during the current period.

Period 3: A holder who purchased product 1 in the first period has two choices in the third period: either to trade in the old product for product 2, or keep the old product and do not purchase product 2.

4.3 Equilibrium analyses

Following standard backward induction, we first analyze the seller's third-period problem regarding the second-generation product, then the problems in the first two stages regarding the first-generation product.

In the third period, only customers who participated in the first period presale activities are eligible to utilize the trade-in program offering value t. If the seller adopts AT strategy, it determines the price p and trade-in value t to incentivize informed consumers to participate in the program, resulting in N people who can trade in. As mentioned in Section 3, the consumer trades only if $v \geqslant \frac{p-t}{1-\delta}$. Hence, in the third period, the number of customers who want to trade in is $N\left(1-\frac{p-t}{1-\delta}\right)$. The retailer's profit from the trade-in period is

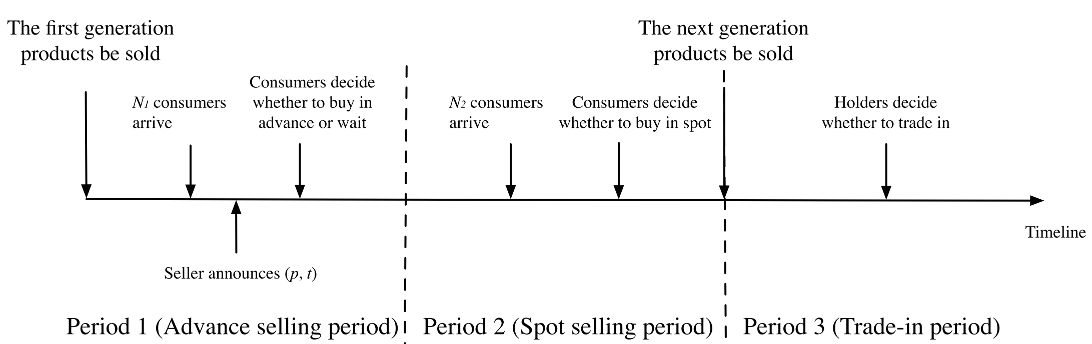

Fig. 2. Sequence of events.

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$N(p-t+s-c)\left(1-\frac{p-t}{1-\delta}\right)$.

If informed consumers buy early, only uninformed consumers remain in the market. Demand during the second period is $N(1-p)$, and the seller's profit in the second period is $N(p-c)(1-p)$.

In the first period, the seller must set the advance price p and trade-in value t. Informed consumers observe both and compare the expected utilities from preordering and waiting. When $U_A \geqslant U_W$ and $U_A \geqslant 0$, informed consumers buy early. The seller's profit in the advance-selling period is $N(p-c)$.

The optimal price p and trade-in value t maximize the retailer's total expected profit from the three periods, denoted by Π, while motivating informed consumers to buy early. Hence, the seller solves the following problem for the optimal pricing decision as:

$$\Pi = \max\left\{N(p-t+s-c)\left(1-\frac{p-t}{1-\delta}\right)+N(p-c)(1-p)+N(p-c)\right\}$$

$$\text{s.t.}\begin{cases} U_A \geq U_W, & \text{(Consumers prefer purchasing early)} \\ U_A \geqslant 0, & \text{(Positive profit)} \\ 0 \leqslant \dfrac{p-t}{1-\delta} \leqslant 1, & \text{(Consumers prefer participating in the trade-in program)} \\ \dfrac{p}{1-\delta} \geqslant 1. & \text{(Consumers prefer keeping the old product)} \end{cases}$$

Proposition 4.1. The seller's optimal prices and profits satisfy the following conditions:

(i) When $s \leqslant 1-5\delta+2\delta^2$, if $c \in \left(0, \frac{1+s+\delta-s\delta-3\delta^2+\delta^3}{2}\right)$, then $p = 1-\delta$ and $t = (\delta-1)^2$. Correspondingly, $\Pi = 1-2c+s+\delta-s\delta-3\delta^2+\delta^3$. If $c \geqslant \frac{1+s+\delta-s\delta-3\delta^2+\delta^3}{2}$, it is not optimal to employ the AT strategy.

(ii) When $1-5\delta+2\delta^2 \leqslant s \leqslant 1-\delta$, if $c \in \left(0, \frac{(3+s-\delta)^2}{8(2-\delta)}\right)$, then $p = \frac{3+s-\delta}{2(2-\delta)}$ and $t = \frac{1+s+\delta}{2}$. Correspondingly, $\Pi = \frac{3+s-\delta}{4(2-\delta))} - 2c$. If $c \geqslant \frac{(3+s-\delta)^2}{8(2-\delta)}$, it is not optimal to employ the AT strategy.

(iii) When $s \geqslant 1-\delta$, if $c \in \left(0, \frac{1+s}{2}\right)$, then $p = 1$ and $t = 1$. Correspondingly, $\Pi = 1-2c+s$. If $c \geqslant \frac{1+s}{2}$, it is not optimal to employ the AT strategy.

Proposition 4.1 shows that the seller does not have an incentive to adopt the AT strategy when the product cost c is high. This is consistent with intuition. When the cost is very high, the seller's profit decreases until it becomes negative. When $s \leqslant 1-5\delta+2\delta^2$, the sale price p and trade-in value t are determined only by δ. This is because when δ is sufficiently small, consumers are willing to trade in; thus, the price and sales volume are sensitive to the value of δ. When $1-5\delta+2\delta^2 \leqslant s \leqslant 1-\delta$, if δ is moderate, then the selling price p and trade-in value t depend on both s and δ. At this time, consumers' willingness to trade in is not as strong as in Proposition 4.1(i). To obtain more profit, it is necessary to comprehensively consider the durability and salvage value of the old products.

Notably, when $s \geqslant 1-\delta$, even if the price and trade-in value are extremely high (i.e., $p = 1$ and $t = 1$), it is still profitable for the seller to adopt the AT strategy. The reasons for this are as follows. For strategic consumers, future trade-in benefits are calculated when considering whether to purchase in advance in the first stage. Therefore, even if consumer utility is negative in the first stage, consumers are willing to participate in the presale program because the total income from buying two generations of products is positive. For the seller, setting t too high is obviously unfavorable; however, the seller may still adopt this strategy for two reasons. First, owing to the high price, the marginal profit can compensate for the loss from excessive t. Second, when s is high, the seller's income from recycling the old products is also considerable. When δ is sufficiently high, consumers' motivation to update products is relatively weak, and a relatively large t can encourage them to buy products and increase sales volume.

5 Two benchmark programs: traditional advance selling strategy and traditional trade-in strategy

To examine the effectiveness of the AT strategy, in this section, we present two benchmark programs. The first is the AS program that does not include a trade-in option. The second is the TN program that does not include advance selling.

5.1 Traditional advance selling strategy

Fig. 3 shows the three periods: the advance selling period, spot selling period for product 1, and spot selling period for product 2. In the advance selling period, consumers are uncertain about their valuations of the product and first-generation

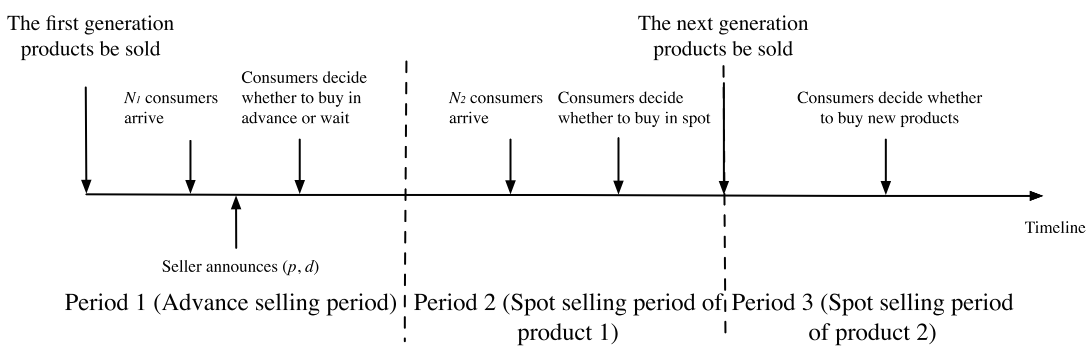

Fig. 3. Sequence of events.

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products are sold. The seller announces price p and discount d. Then, informed consumers arrive and make their advance selling purchase decisions. If they choose to buy early, their expected utility is $EV-p+d$. In the spot selling period, consumers realize their valuations. They can only buy products at the regular price p (without the discount); thus, their surplus is $v-p$. In the third period, the next-generation products are released, and all consumers decide whether to buy them at the regular price p. As mentioned in Section 4, we assume that a holder who purchased product 1 in the second period will not update the product, and that a nonholder will still not purchase product 2. Hence, it suffices to focus on the case in which $p \geqslant 1-\delta$.

Buyers purchase in advance only when the expected surplus from advance buying (U_{A}) exceeds that from waiting (U_{W}). U_{A} and U_{W} can be expressed as

$$U_{\mathrm{A}} = EV-p+d+\int_0^1 \delta v \mathrm{d}v = d-p+\frac{1+\delta}{2},$$

$$U_{\mathrm{W}} = E(v-p)^+ + \int_p^1 \delta v \mathrm{d}v = (1-\delta)\frac{(1-p^2)}{2}-(1-p)p.$$

Accordingly, the seller's maximum total expected profit is

$$\varPi_{\mathrm{AS}} = \max\{N(p-c-d)+N(1-p)(p-c)\}.$$

The optimal product price p and AS discount d maximize $\varPi_{\mathrm{AS}}$ while motivating informed consumers to buy early. Given this motivation, we can derive the relationship between p and d.

Lemma 5.1. The advance selling discount d is a function of p, that is, $d=\dfrac{p^2(1-\delta)}{2}$.

Jointly considering the above situations and Lemma 5.1, we present the seller's optimal profit performance under AS in the following proposition:

Proposition 5.1. (ⅰ) When $\delta \in (0, 2-\sqrt{3})$, it is optimal for the seller to charge the optimal product price and advance selling discount d at

$$(p,d)=\begin{cases} \left((1-\delta),\dfrac{(1-\delta)^3}{2}\right), & \text{if } c \in (0, 1-4\delta+\delta^2); \\ \left(\dfrac{2+c}{3-\delta},\dfrac{(1-\delta)(2+c)^2}{2(3-\delta)^2}\right), & \text{if } c \in (1-4\delta+\delta^2, 1-\delta); \\ \left(1,\dfrac{1-\delta}{2}\right), & \text{if } c \in (1-\delta, 1). \end{cases}$$

Accordingly, the seller's optimal profit is

$$\varPi_{\mathrm{AS}}=\begin{cases} \dfrac{1+3\delta-5\delta^2+\delta^3-2c(1-\delta)}{2}, & \text{if } c \in (0, 1-4\delta+\delta^2); \\ \dfrac{4+c^2-4c(2-\delta)}{2(3-\delta)}, & \text{if } c \in (1-4\delta+\delta^2, 1-\delta); \\ \dfrac{1-2c+\delta}{2}, & \text{if } c \in (1-\delta, 1). \end{cases}$$

(ⅱ) When $\delta \in (2-\sqrt{3}, 1)$, it is optimal for the seller to charge the optimal product price and advance selling discount d at

$$(p,d)=\begin{cases} \left(\dfrac{2+c}{3-\delta},\dfrac{(1-\delta)(2+c)^2}{2(3-\delta)^2}\right), & \text{if } c \in (0, 1-\delta); \\ \left(1,\dfrac{1-\delta}{2}\right), & \text{if } c \in (1-\delta, 1). \end{cases}$$

Accordingly, the seller's optimal profit is

$$\varPi_{\mathrm{AS}}=\begin{cases} \dfrac{4+c^2-4c(2-\delta)}{2(3-\delta)}, & \text{if } c \in (0, 1-\delta); \\ \dfrac{1-2c+\delta}{2}, & \text{if } c \in (1-\delta, 1). \end{cases}$$

Proposition 5.1 indicates that whether the AS strategy is profitable for the seller depends on the product cost c and durability δ. The profit is negative if the cost is too high. Product durability δ has a negative effect on the seller's advance selling discount d while having a positive impact on profit performance. An increase in δ indicates more durability of product 1, resulting in a weaker motivation for consumers to update their old products. Thus, the seller is better off paying a lower discount to obtain a greater surplus.

5.2 Traditional trade-in strategy

We now examine a TN program consisting of only two periods for two generations of products: spot selling and trade-in. To facilitate subsequent comparisons, the market size is also $2N$. In the first period, the seller announces the price of the product and consumers decide whether to buy it. In the second period, the seller announces the trade-in value t'. The

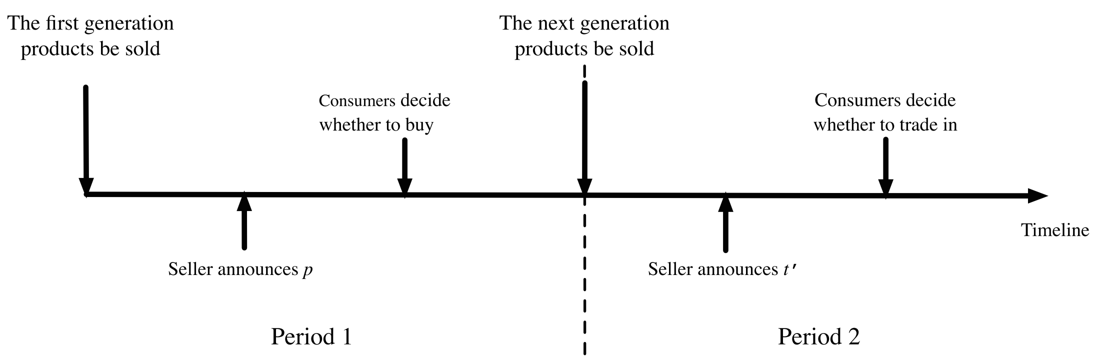

Fig. 4. Sequence of events.

price of the second-generation product is the same as that of the first-generation product; however, consumers who own the old product can trade it in at value t'. We assume that consumers who did not buy first-generation products in Period 1 will not purchase the new-generation products in Period 2 because the surplus $(v-p)$ of such consumers is negative in both periods. The sequence of events is shown in Fig. 4.

Based on backward induction, we first analyze the seller's second-period problem. The consumer surplus from participating in the trade-in program is $v-p+t'$. The corresponding surplus from keeping the old product is δv. Hence, the demand in the second period is $2N(1-\frac{p-t'}{1-\delta})$. The seller maximizes its second-period profit by solving the following problem:

$$\Pi_2 = \max\left\{2N(p-c+s-t')\left(1-\frac{p-t'}{1-\delta}\right)\right\},$$

$$\text{s.t.}\begin{cases} 0 \leqslant \dfrac{p-t'}{1-\delta} \leqslant 1, & \text{(The demand for the trade-in program is positive)} \\ 1-\dfrac{p-t'}{1-\delta} \leqslant 1-p. & \text{(The number of holders is greater than the number of customers who trade in)} \end{cases}$$

The following lemma establishes the seller's optimal second-period decisions.

Lemma 5.2. The trade-in value t' is a function of p:

(i) When $c \geqslant 1+s-\delta$, $t' = p-1+\delta$;

(ii) when $c < 1+s-\delta$ and $p \leqslant \dfrac{1+c-s-\delta}{2(1-\delta)}$, $t' = \dfrac{(-1-c+2p+s+\delta)}{2}$;

(iii) when $c < 1+s-\delta$ and $p > \dfrac{1+c-s-\delta}{2(1-\delta)}$, $t' = \delta p$.

The seller's problem in the first period can be stated as maximizing the total profit from both periods:

$$\Pi_{\rm TN} = \max\left\{2N(p-c+s-t')\left(1-\frac{p-t'}{1-\delta}\right)+2N(p-c)(1-p)\right\}.$$

To solve the above problem, we present the seller's optimal TN profit in the following proposition.

Proposition 5.2. When $s-\delta \leqslant 0$, it is optimal for the seller to set the product price p and trade-in value t' as follows:

$$(p,t') = \begin{cases} \left(\dfrac{2+2c-s-\delta}{2(2-\delta)}, \dfrac{\delta(2+2c-s-\delta)}{2(2-\delta)}\right), & \text{if } c \in \left(0, \dfrac{s}{\delta}\right); \\ \left(\dfrac{1+c}{2}, \dfrac{s+\delta}{2}\right), & \text{if } c \in \left(\dfrac{s}{\delta}, 1+s-\delta\right); \\ \left(\dfrac{1+c}{2}, \dfrac{-1+c+2\delta}{2}\right), & \text{if } c \in (1+s-\delta, 1). \end{cases}$$

Accordingly, the seller's optimal profit is

$$\Pi_{\rm TN} = \begin{cases} \dfrac{(2+2c-s-\delta)^2}{2(2-\delta)}, & \text{if } c \in \left(0, \dfrac{s}{\delta}\right); \\ \dfrac{2+2s-2c(2+s-2\delta)+(s-\delta)^2+c^2(2-\delta)-3\delta}{2(1-\delta)}, & \text{if } c \in \left(\dfrac{s}{\delta}, 1+s-\delta\right); \\ \dfrac{(1-c)^2}{2}, & \text{if } c \in (1+s-\delta, 1). \end{cases}$$

Otherwise, when $s-\delta \geqslant 0$, the optimal profit is $(p_{\rm TN}, t') = \left(\dfrac{2+2c-s-\delta}{2(2-\delta)}, \dfrac{\delta(2+2c-s-\delta)}{2(2-\delta)}\right)$, generating profit $\Pi_{\rm TN} = \dfrac{(2+2c-s-\delta)^2}{2(2-\delta)}$.

Proposition 5.2 indicates that whether the traditional trade-in strategy is profitable for the seller depends on the cost c, durability δ, and salvage value s. It also provides a feasible solution for the price-decision problem of a seller providing customers with trade-in services.

Although the price and profit functions are monotonic with respect to cost c, the impacts of product durability and salvage value of the trade-in items on the optimal price and profit are too complicated to analyze. Hence, in Section 7, we numerically examine the impact of the model parameters.

6 Comparisons across different strategies

Using the results presented in the previous two sections, we compare the seller's expected profit from adopting the AT, AS, and TN strategies to explore the optimal strategy under different conditions.

6.1 Comparison between the AT and AS strategies

Propositions 4.1 and 5.1 summarize the optimal AT and AS equilibria, respectively. Next, we compare the profits of the two strategies to determine the conditions under that AT strategy outperforms the AS strategy.

Proposition 6.1. The optimal strategy is as follows:

(i) AT strategy dominates AS strategy when any of the following three conditions are satisfied:

① $s+\delta$ is high and δ is sufficiently low (i.e., $s+\delta \geqslant 1$ and $\delta \in (0, 2-\sqrt{3})$),

② s is sufficiently high, or

③ c is sufficiently low.

(ii) When s is relatively low and c is relatively high, AS strategy dominates AT strategy; otherwise, AT strategy is more profitable than AS strategy.

Proposition 6.1 provides the sufficient conditions under which AT strategy dominates AS strategy. First, Proposition 6.1(i) indicates that AT strategy is better than AS strategy for the seller when product durability δ is sufficiently low that consumers are willing to purchase product 2. When product durability is high, consumers are content with the old product and reluctant to purchase product 2. This makes the trade-in less attractive and results in a lower sales volume. This result is consistent with JD's current practice; the product durability of iPhone SE is low because it is a low-end product among iPhone models. Second, when the salvage value of the trade-in products is sufficiently high for the sellers, there is sufficient incentive for them to offer trade-in programs. Hence, the sellers should adopt an AT strategy, which can motivate consumers to buy new products and increase the sales volume of product 2 through trade-in items, thereby increasing the overall profits.

Proposition 6.1(ii) indicates that when the salvage value s is sufficiently low and product cost is not too small, the seller should choose AS strategy rather than AT strategy to attract more customers. A relatively low salvage value will lead to

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lower profits when the seller provides a trade-in program. Meanwhile, a high product cost will reduce the seller's unit profit and increase the price, resulting in a lower sales volume. The relatively small sales volume will make it difficult to create economies of scale. Therefore, the AS strategy, which relies less on sales volume, is better than the AT strategy in this case.

6.2 Comparison between the AT and TN strategies

Proposition 4.1 describes the AT equilibrium, while Proposition 5.2 identifies the optimal price and profit when the seller only offers a TN program. Comparing the optimal profits of the two strategies, we can determine the conditions under which AT strategy dominates TN strategy and the conditions under which TN strategy should be adopted.

Proposition 6.2. The optimal strategy is as follows:

(i) If $s \geqslant \delta$, AT dominates TN when ① both s and c are low or ② s is high and c is moderate.

(ii) If $s \leqslant \delta$, AT dominates TN when c is low.

Proposition 6.2 provides the necessary and sufficient conditions under which a seller definitely prefers AT strategy or TN strategy. Proposition 6.2(i) indicates that if s is higher than δ and the salvage value s and product cost c are relatively low, the seller should adopt the AT strategy. The main reasons for this are as follows. First, although a lower salvage value s provides lower recovery profits for the seller, the seller can charge a higher price p under the AT strategy, which will yield a greater unit profit at a lower cost. Second, under normal circumstances, increased prices will reduce the sales volume; however, the AT strategy encourages consumers to think strategically about the total valuation of the two generations of products because it has an advance selling period. Based on this, strategic consumers will not forgo buying products, even if the price is high. Their long-term valuations are positive due to the existence of a large t. Therefore, an increase in price will not reduce the sales volume but will increase the marginal profit, resulting in an increase in the seller's total profit. Third, when s is small and δ is lower than s, consumers have greater incentives to choose to trade in. When the trade-in is easily accepted, the AT strategy is more advantageous than TN strategy. This is because the AT strategy not only provides a trade-in option but also supports advance selling services, which can utilize consumer valuation uncertainty to increase the sales volume. Furthermore, when the salvage value s of the product is relatively high and the cost is moderate, the seller tends to adopt the AT strategy. When the salvage value of the product is high, the seller's income from recycling old products is relatively large, which is sufficient to compensate for the loss caused by the increased cost.

Proposition 6.2(ii) indicates that if s is lower than δ, the AT strategy is better when the product cost c is sufficiently small. On the one hand, when the product is sufficiently durable ($\delta > s$), customers are reluctant to buy the new generation of the product in Period 3. Therefore, the trade-in options are not sufficiently attractive to the consumers. At this point, the advantages of advance selling come into play at a lower product cost. On the other hand, the AT strategy price is generally higher than the TN strategy price. If the cost is sufficiently large, the seller prefers the TN strategy because an excessively high cost will reduce the upside potential of the product price while the AT strategy will not be sufficiently competitive.

7 Numerical studies

This section examines how a seller's strategy changes using three key parameters: c (product cost), s (salvage value of the used product), and δ (durability measure). To do so, numerical examples are constructed to illustrate the optimal profits and prices under each strategy. These optima allow the seller to choose the strategy that generates the highest profits. As the potential market size N causes scale changes in the market, we set $N = 1$ without loss of generality. Our numerical studies demonstrate that the main insights from the analytical results continue to hold for these extensions.

We first investigate the influence of product cost. We set the negative profit to zero in the figures because the profit may be negative when the cost is too high. Intuitively, the marginal cost negatively influences the seller's profit performance in the three strategies. Fig. 5 shows that profit performance monotonically decreases with product cost c.

Fig. 5a reaffirms the analytical result stated in Proposition 6.1: when the product cost is high and the salvage value of the used product is sufficiently small, AS strategy performs much better than AT strategy.

Similar to the previous findings stated in Proposition 6.2, under most circumstances, when the cost is not too high, AT strategy is better than both AS and TN strategies (see Fig.5b and 5c). Furthermore, when the product durability is low, the production cost is moderate and the salvage value of the used product is sufficiently high, the AT strategy dominates the other two (see Fig. 5d).

Next, we consider how the salvage value of a used product (s) and durability (δ) affect the seller's profit and pricing under these three strategies.

Fig. 6 shows the impact of salvage value s on optional profit and pricing under different programs. The seller's total profit increases with s under the AT and TN programs (see Fig. 6c). This means that a higher salvage value provides additional incentives for the seller to accept trade-ins. Under the TN strategy, when s is low, the price is not affected by s, and an increase in salvage value s leads to higher profits. However, when s is sufficiently high, the price and trade-in value decrease with s (see Fig. 6a and 6b). Given the lower prices, more customers will purchase first-generation products, and subsequently, more holders can trade in during the third period. The increase in total profit comes from the increase in both sales volume and salvage value of the used product. Furthermore, a lower trade-in value helps reduce losses. AT strategy enables the seller to set a higher price p and trade-in value t than those under the TN strategy. The pricing decision can entice customers to buy both generations of products without sacrificing the seller's profit margin because of consumer valuation uncertainty in the first period. The increase in total profit comes from increases in both the price and salvage value.

When the product is more durable, customers are more reluctant to update their used products in Period 3. Hence, the

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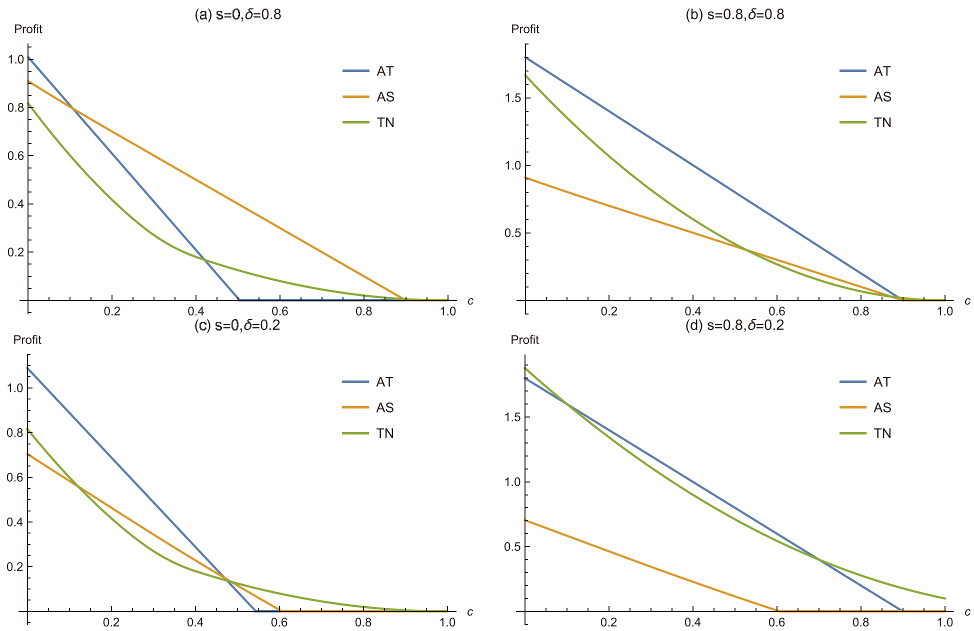

Fig. 5. Impact of product cost c on optimal profit under different programs.

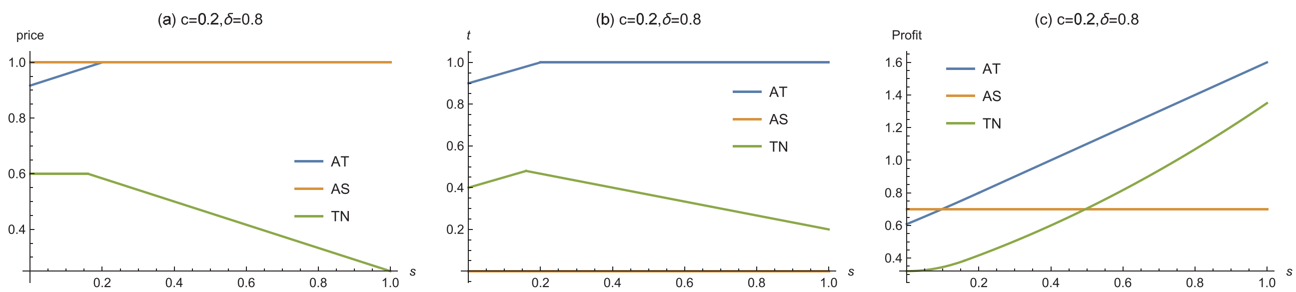

Fig. 6. Impact of salvage value s on optimal profit and pricing under different programs.

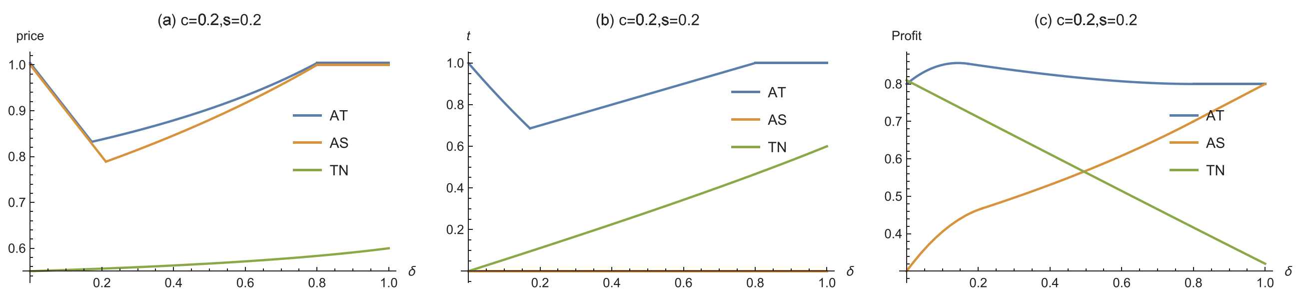

Fig. 7. Impact of product durability δ on optimal profit and pricing under different programs.

number of consumers who are willing to trade in decreases in δ. Under the TN strategy, to compensate for the negative impact of the decrease in sales volume, the seller can increase prices to obtain higher unit profits and increase the trade-in value to encourage consumers to trade in (Fig. 7a and 7b), but the total profit will still decrease in δ (Fig. 7c). Unlike TN, the impact of δ on the seller's profit is nonmonotonic under the AT strategy. Fig. 7c shows that the seller's profit first increases, but then decreases as δ increases. This behavior is counterintuitive because conventional wisdom suggests that uncertainty increases profits. This is an interesting effect of the interaction between advance selling and the trade-in option. As mentioned in Section 3, consumers arriving in advance know the distribution of valuations, but not their valuations. Owing to valuation uncertainty, the seller can substantially increase the sales volume by lowering prices, thereby increasing total profits when the product is not too durable. However, when product durability is high, the number of consumers willing to trade in decreases rapidly. Therefore, the seller should raise the price to compensate for the loss caused by the increase in δ.

By comparing the AS and AT strategies, we find that the interaction of advance selling and the trade-in option makes the sales volumes of the two generations of products under the AT strategy much higher than those under AS strategy. Consequently, there is a large profit gap between the two

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strategies, as shown in the above figures.

8 Conclusions

This study explores an emerging advance selling strategy that incorporates a trade-in program. It examines the seller's optimal pricing decisions and expected profits under different strategies: AT, AS, and TN. We find that, to determine an optimal price, sellers must consider the durability and salvage value along with the basic cost of the product. Intuitively, the profit performance monotonically decreases with the product cost under all strategies and increases with the salvage value of the used product under the AT and TN strategies. An interesting result is that the impact of product durability on the seller's profit is nonmonotonic under the AT strategy. This means that the seller can obtain the maximum total expected profit when product durability is moderate.

Our research provides several managerial insights and guidance for sellers in practice. We explore the necessary and sufficient conditions under which sellers should adopt the AT strategy. Depending on the product's durability, salvage value of the used product, and product cost, it may be optimal to offer an AT program for one product but not for another. Compared to AS, the seller is more inclined to adopt an AT strategy when the product durability is sufficiently low and the salvage value of the trade-in product is sufficiently high, or the product cost is relatively low. Compared to TN, it is more profitable for the seller to employ the AT strategy when the salvage value and product cost are relatively low; simultaneously, the product durability is lower than the salvage value. However, we also find that in some cases, the AS and TN strategies outperform AT. In particular, when the product cost is high and salvage value is sufficiently low, the seller should choose AS instead of the AT strategy. When the salvage value and product cost are relatively high, the seller is less inclined to adopt AT than the TN strategy.

Our results explain JD.com's current practice of offering an AT program to consumers. The product durability of the iPhone SE is relatively low because it is a low-end product among iPhone models, which is the main factor driving the seller to adopt an AT strategy. Furthermore, it is plausible that the optimal price could be extremely high when the product cost is low or moderate. If so, the seller can set high prices to generate higher total profit, which explains the extreme situation in Fig. 1, in which the trade-in value equals the original price of the iPhone.

As an initial attempt to examine the implications of a new advance selling strategy incorporating a trade-in program, our model has several limitations that require further examination. First, our study assumes that consumers have a homogeneous belief regarding the valuation distribution. Therefore, future work should model heterogeneous beliefs regarding the valuation. Second, the AT strategy is investigated from the viewpoint of a natural research question, namely whether and when AT can benefit the seller. Future studies should extend our work by including additional consumer behavioral characteristics, such as trust and regret. Finally, as this study considers a monopolist seller, it would be interesting to examine the performance of the AT strategy in a competitive setting.

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Conflict of interest

The authors declare that they have no conflict of interest.

Biographies

Jie Wu received his Ph.D. degree from the University of Science and Technology of China (USTC). He is currently a Professor at School of Management, USTC. His research interests focus on operation management, supply chain management, and optimization.

Zhixin Chen is currently a Ph.D. candidate at the School of Management, University of Science and Technology of China. His research interests focus on operation management and marketing interface.

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Appendix

Proof of Proposition 4.1. As shown in Section 4, the problem can be rewritten as follows:

$$\Pi=\min\left\{-N(p-t+s-c)\left(1-\frac{p-t}{1-\delta}\right)-N(p-c)(1-p)-N(p-c)\right\}$$

$$\text{s.t.}\begin{cases} 2p(1+t-\delta)-(1+t-\delta)^2+p^2(\delta-2)\delta\leqslant 0,\\ t-p\leqslant 0,\\ p-t-1+\delta\leqslant 0,\\ -p+1-\delta\leqslant 0,\\ p-1\leqslant 0,\\ -t\leqslant 0.\end{cases}$$

This is a nonlinear programming problem, but we can use the Karush-Kuhn-Tucker (KKT) conditions to find its feasible solutions. We then exclude solutions that do not satisfy the following constraints: $U_A\geqslant U_W$, $U_A\geqslant 0$, $0\leqslant\frac{p-t}{1-\delta}\leqslant 1$, $\frac{p}{1-\delta}\geqslant 1$, $p\leqslant 1$, $t\geqslant 0$. In particular, we must ensure that profits are greater than 0. Therefore, we can determine the optimal product price and trade-in value t as:

(ⅰ) When $1-s-5\delta+2\delta^2\geqslant 0$, if $c\in\left(0,\frac{1+s+\delta-s\delta-3\delta^2+\delta^3}{2}\right)$, then $p=1-\delta$ and $t=(\delta-1)^2$. Correspondingly, $\Pi=N(1-2c+s+\delta-s\delta-3\delta^2+\delta^3)$. Otherwise, $\Pi\leqslant 0$;

(ⅱ) When $s+\delta\leqslant 1$ and $1-s-5\delta+2\delta^2\leqslant 0$, if $c\in\left(0,\frac{(3+s-\delta)^2}{8(2-\delta)}\right)$, then $p=\frac{3+s-\delta}{2(2-\delta)}$ and $t=\frac{1+s+\delta}{2}$. Correspondingly, $\Pi=\frac{N(3+s-\delta)}{4(2-\delta))}-2Nc$. Otherwise, $\Pi\leqslant 0$;

(ⅲ) When $s+\delta\geqslant 1$, if $c\in\left(0,\frac{1+s}{2}\right)$, then $p=1$ and $t=1$. Correspondingly, $\Pi=N(1-2c+s)$. Otherwise, $\Pi\leqslant 0$.

Proof of Lemma 5.1. Consumers buy early if and only if $U_A\geqslant U_W$ and $U_A\geqslant 0$.

Since $U_A-U_W=d-\frac{p^2(1-\delta)}{2}$, we have $d\geqslant\frac{p^2(1-\delta)}{2}$. Since $p\geqslant 0$, d is linearly increasing in p. Then,

$$\min\{U_A\}=\frac{(1-p)(1-p(1-\delta)+\delta)}{2}\geqslant 0.$$

$\frac{\partial\Pi_{AS}}{\partial d}\leqslant 0$, Π_{AS} is strictly decreasing in d, implying that the optimal discount d is $\frac{p^2(1-\delta)}{2}$.

Proof of Proposition 5.1. Utilizing Lemma 5.1, we can re-

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write the expected profit function as follows:

$$\Pi_{\mathrm{AS}} = N\left((2-p)(p-c) - \frac{p^2(1-\delta)}{2}\right).$$

First, Π_{AS} is strictly concave in p and $1-\delta \leqslant p \leqslant 1$. By comparing the first-order condition (FOC) point to the boundary points, we can determine the optimal price and discount d for the case with $1-4\delta+\delta^2 \geqslant 0$ as

$$(p,d) = \begin{cases} \left(1-\delta, \dfrac{(1-\delta)^3}{2}\right), & \text{if } c \in (0, 1-4\delta+\delta^2); \\ \left(\dfrac{2+c}{3-\delta}, \dfrac{(1-\delta)(2+c)^2}{2(3-\delta)^2}\right), & \text{if } c \in (1-4\delta+\delta^2, 1-\delta); \\ \left(1, \dfrac{1-\delta}{2}\right), & \text{if } c \in (1-\delta, 1). \end{cases}$$

The corresponding profit is given by

$$\Pi_{\mathrm{AS}} = \begin{cases} \dfrac{1+3\delta-5\delta^2+\delta^3-2c(1-\delta)}{2}, & \text{if } c \in (0, 1-4\delta+\delta^2); \\ \dfrac{4+c^2-4c(2-\delta)}{2(3-\delta)}, & \text{if } c \in (1-4\delta+\delta^2, 1-\delta); \\ \dfrac{1-2c+\delta}{2}, & \text{if } c \in (1-\delta, 1). \end{cases}$$

Second, we consider the case $1-4\delta+\delta^2 < 0$.

$$(p,d) = \begin{cases} \left(\dfrac{2+c}{3-\delta}, \dfrac{(1-\delta)(2+c)^2}{2(3-\delta)^2}\right), & \text{if } c \in (0, 1-\delta) \\ \left(1, \dfrac{1-\delta}{2}\right). & \text{if } c \in (1-\delta, 1) \end{cases}$$

The corresponding profit is:

$$\Pi_{\mathrm{AS}} = \begin{cases} \dfrac{4+c^2-4c(2-\delta)}{2(3-\delta)}, & \text{if } c \in (0, 1-\delta); \\ \dfrac{1-2c+\delta}{2}, & \text{if } c \in (1-\delta, 1). \end{cases}$$

Therefore, the optimal price, discount, and profit can be expressed as follows:

Proof of Lemma 5.2. By differentiating the second-period profit function of the TN strategy, we obtain

$$\frac{\partial \Pi_2}{\partial t'} = \frac{2N(2p+s+\delta-2t'-c-1)}{1-\delta},$$

$$\frac{\partial^2 \Pi_2}{\partial t'^2} = \frac{4N}{\delta-1} < 0.$$

Therefore, the second-period profit is strictly concave in t', and the FOC point is $t' = \dfrac{2p+s+\delta-1-c}{2}$. The maximization problem of the second-period profit is subject to

$$\begin{cases} 0 \leqslant \dfrac{p-t'}{1-\delta} \leqslant 1, \\ 1 - \dfrac{p-t'}{1-\delta} \leqslant 1-p. \end{cases}$$

The constraint conditions can be rewritten as $p-1+\delta \leqslant t' \leqslant \delta p$. By comparing the boundary points or FOC, we can summarize the optimal trade-in value t' as stated in Lemma 5.2.

Proof of Proposition 5.2. Based on Lemma 5.2, there are three cases:

(i) When $\dfrac{(-1-c+2p+s+\delta)}{2}$ (FOC point of t') $\leqslant p-1+\delta$ (i.e., $c \geqslant 1+s-\delta$), then $t' = p-1+\delta$. Substituting t' in the total profit function, we can rewrite the function as

$$\Pi^1_{\mathrm{TN}} = 2N(p-c)(1-p).$$

Π^1_{TN} is strictly concave in p. By comparing the boundary points or FOC, we can determine the optimal price and trade-in value for this case as follows:

When $s-\delta \leqslant 0$, if $c \in (1+s-\delta, 1)$, then

$$(p^1_{\mathrm{TN}}, t'^1) = \left(\frac{1+c}{2}, \frac{-1+c+2\delta}{2}\right),$$

and the corresponding profit is

$$\Pi^1_{\mathrm{TN}} = \frac{N(1-c)^2}{2}.$$

(ii) When $p-1+\delta \leqslant \dfrac{(-1-c+2p+s+\delta)}{2} \leqslant \delta p$ (i.e., $c \leqslant 1+s-\delta$ and $p \leqslant \dfrac{1+c-s-\delta}{2(1-\delta)}$), then $t' = \dfrac{(-1-c+2p+s+\delta)}{2}$. Substituting t' in the total profit function, we can rewrite the function as

$$\Pi^2_{\mathrm{TN}} = \frac{N(c^2-2c(3+s-2p(1-\delta)-3\delta)+(1+s-\delta)^2+4p(1-\delta)-4p^2(1-\delta))}{2(1-\delta)}.$$

Π^2_{TN} is strictly concave in p. By comparing the boundary points or FOC, we can determine the optimal price and trade-in value for this case as follows:

When $s-\delta \leqslant 0$, if $c \in \left(\dfrac{s}{\delta}, 1+s-\delta\right)$,

$$(p^2_{\mathrm{TN}}, t'^2) = \left(\frac{1+c}{2}, \frac{s+\delta}{2}\right),$$

and the corresponding profit is

$$\Pi^2_{\mathrm{TN}} = \frac{N(2+2s-2c(2+s-2\delta)+(s-\delta)^2+c^2(2-\delta)-3\delta)}{2(1-\delta)}.$$

(iii) When $\dfrac{(-1-c+2p+s+\delta)}{2} \geqslant \delta p$ (i.e., $c \leqslant 1+s-\delta$ and $p \geqslant \dfrac{1+c-s-\delta}{2(1-\delta)}$), then $t' = \delta p$. Substituting t' in the total profit function, we can rewrite the function as

$$\Pi^3_{\mathrm{TN}} = 2N(1-p)(s-2c+p(2-\delta)).$$

It is easy to prove that Π^3_{TN} is strictly concave in p. By comparing the boundary points or FOC, we can determine the optimal price and trade-in value for this case:

When $s-\delta \leqslant 0$, if $c \in (0, \dfrac{s}{\delta})$,

$$(p^3_{\mathrm{TN}}, t'^3) = \left(\frac{2+2c-s-\delta}{2(2-\delta)}, \frac{\delta(2+2c-s-\delta)}{2(2-\delta)}\right).$$

The corresponding profit is given by

$$\Pi^3_{\mathrm{TN}} = \frac{N(2+2c-s-\delta)^2}{2(2-\delta)}.$$

When $s-\delta \geqslant 0$, for any $c \in (0, \dfrac{s}{\delta})$,

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Table A1. Definitions of the regions for Proposition 6.1.

Region	Definition
1	$s+\delta \geqslant 1$ and $\delta \in (0, 2-\sqrt{3})$
2	$s+\delta \leqslant 1$, $1-s-5\delta+2\delta^2 \leqslant 0$ and $\delta \in (0, 2-\sqrt{3})$
3	$1-s-5\delta+2\delta^2 \geqslant 0$ and $\delta \in (0, 2-\sqrt{3})$
4	$s+\delta \geqslant 1$ and $\delta \in (2-\sqrt{3}, 1)$
5	$s+\delta \geqslant 1$ and $\delta \in (2-\sqrt{3}, 1)$

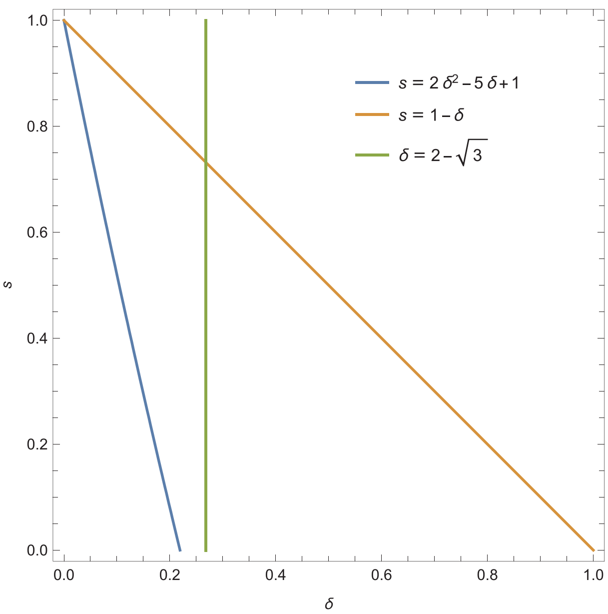

Fig. A1. $\delta-s$ plane.

$$(p_{\rm TN}^3, t'^3) = \left(\frac{2+2c-s-\delta}{2(2-\delta)}, \frac{\delta(2+2c-s-\delta)}{2(2-\delta)}\right),$$

and the corresponding profit is

$$\Pi_{\rm TN}^3 = \frac{N(2+2c-s-\delta)^2}{2(2-\delta)}.$$

Proof of Proposition 6.1. To facilitate our demonstration, the $\delta-s$ plane is divided into five mutually exclusive and collectively exhaustive regions, as shown in Table A1 and Fig. A1. Then, for each region, we compare a firm's profit performance when adopting the AT and AS strategies, as presented in Propositions 4.3 and 5.1, respectively. We summarize the equilibrium results and the optimal strategy for each region as follows:

(i) In Region 1, $p=1$ and $t=1$. Correspondingly, $\Pi = N(1-2c+s)$ (the AT strategy).

(ii) In Regions 2 and 5, there are three cases:

When $s < -3+\delta+\sqrt{2(2-\delta)(3-\delta)}$, if $c \in \left(0, -2+\frac{(3-\delta)(3+s-\delta)}{\sqrt{2(2-\delta)(3-\delta)}}\right)$, then $p=\frac{3+s-\delta}{2(2-\delta)}$ and $t=\frac{1+s+\delta}{2}$. Correspondingly, $\Pi = N\left(\frac{(3+s-\delta)^2}{4(2-\delta)}-2c\right)$ (the AT strategy). If $c \in \left(-2+\frac{(3-\delta)(3+s-\delta)}{\sqrt{2(2-\delta)(3-\delta)}}, 1-\delta\right)$, then $p=\frac{2+c}{3-\delta}$ and $d=\frac{(2+c)^2(1-\delta)}{2(3-\delta^2)}$. Correspondingly, $\Pi=\frac{N(4+c^2-4c(2-\delta))}{2(3-\delta)}$ (the AS strategy). If $c\in(1-\delta, 1)$, then $p=1$ and $d=\frac{1-\delta}{2}$. Correspondingly, $\Pi=\frac{N(1-2c+\delta)}{2(3-\delta)}$ (the AS strategy).

When $-3+\delta+\sqrt{2(2-\delta)(3-\delta)} \leqslant s < -3+\delta+\sqrt{2(2-\delta)(3+\delta)}$, if $c \in \left(0, \frac{5+6s+s^2-8\delta-2s\delta+3\delta^2}{4(2-\delta)}\right)$, then $p=\frac{3+s-\delta}{2(2-\delta)}$ and $t=\frac{1+s+\delta}{2}$. Correspondingly, $\Pi = N\left(\frac{(3+s-\delta)^2}{4(2-\delta)}-2c\right)$ (the AT strategy). If $c \in \left(\frac{5+6s+s^2-8\delta-2s\delta+3\delta^2}{4(2-\delta)}, 1\right)$, then $p=1$ and $d=\frac{1-\delta}{2}$. Correspondingly, $\Pi=\frac{N(1-2c+\delta)}{2}$ (the AS strategy).

When $s \geqslant -3+\delta+\sqrt{2(2-\delta)(3+\delta)}$, $p=\frac{3+s-\delta}{2(2-\delta)}$ and $t=\frac{1+s+\delta}{2}$. Correspondingly, $\Pi = N\left(\frac{(3+s-\delta)^2}{4(2-\delta)}-2c\right)$ (the AT strategy).

(iii) In Region 3, there are three cases:

When $s<\frac{1-\delta(8-3\delta)}{2}$, if $c\in\left(0,\frac{1+2s-\delta^2}{2}\right)$, then $p=1-\delta$ and $t=(1-\delta)^2$. Correspondingly, $\Pi = N(1-2c+s+\delta-s\delta-3\delta^2+\delta^3)$ (the AT strategy). If $c\in\left(\frac{1+2s-\delta^2}{2}, 1-4\delta+\delta^2\right)$, then $p=1-\delta$ and $d=\frac{(1-\delta)^3}{2}$. Correspondingly, $\Pi=\frac{N(1+3\delta-5\delta^2+\delta^3-2c(1-\delta))}{2}$ (the AS strategy). If $c\in(1-4\delta+\delta^2, 1-\delta)$, then $p=\frac{2+c}{3-\delta}$ and $d=\frac{(2+c)^2(1-\delta)}{2(3-\delta^2)}$. Correspondingly, $\Pi=\frac{N(4+c^2-4c(2-\delta))}{2(3-\delta)}$ (the AS strategy). If $c\in(1-\delta,1)$, then $p=1$ and $d=\frac{1-\delta}{2}$. Correspondingly, $\Pi=\frac{N(1-2c+\delta)}{2}$ (the AS strategy).

When $\frac{1-\delta(8-3\delta)}{2} \leqslant s < \frac{1-\delta+6\delta^2-2\delta^3}{2(1-\delta)}$, if $c\in\left(0,\frac{1+2s+\delta-2s\delta-6\delta^2+2\delta^3}{2}\right)$, then $p=1-\delta$ and $t=(1-\delta)^2$. Correspondingly, $\Pi = N(1-2c+s+\delta-s\delta-3\delta^2+\delta^3))$ (the AT strategy). If $c\in\left(\frac{1+2s+\delta-2s\delta-6\delta^2+2\delta^3}{2}, 1\right)$, then $p=1$ and $d=\frac{1-\delta}{2}$. Correspondingly, $\Pi=\frac{N(1-2c+\delta)}{2}$ (the AS strategy).

When $s\geqslant\frac{1-\delta+6\delta^2-2\delta^3}{2(1-\delta)}$, $p=1-\delta$ and $t=(1-\delta)^2$. Correspondingly, $\Pi = N(1-2c+s+\delta-s\delta-3\delta^2+\delta^3)$ (the AT strategy).

(iv) In Region 4, there are two cases:

When $s<\frac{1+\delta}{2}$, if $c\in\left(0,\frac{1+2s\delta}{2}\right)$, then $p=1$ and $t=1$. Correspondingly, $\Pi=N(1-2c+s)$ (the AT strategy). If

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$c \in \left(\frac{1+2s\delta}{2}, 1\right)$, then $p = 1$ and $d = \frac{1-\delta}{2}$. Correspondingly, $\Pi = \frac{N(1-2c+\delta)}{2}$ (the AS strategy).

When $s \geqslant \frac{1+\delta}{2}$, $p = 1$ and $t = 1$. Correspondingly, $\Pi = N(1-2c+s)$ (the AT strategy).

Proof of Proposition 6.2. This proof is similar to that of Proposition 6.1. The $\delta - s$ plane is divided into six regions, as shown in Table A2 and Fig. A2. Then, for each region, we compare the firm's profit performance when adopting the AT and TN strategies, as presented in Propositions 4.1 and 5.2, respectively. The proof is similar to that of Proposition 6.1, therefore, we omit the details.

Table A2. Definitions of the regions for Proposition 6.2.

Region	Definition
1	$s+\delta \geqslant 1$ and $s-\delta \geqslant 0$
2	$s+\delta \leqslant 1$, $1-s-5\delta+2\delta^2 \leqslant 0$ and $s-\delta \geqslant 0$
3	$1-s-5\delta+2\delta^2 \geqslant 0$ and $s-\delta \geqslant 0$
4	$1-s-5\delta+2\delta^2 \geqslant 0$ and $s-\delta \leqslant 0$
5	$s+\delta \leqslant 1$, $1-s-5\delta+2\delta^2 \leqslant 0$ and $s-\delta \leqslant 0$
6	$s+\delta \geqslant 1$ and $s-\delta \leqslant 0$

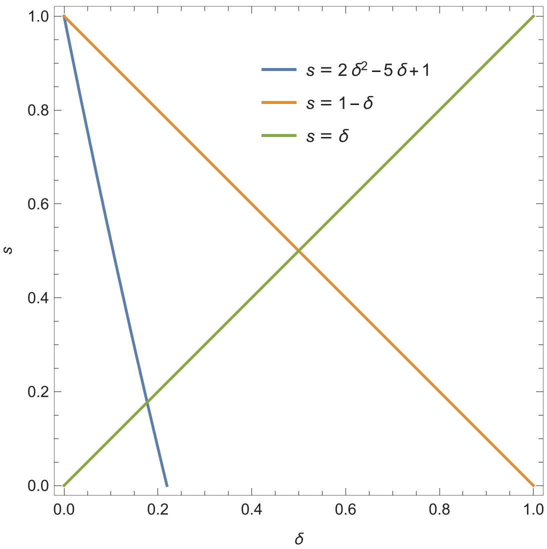

Fig. A2. $\delta - s$ plane.

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