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Spontaneous symmetric breaking in two-lane totally asymmetric simple exclusion process with narrow entrances

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Abstract: Two-lane totally asymmetric simple exclusion process (TASEP) with narrow entrances was studied. The N-cluster mean field analysis was carried out for spontaneous symmetric breaking, in which correlation of $2N$ sites was considered. The analysis indicates that asymmetric LD/LD (low density/low density) phase does not exist. With increase of N, the analytical results are getting closer to the Monte Carlo simulation ones. In addition, it was found that the boundaries between the phases all exhibit exponential change, which enables us to predict the boundaries in the exact solution, and it was shown that the predicted results are in good agreement with the large size simulation ones. The spontaneous symmetric breaking exists when interaction between particles represented by parameter p is strong enough. It was found that the maximum values of entrance rate of asymmetric HD/LD (high density/low density) phase obey exponential decay with increase of p, and the critical value beyond which the asymmetric HD/LD phase disappears can be obtained in every cluster mean field analysis. What's more, the critical values exhibit exponential increase with increase of N, and the exact solution of the critical value was also predicted. It was shown that the predicted critical value is also in good agreement with that obtained from simulations.

Key words: spontaneous symmetry breaking; simple mean field analysis; cluster mean field analysis; asymmetric simple exclusion process

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双道窄入口完全非对称排他过程中自发对称破缺现象的研究

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摘要:研究了双道窄入口完全非对称排他过程.运用了集簇平均场理论来研究了自发对称破缺现象,该方法

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考虑了 $2N$ 个格点之间的相关性. 研究结果显示非对称低密度/低密度相是不存在的；随着每道格点数 N 的增加，理论分析结果越来越接近蒙特卡罗模拟结果。此外，系统中各相的边界均呈现指数变化，据此变化得到了各相边界理论分析的精确值，这些精确值与大尺寸系统下的模拟结果非常吻合. 在该模型中，入口和出口处粒子之间相互作用的强度用 p 来表示，当 p 比较大的时候，该系统才会产生自发对称破缺现象. 在集簇平均场理论分析中，非对称的高密度/低密度相对应的最大入口概率值随着 p 的增加而呈现指数衰减的特征，据此可以得到非对称的高密度/低密度相消失时对应的临界 p 值. 此外，随着 N 的增加，得到的不同的临界 p 值也呈现指数增加的特征，据此可以得到临界 p 值的精确分析值，该分析值与模拟值也非常吻合.
关键词：自发对称破缺；简单平均场分析；集簇平均场分析；完全非对称排他过程

0　Introduction

The totally asymmetric simple exclusion process (TASEP) is a prominent model in study of non-equilibrium system, and its status is similar to the Ising model in equilibrium system[1]. It was initially introduced in the modelling of biopolymerization kinetics[2], and it has received much attention in recent decades because it can be applied in various situations, including microscopic scale such as intracellular transportation[3-8], macroscopic scale such as traffic and pedestrian flow[10-13] and so on. Despite its simplicity, TASEP can reproduce some complex phenomena, such as spontaneous symmetric breaking (SSB)[14-23], phase transition[24-25], phase separation[17,26], shock formation[27-28] and so on.

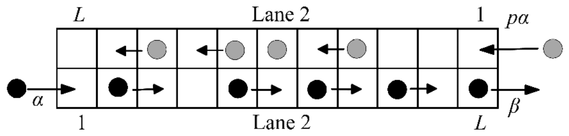
SSB is an intriguing phenomenon which was first found in "bridge model"[14], in which two types of particles move in the same lane with opposite directions. Both particles interact with each other at every site. It is shown that symmetry broken phases exist corresponding to the symmetrical conditions for both particles. However, occurrence of this phenomenon is a not well comprehended. To investigate the influence of location of interaction on SSB, Pronina et al. has studied two-lane TASEP with narrow entrances[15]. In Ref. [15], there are also two species of particles, but they move in two parallel lanes with opposite directions. In the model, changing lanes is forbidden, and the interaction of both particles only exists at the entrance and exit sites. Monte Carlo simulations have shown that one asymmetric phase may disappear in the large size system. The simple mean field (SMF) analysis adopted in Ref. [15] has indicated that the asymmetric phase exists. However, correlation of sites was ignored. Later, cluster mean field (CMF) analysis has been carried out, and the correlation was considered. The analysis has verified the nonexistence of asymmetric LD/LD (low density/low density) phase[18-19]. However, understanding SSB is still a question.

Two-parallel-lane TASEP has also been studied in Ref. [29]. In the model, a parameter p was introduced, and the parameter can control the strength of interaction between particles at the entrance and exit sites. The model in Refs. [15, 18] is the special case of that in Ref. [29]. Simulations have indicated that SSB occurs only when the interaction is strong enough. However, the SMF method was adopted, and existence of one asymmetric phase (i. e., asymmetric LD/LD) can not be validated by the analysis. The theoretical analytical results of phases boundaries deviate remarkably from the simulation ones, especially when the correlation is strong. In addition, the critical value of p obtained from analysis, beyond which the asymmetric phases disappear, also deviates from that obtained from simulation.

In the present paper, we also study the model mentioned in Ref. [29]. We focus on the SSB using N-cluster mean field (N-CMF) method. We not only investigate the existence of asymmetric LD/LD phase, but also study the boundaries between phases. It is found that the boundaries exhibit a charateristic of exponential change, and the results of thermodynamic limit can be predicted. The analytical results and simulation ones are also discussed.

1 Model description

The model consists of two parallel one-dimensional lanes composed of L sites. The sites are numbered from 1 to L, see Fig. 1. In the system, each site has two states: empty or occupied by one particle. Two species of particles move in different lanes, and changing lanes is forbidden. One species of particles move from left to right, and the other move oppositely. The update rule is randomly updated, which means that only one site is chosen at every time step. In the bulk of the system, the particles can move forward if the next site is empty. The particles of exit sites can leave the system with a rate β. The entrance rate depends on states of entrance site of one lane and the exit site of the other lane. When the entrance site is empty and the exit site is empty (occupied), one particle can enter the system with a rate α ($p\alpha$). Here, parameter p represents the strength of interaction between particles. The model of Refs. [15,18] is the special case of the present model in which $p=0$. With increase of p, the interaction deceases, and when $p=1$, the two lanes are independent.



Filled circles represent particles, and arrows show allowed hopping. Exit rates of particles are equal to β. When the entrance site is empty and the exit site on the other lane is empty (occupied), the entrance rate is α ($p\alpha$).

Fig. 1 Illustration of model

2 Analytical and simulation results

In Ref. [29], Monte Carlo simulations have shown that five phases (i. e., asymmetric LD/LD, asymmetric HD/LD (high density/low density), symmetric HD, symmetric MC (maximum current) and symmetric LD phases) exist in the system when p is small. With increase of p, asymmetric LD/LD and asymmetric HD/LD phases shrink, symmetric HD and symmetric MC phases expand, symmetric LD phase expands horizontally and shrinks vertically. Large system size ($L=10\,000$) simulations show that when $p>p_{cr}\approx 0.56$, the two asymmetric phases disappear, which means that SSB does not exist when $p>0.56$.

In Ref. [29], the SMF method was adopted, and the system was considered as two independent single lanes. The correlation of particles was ignored. It was found that the asymmetric LD/LD phase is determined by

$$\frac{\alpha-p\alpha-p\alpha^2+p^2\alpha^2}{1+\alpha+\alpha^2-p\alpha-2p\alpha^2+p^2\alpha^2}<\beta<\frac{1}{6}\Big[2\alpha-4\alpha^2-2p\alpha+4p\alpha^2+\sqrt{(2\alpha-4\alpha^2-2p\alpha+4p\alpha^2)^2+12(\alpha^2-2p\alpha^2+p^2\alpha^2)}\Big] \tag{1}$$

The asymmetric HD/LD phase exists when

$$\frac{p\alpha}{1-\alpha(1-p)}<\beta<\frac{\alpha-p\alpha-p\alpha^2+p^2\alpha^2}{1+\alpha+\alpha^2-p\alpha-2p\alpha^2+p^2\alpha^2} \tag{2}$$

The existence of symmetric HD phase reqiures

$$\beta<\min\left(\frac{p\alpha}{1-\alpha(1-p)},\frac{1}{2}\right) \tag{3}$$

When

$$\beta>\frac{1}{2} \tag{4}$$

and

$$\alpha>\frac{2\beta}{4\beta-1+p} \tag{5}$$

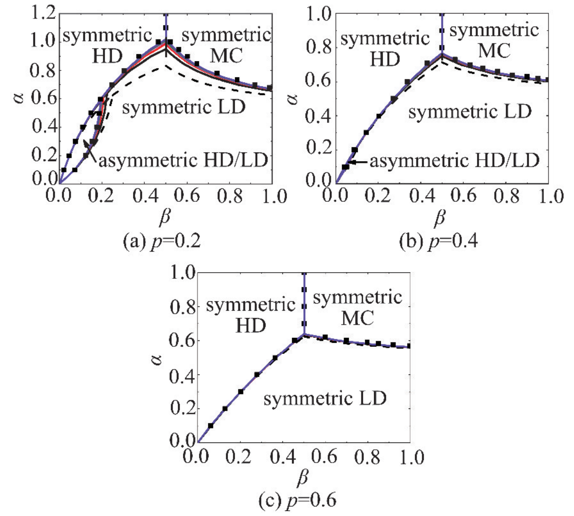
are satisfied, the symmetric MC phase exists.

When

$$\alpha<\frac{2\beta}{4\beta-1+p} \tag{6}$$

the symmetric LD phase exists.

However, because of the ignorance of correlation, the analytical results of phases boundaries deviate remarkably from the simulation ones, especially when p is small in which the correlation is strong, see Fig. 2. In addition, the critical values of p beyond which SSB disappears were obtained from the analysis and simulation, and deviation also exists between the two values.



(a) Monte Carlo simulation (system size $L=10\ 000$) is represented by the scattered points. The SMF analysis is represented by dashed lines, and 1-cluster, 2-cluster, 3-cluster and 4-cluster mean field analysis are represented by black, red, magenta and olive solid lines respectively. The violet solid lines are predicted analytical results in limit.

Fig. 2　The phase diagrams of the model with different *p*

Motivated by these, N-CMF analysis is carried out in this paper, and correlation of $2N$ sites is considered. The $2N$ sites are consecutive, which consist of N consecutive sites on each lane. The N sites of Lane 1 (2) start from the exit (entrance) site.

We first present an analysis of 1-cluster. In the analysis, the entrance site and the exit site of different lanes are considered, see Fig. 3. P_{00}, P_{01}, P_{10} and P_{11} denote four probabilities corresponding to four states in the cluster. P_{00} and P_{11} correspond to the states that two sites are both empty and occupied respectively, P_{01} and P_{10} correspond to the states in which only one site is occupied. p_1 denotes the effective injection probability to the exit site of Lane 1, p_2 denotes the effective removal probability from the entrance site of Lane 2.

We can obtain four master equations.

$$\frac{dP_{00}}{dt}=-(\alpha+p_1)P_{00}+\beta P_{01}+p_2P_{10} \tag{7}$$

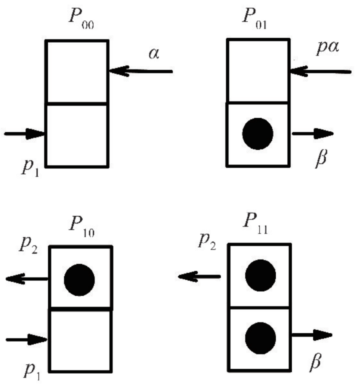


Fig. 3　Four states in the 1-CMF analysis

$$\frac{dP_{01}}{dt}=p_1P_{00}-(\beta+p\alpha)P_{01}+p_2P_{11} \tag{8}$$

$$\frac{dP_{10}}{dt}=\alpha P_{00}-(p_1+p_2)P_{10}+\beta P_{11} \tag{9}$$

$$\frac{dP_{11}}{dt}=p\alpha P_{01}+p_1P_{10}-(p_2+\beta)P_{11} \tag{10}$$

When the system arrives at a stationary state, $dP_{00}/dt=dP_{01}/dt=dP_{10}/dt=dP_{11}/dt=0$. However, only three of the equations are independent. For conservation of the four probabilities, we have

$$P_{00}+P_{01}+P_{10}+P_{11}=1 \tag{11}$$

We first analyze the asymmetric LD/LD phase. The current and bulk density of Lane 1 are denoted by J_1 and ρ_1 respectively, then ρ_1 and J_1 satisfy

$$J_1=\rho_1(1-\rho_1) \tag{12}$$

Let ρ_L denote the density of site L of Lane 1, then we have

$$J_1=\beta\rho_L \tag{13}$$

Furthermore J_1 can be given by

$$J_1=p_1(1-\rho_L) \tag{14}$$

The bulk density of Lane 2 is calculated by

$$\rho_2=P_{10}+P_{11} \tag{15}$$

Then we can obtain

$$p_2=1-\rho_2 \tag{16}$$

Now we have ten variables, i. e., P_{00}, P_{01}, P_{10}, P_{11}, p_1, p_2, ρ_1, ρ_2, ρ_L and J_1. However, we only have nine equations, i. e., Eqs. (7)～(9), (11)～(16). In order to solve the equations, ρ_1 is given, and we can obtain the numerical result of ρ_2. With change of ρ_1, $\rho_2=f(\rho_1)$ can be obtained. At the other end of the system, 1-cluster including the entrance site of Lane 1 and the exit site of Lane 2 satisfies similar conditions. Under this circumstance, when ρ_2 is given, $\rho_1 = f(\rho_2)$ can also be obtained. The boundaries of the left and right can be obtained by the difference of curves of $\rho_2 = f(\rho_1)$ and $\rho_1 = f(\rho_2)$, as has been studied in Ref. [18].

Let J_A, J_B and J_C denote the currents of the system when the system is in asymmetric HD/LD, symmetric LD and asymmetric LD/LD phases respectively, and they are all the sum of currents of the two lanes. The currents of Lane 1 and Lane 2 are $\beta(P_{11} + P_{01})$ and $\alpha P_{00} + p\alpha P_{01}$. Then J_A, J_B and J_C all satisfy

$$J = \beta(P_{11} + P_{01}) + \alpha P_{00} + p\alpha P_{01} \quad (17)$$

When parameters α and β are fixed, values of P_{00}, P_{01} and P_{11} are not the same when the system is in the three phases. Then J_A, J_B and J_C can be compared. In Ref. [18], $p = 0$, correlation is stronger than that when $p > 0$. It has been demonstrated that the asymmetric LD/LD phase does not exist when $p = 0$, nor does it exist when $p > 0$.

In the analysis of the asymmetric HD/LD phase, it is supposed that Lane 1 and Lane 2 are in high density and low density respectively. The Eqs. (7)～(11), (15) and (16) are still valid, and Eqs. (12)～(14) can be replaced by

$$p_1 = 1 - \beta \quad (18)$$

Under this circumstance, there are eight variables, i. e., P_{00}, P_{01}, P_{10}, P_{11}, p_1, p_2, ρ_1, ρ_2, and there are also eight equations, i. e., Eqs. (7)～(11), (15), (16) and (18). When p and α are given, ρ_2 increases with decrease of β. When $\rho_2 = \beta$, the β corresponds to the boundary between symmetric HD and asymmetric HD/LD phases. In addition, the boundary between symmetric LD and asymmetric HD/LD phases can also be obtained when the currents of two phases are equal.

In the symmetric LD phase, the densities of two lanes are equal, $\rho_1 = \rho_2$. In this case, there are nine variables, i. e., P_{00}, P_{01}, P_{10}, P_{11}, p_1, p_2, $\rho_1(\rho_2)$, ρ_L and J_1, and there are also nine equations, i. e., Eqs. (7)～(9), (11)～(16). When p and α are given, $\rho_1(\rho_2)$ increases with increase of β. When $\rho_1 = \rho_2 = \beta(0.5)$, the β corresponds to the boundary between symmetric LD and symmetric HD(MC) phases.

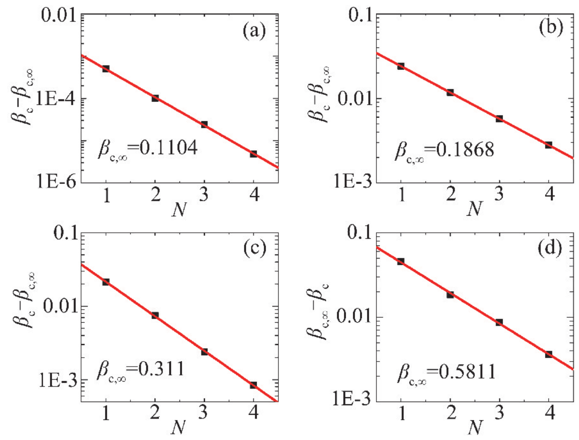
The 1-CMF analytical results is shown in Fig. 2. It can be seen that the results are closer to the simulation ones than those of the SMF analysis. However, deviation between analysis and simulation still exists, especially when p is small. Motivated by this, the correlation of more sites is considered, and N-CMF analysis ($N = 2, 3, 4$) are carried out. In the analysis, there are 4^N master equations with the change of N.

From Eqs. (12)～(14), $p_1 = \beta\rho_1(1-\rho_1)/[\beta - \rho_1(1-\rho_1)]$ is satisfied. When we analyze the asymmetric LD/LD phase in the 1-CMF analysis, ρ_1 is given, and p_1 can be obtained. However, a similar relationship between p_1 and ρ_1 can not be satisfied in N-CMF analysis ($N > 1$) because of correlation. Then p_1 is directly given in the analysis of the phase.

With increase of N, the analysis also verifies the nonexistence of the asymmetric LD/LD phase. The boundaries of other phases are also obtained, see Fig. 2. When $p = 0.2$, it is shown that the results of the analysis are remarkably getting closer to those of simulation with the increase of N. This is because of the the strong correlation. When $p = 0.6$, there is very small difference between the CMF and SMF due to the weak correlation.

In addition, it is found that the boundaries exhibit exponential change with the change of N, see Fig. 4, in which the parameters p and α are fixed, Fig. 4 (a), (b) and (c) show that the values of boundaries are consistent with exponential decay $\beta_c - \beta_{c,\infty} \propto e^{a_1 N + b_1}$, and $a_1 < 0$, $b_1 < 0$. The values decrease with the increase of N. When $N \to \infty$, β_c is equal to $\beta_{c,\infty}$. Fig. 4(d) shows that the values of boundaries are also consistent with exponential decay $\beta_{c,\infty} - \beta_c \propto e^{a_2 N + b_2}$, and $a_2 < 0$, $b_2 < 0$. The values increase with the increase of N. When $N \to$

∞, β_c is also equal to $\beta_{c,\infty}$. When $N\to\infty$, the analytical results tend to be the exact solutions. Fig. 2 shows that the predicted results and Monte Carlo simulation ones in which system size L is 10 000 are in good agreement. In order to validate the prediction, larger system size simulations are needed. However, in very large system simulations, the random-number generator can produce a pseudorandom number series rather than a true random number series[18-19].



(a) The boundary separating the symmetric HD and asymmetric HD/LD phases with parameter $\alpha=0.4$. (b) The boundary separating the asymmetric HD/LD and symmetric LD phases with parameter $\alpha=0.5$. (c) The boundary separating the symmetric LD and symmetric HD phases with parameter $\alpha=0.8$. (d) The boundary separating the symmetric LD and symmetric MC phases with parameter $\alpha=0.9$.

Fig. 4　The relationship between the boundaries and N in the CMF analysis (color online). The parameter $p=0.2$

Simulations show that when p exceeds a critical value p_{cr}, asymmetric phases disappear, and $p_{cr}\approx 0.56$. In the analysis, we also investigate the critical value p_{cr}. In every CMF analysis, it is found that the maximal values of α (i. e. α_{max}) of asymmetric HD/LD phase exhibit exponential decay with the increase of p. Take 1-CMF analysis for example, the α_{max} is consistent with $\alpha_{max}-\alpha_{max,\infty}\propto\exp(ap+b)$, $\alpha_{max,\infty}=-0.66$, $a=-2.1129$, $b=0.69517$, see Fig. 5. When the asymmetric HD/LD disappears, $\alpha_{max}=0$, then we can obtain that $p_{cr}=0.525669$. In addition, it is shown that p_{cr} obtained from N-CMF analysis exhibit exponential increase with the increase of N, see Fig. 6. When $N\to\infty$, p_{cr} is equal to $p_{cr,\infty}=0.555$. The predicted value is in good agreement with that obtained from large size simulations.

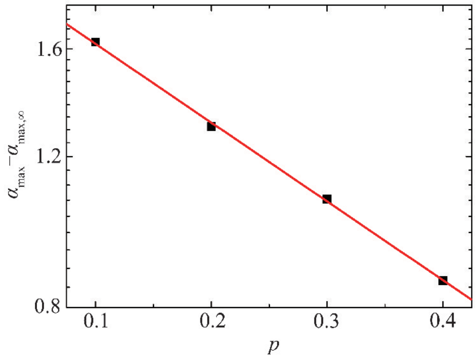


Fig. 5　The relationship between α_{max} and p in the 1-CMF analysis (color online)

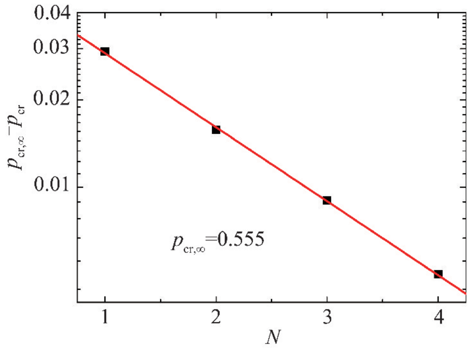


Fig. 6　The relationship between p_{cr} and N in the CMF analysis (color online)

3　Conclusion

To summarize, we have studied TASEP in two parallel lanes with narrow entrances. Two species of particles move in different lanes with opposite directions, and changing lanes is forbidden. The parameter p is introduced, which represents strength of interaction between particles at the entrance and exit sites. Monte Carlo simulations show that five phases (i. e. asymmetric LD/LD, asymmetric HD/LD, symmetric HD, symmetric MC and symmetric LD) exist in the system when p is small. The interaction decreases with the increase of p. Large size system simulations indicate that asymmetric phases disappear when $p>0.56$. It means that SSB exists only when $p\leqslant 0.56$.

The SMF method can be adopted in the

theoretical analysis. However, the analytical results deviate remarkably from the simulation ones. This is because of the ignorance of correlation. Motivated by this, N-CMF analysis ($N=1,2,3,4$) is carried out in the present paper. The N-cluster consists of N consecutive sites on each lane, and the consecutive sites start from the exit site on one lane and the entrance site on the other lane.

As expected, the analysis indicates that the asymmetric LD/LD phase does not exist. With the increase of N, the analytical results are getting closer to simulation ones, especially when p is small in which the correlation is strong. With the increase of p, the difference between the results of SMF and CMF analysis becomes smaller due to the decrease of correlation. It is found that the boundaries separating phases exhibit exponential change with the change of N. The boundaries are predicted in the thermodynamic limit, which correspond to the exact solutions, and they are in good agreement with simulations obtained from large size systems.

It is found that the maxima α of asymmetric HD/LD phase obey exponential decay with the change of p, which enables us to obtain critical value p_{cr} beyond which the asymmetric HD/LD disappears. What's more, the critical values obtained form N-CMF analysis also exhibit the characteristic of exponential change. The exact solution of the critical value is obtained, and it is also in good agreement with that obtained from large size system simulations.

This study has indicated that the results are more accurate when the correlation is considered. The analysis can be adopted in the other complex systems, and this will be our future work.

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