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# A new generation mechanism of three-band chorus waves in the Earth's magnetosphere

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Abstract: A new generation mechanism of three-band chorus waves in the Earth's magnetosphere was proposed with Van Allen Probes data, where two obvious power gaps can be observed in the spectrograms. The satellite observations reveal that there usually exist two separated electron beams in the direction opposite wave propagation along with three-band chorus waves. The linear theoretical results confirm that the existence of electron beams can indeed cause two minima of growth rates of chorus waves propagating in the opposite direction of electron beams. Therefore, the two power gaps should be formed due to the severe damping of chorus waves at gap frequencies resulting from the two electron beams. This study not only gives a new mechanism to explain the formation of three-band chorus event, but also provides new insights into understanding the well-known power gap at half electron gyrofrequency.

**Key words:** Earth's magnetosphere; three-band chorus waves; power gaps; electron beams; wave-particle interaction

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## 地球磁层中三波段合声波的激发机制

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摘要:地球磁层中存在着三波段合声波,它的频谱中有两个明显的能量间断.利用范艾伦探针的观测数据,

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提出了一种新的激发机制来解释三波段合声波的产生.在观测到三波段合声波时,通常伴随着两束与合声波传播方向相反的电子束流.根据线性理论计算出的合声波的增长率,发现两束电子束流的存在确实能使反方向传播的波动的增长率出现两个极小值.因此,两个特定频率处的合声波被阻尼,从而形成两个能量间断,并导致合声波呈现三个波段的特征.这一工作不仅提出了一种新的机制来解释三波段合声波的产生,还为磁层中经常观测到的合声波在 0.5 个电子回旋频率处的能量间断提供了新的见解.

关键词:地球磁层;三波段合声波;能量间断;电子束流;波粒相互作用

### 0 Introduction

Whistler-mode chorus wave is the most intense electromagnetic emission in the Earth's magnetosphere falling within the frequency range of 0.  $1 \sim 1$   $f_{ce}$ , where the  $f_{ce}$  is the equatorial electron gyrofrequency[1-4]. Chorus waves have long been a hot topic in the magnetospheric physics since they have played an important role in regulating electron dynamics the magnetosphere<sup>[5-8]</sup>. They can scatter low-energy  $(\sim 1 \text{ keV})$  electrons into the loss cone and cause electron precipitation into the upper atmosphere, and this is the dominant cause of diffuse aurora [6-7]. Moreover, they can also accelerate ~100 keV electrons to relativistic energies through the cyclotron resonance, which is the dominant source of relativistic electrons in the radiation belt<sup>[5,8]</sup>. In the inner magnetosphere, chorus waves are commonly believed to be excited by the anisotropy instability of hot electrons injected from the magnetotail plasma sheet[9-13]. Their source region is mainly near the magnetic equator<sup>[14-15]</sup>, but there also exists the high-latitude wave source during the period of strong solar wind pressure[16-17].

One remarkable property of chorus waves is the power gap around 0.5  $f_{\rm ce}$  in the spectrogram<sup>[2,4]</sup>, which remains a mystery since the first in situ observation of chorus wave in 1960s. Generally, chorus waves are divided into two frequency bands by this gap: lower band (0.1  $\sim$ 0.5  $f_{\rm ce}$ ) and upper band (0.5 $\sim$ 1  $f_{\rm ce}$ ). Based on long-term Time History of Events and Macroscale Interactions during Substorm (THEMIS)

observations, Gao et al. [4] has provided a comprehensive description of the power gap. They found that there are about 2/3 of chorus events having a power gap, and the distribution of gap positions indeed peaks at the frequency of  $\sim 0.49$  $f_{\rm ce}$  but covers a broad frequency range from 0.46 to 0.7  $f_{ce}$ . Although there has been some progress made on the generation mechanism of the power gap<sup>[18-23]</sup>, there is still no consensus on this issue. It is naturally thought that the upper band and lower band of the whistler-mode waves are excited by two different anisotropic electron populations, leaving a power gap between them<sup>[19-20]</sup>. Besides, Omura et al. [18,23] suggested that the power gap can be generated by the efficient Landau damping while chorus waves propagate toward the poles after their excitation. The idea of nonlinear wavewave interactions has also been introduced to explain such banded whistler-mode waves, where the harmonics of whistler-mode waves are produced due to lower band cascade, and the 'multiband chorus' commonly observed in the magnetosphere is then formed [13,21-22].

Recently, with both the linear theory and particle-in-cell simulation, Chen et al. [24] proposed that the electron beam/plateau distribution leads to a severe damping of chorus waves around 0.5  $f_{ce}$ . Satellite observations reveal that chorus waves are usually detected along with the electron beam/plateau distribution [25-26]. A statistical study conducted by Chen et al. [26] pointed out that this electron beam/plateau distribution can be caused by either the Landau resonance between chorus waves and electrons or some other processes, like TDS, etc.

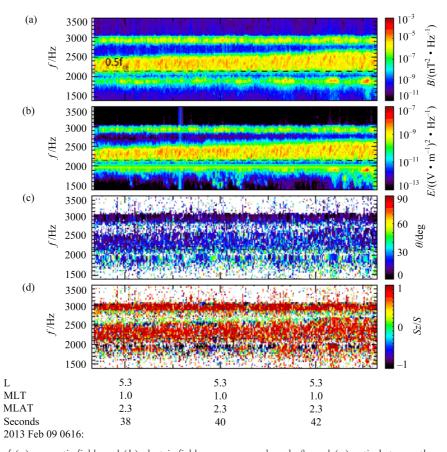
Beside the typical two-band chorus waves, there are also three-band chorus events observed in the Earth's magnetosphere [22,27], meaning there exist two power gaps in the spectrogram. Some of them have been well explained by the resonant interactions between chorus waves. Subcyclotron resonances between chorus waves and electrons were also proposed to explain the formation of the two gaps[22], but this mechanism is not easy to confirm in data. In this paper, we present a novel generation mechanism of three-band chorus waves with Van Allen Probes data. The observation indicates that along with three-band chorus events, there are two beam-like electron populations in the opposite direction of wave propagation, which will lead to the strong damping of chorus waves at two gap frequencies. This mechanism is also confirmed by the theoretical analysis [24,28]. In Section 1, we will describe the instruments onboard Van Allen Probes. The observational results are presented in Section 2. In Section 3, we summarize and further discuss our principal results.

#### 1 Instruments and data

Van Allen Probes, including twin satellites, is operating in a near-equatorial, highly elliptical, and low-inclination orbit with perigees of 1.  $1R_{\rm E}$ and apogees of 5.  $8R_{\rm E}$ . These satellites provide in situ measurements accurate of wave environment and particle fluxes in the Earth's inner magnetosphere. The satellite position information is provided by Science Operations Center in Energetic Particle, Composition, and Thermal Plasma Suite, and the corresponding L, magnetic local time (MLT), and magnetic latitude (MLAT) values are estimated by TS04D model. The Electric and Magnetic Field Instrument Suite and Integrated Science (EMFISIS) investigation<sup>[28]</sup> onboard two probes can provide the high-resolution fields  $(35\ 000\ S/s)$ , electromagnetic resolution magnetic fields (64 S/s), and highfrequency power spectra (1/6 S/s). The lowresolution magnetic fields are treated as the background magnetic field, while the high-resolution magnetic fields are analyzed to obtain polarization information (such as the wave normal angle and pointing flux) of chorus waves with the Means' method<sup>[29]</sup>. The background plasma density is estimated by the upper-hybrid band<sup>[30]</sup> shown in the high-frequency power spectrum. The Helium Oxygen Proton Electron (HOPE) Mass Spectrometer<sup>[31]</sup> provides electron spectra from ~10 eV to tens of keV with the sampling rate of ~1/20 S/s.

#### 2 Observational results

An overview plot of three-band chorus events is shown in Fig. 1, including the power spectrum of (a) magnetic fields and (b) electric fields, wave normal angle  $\theta$ , and (c) ratio between the parallel component of Poyning flux and its total intensity  $S_z/S$ . Here, the positive (or negative)  $S_z/S$ means the wave propagates in the parallel (or antiparallel) direction with respect to the background magnetic field. In Figs. 1(a) and 1(b), we can clearly find there are three separated frequency bands in the spectrograms. Besides the well-known 0.5  $f_{ce}$  power gap, another power gap can also be observed in the upper band (~2 700 Hz). All three bands have small wave normal angles with  $\theta \leq 30^{\circ}$  (Fig. 1(c)), and the same propagating direction, i. e., parallel to the background magnetic field (Fig. 1(d)). For clarity, we have integrated the magnetic power over the entire time interval and present the integrated power as a function of normalized frequency in Fig. 2. Two power gaps can be easily identified, which have been marked by two red diamonds. Note that we use the frequency of power minimum to represent the frequency of the power gaps. So, the gap frequencies of the two gaps are estimated as 0.477 and 0.644  $f_{\rm ce}$ , respectively. Furthermore, the width of Gap 2 is relatively larger than that of Gap 1.



The power spectrum of (a) magnetic fields and (b) electric fields, wave normal angle  $\theta$ , and (c) ratio between the parallel component of Poyning flux and its total intensity  $S_Z/S$ . Here, the positive (or negative)  $S_Z/S$  means the wave propagates in the parallel (or antiparallel) direction with respect to the background magnetic field. In each panel, the dashed lines in black represent 0.5  $f_{ce}$ , where  $f_{ce}$  represents electron cyclotron frequency.

Fig. 1 An overview plot of three-band chorus event

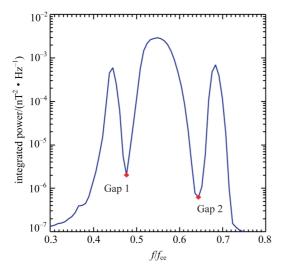


Fig. 2 The integrated power as a function of normalized frequency of the three-band chorus event

To understand the formation mechanism of power gaps, we have also analyzed the electron distributions simultaneously measured by Van Allen Probes. Fig. 3 displays the phase space density (PSD) of electrons as a function of energy in the anti-parallel (black) and perpendicular (red) directions. The cross denotes the measured PSD, while the line is the fitted distribution curve. Two things should be mentioned. Firstly, the electron distribution shown here is recorded at the time of  $06:17:10 \ (\sim 30 \text{ s after } 06:16:43 \text{ UT}), \text{ because}$ the time resolution of electron measurement is relatively low ( $\sim 1/20$  S/s). Secondly, the antiparallel and perpendicular electron PSD are given by the measured electron fluxes in pitch-angle channels of 175.5° and 90°, respectively. Interestingly, there are two clear electron beams located at  $\sim 300$  eV and  $\sim 1.5$  keV in the antiparallel direction, while no clear electron beam is found in the parallel direction (not shown). To calculate the linear growth of chorus waves, we have fitted the electron distribution by summing five bi-Maxwellian distributions and two drifting bi-Maxwellian distributions, and listed the fitting parameters in Tab. 1. As shown in Fig. 3, the fitted electron distribution can well depict the principal structures of the measured distribution.

Tab. 1 The fitting parameters for electron distributions at observation sites (event in Fig. 1)

	$N_{ m e}/{ m cm}^{-3}$	$T_{\parallel}/\mathrm{eV}$	$T_{\perp}/\mathrm{eV}$	$v_{ m b}/v_{ m Ae}$
# 1	3.8	7	7	0.0
#2	1.0	36	120	0.0
#3	0.3	160	700	0.0
# 4	0.2	1000	3000	0.0
# 5	0.3	3300	5000	0.0
# 6	0.006	12	200	-0.21
#7	0.003	20	600	-0.40

[Note]  $B_0 = 153.2 \text{ nT}$ ,  $v_{\text{Ae}} = 6.05 \times 107 \text{ m} \cdot \text{s}^{-1}$ .

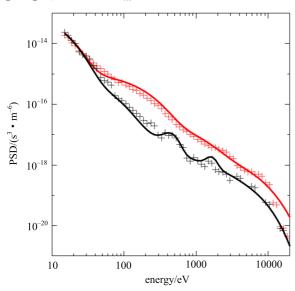


Fig. 3 Parallel (black) and perpendicular (red) electron
PSD measured by HOPE ("+") and
corresponding multicomponent fits (solid lines)

With the linear theory model BO<sup>[32]</sup>, we have calculated the linear growth rate and dispersion relation of chorus waves using the parameters listed in Tab. 1. The results are given in Fig. 4. In Fig. 4(a), it is shown that there are two minima of the linear growth rate at  $kc/\omega_{\rm pe}\approx 1.44$  and 1.91, corresponding to the frequencies of 0.564 and

0.690  $f_{ce}$ , respectively (Fig. 4 (b)). Although there is some deviation between the observational and theoretical gap frequencies, the theoretical results still provide strong support for the severe damping of chorus waves at gap frequencies due to the two electron beams shown in Fig. 3. Moreover, the different width of the two gaps in Fig. 2 can also be well explained by the profile of the linear growth rate given in Fig. 4(a). The first electron beam with the smaller drift velocity causes the wave damping in a broader frequency range (Fig. 4(a)).

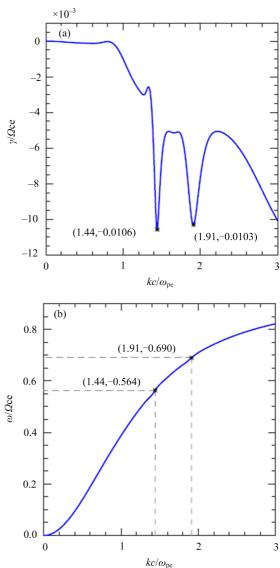
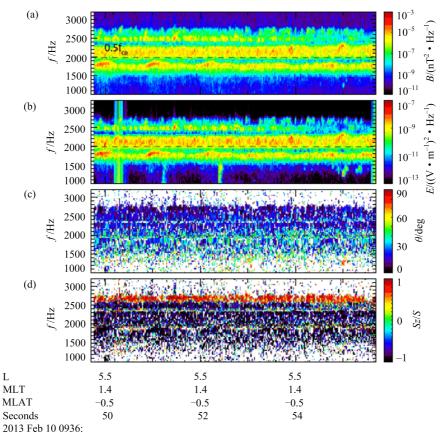


Fig. 4 (a) linear growth rates  $\gamma/\Omega_{\rm ce}$  and (b) wave frequency  $\omega/\Omega_{\rm ce}$  as a function of dimensionless wave number k when propagation direction  $\theta$  is set as  $16^{\circ}$  (calculated by the power-weighted average)

In addition, another three-band chorus event is illustrated in Fig. 5 to further support our proposed mechanism that two electron beams lead to the formation of two gaps. Fig. 5 displays the power spectrum of (a) magnetic fields and (b) electric fields, wave normal angle  $\theta$ , and (c) ratio between the parallel component of Poyning flux and its total intensity  $S_z/S$ . Similarly, we can also observe two clear power gaps in the spectrograms (Figs. 5(a) and 5(b)), which divide the chorus waves into three separated frequency bands. These waves have small wave normal angles with  $\theta \le 30^{\circ}$  (Fig. 5(c)), and also have the same propagating direction (anti-parallel direction; Fig. 5(d)). It is worth noting that there is also a frequency band having the opposite propagating direction (Fig. 5(d)), but it is not relevant to this study. The electron distributions in the parallel and perpendicular directions are shown in Fig. 6(a). Just as expected, we can also observe two separated electron beams along with this three-band chorus event. The measured electron distribution has also been fitted by summing several bi-Maxwellian distributions drifting bi-Maxwellian distributions. Then, with the linear theoretical model, we have also calculated the linear growth rate and dispersion relation of chorus waves and presented results in Figs. 6(b) and 6(c). According to the profile of linear growth rates in Fig. 6(b), the theoretical gaps (i. e., local minima) are at frequencies of 0.523 and 0.668  $f_{\rm ce}$  , which are consistent with the observed gaps, i. e., 0.482 and 0.603  $f_{\rm ce}$ .



The power spectrum of (a) magnetic fields and (b) electric fields, wave normal angle  $\theta$ , and (c) ratio between the parallel component of Poyning flux and its total intensity  $S_Z/S$ . The format is same as Fig. 1.

Fig. 5 Another three-band chorus event

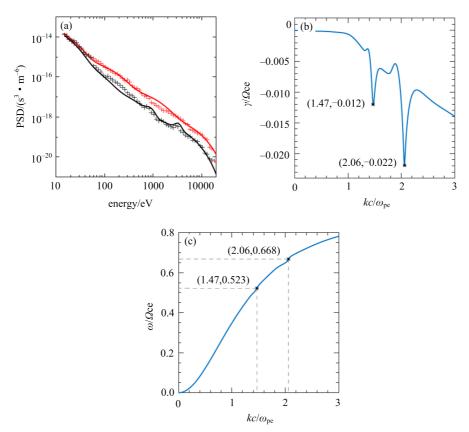


Fig. 6 (a) electron PSD, (b) linear growth rates  $\gamma/\Omega_{cc}(\theta)$  is set as 22°) and (c) wave frequency  $\omega/\Omega_{cc}$  as a function of  $kc/\omega_{pc}$ 

#### 3 Conclusion

With Van Allen Probes data, we have studied three-band chorus waves in magnetosphere, where two obvious power gaps can be identified in the spectrograms. The electron measurements reveal that there usually exist two separated electron beams in the direction opposite wave propagation along with three-band chorus waves. We proposed that the two power gaps are formed due to the severe damping of chorus waves at gap frequencies resulting from the two electron beams. The linear theoretical results confirm that the existence of electron beams can indeed cause two minima of growth rates of chorus waves propagating in the opposite direction of electron beams.

The formation mechanism of the power gap around 0. 5  $f_{ce}$  is a long-standing problem in magnetospheric physics<sup>[2,4]</sup>. Although there have been several potential mechanisms proposed in previous works<sup>[18-23]</sup>, such as the Landau

 $damping^{[18,23]}$ , nonlinear wave-wave coupling^{[21-22]} and different sources[19-20], none of them has been widely accepted. Recently, Chen et al. [24] have proposed that the power gap of chorus waves can be formed by electron beam/plateau distribution, which has been confirmed by both theory and simulations. The three-band chorus events are special events in the Earth's magnetosphere, but figuring out their generation mechanism can help us better understand the mysterious power gap. The satellite observations indicate three-band chorus events are accompanied with two separated electron beams (Fig. 3), which can cause the damping of chorus waves at gap frequencies according to the linear theoretical results (Fig. 4). Therefore, our study not only gives a new mechanism to explain three-band chorus events, but also provides observational evidence for the potential mechanism proposed by Chen et al. [24]. However, more observational evidence is needed to support the mechanism that the power gap can be formed by the electron beam/plateau distribution.

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