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Optimal time-locked trial strategy for software in the presence of piracy

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Abstract, Time-locked trial is one of the commonly used marketing strategies in the commercial software industry. After the trial, one problem facing software companies is possible pirates by users. A model is proposed to study how a monopolistic software supplier should respond to different pirating conditions by use of a time-locked trial strategy. The conditions are determined under which a trial strategy is optimal and was fund that if there is piracy, when the customer's basic perception about the quality of the software is moderate, in the whole piracy region it is better for the company to offer trial while when the basic belief is relatively high or low, the company does not provide trial period unless the piracy cost is relatively tiny. It was also fund that the optimal trial period length decreases with the piracy cost when there is piracy, while the trend is reversed when there is only a threat of piracy. The optimal price increases with the cost of piracy when there is piracy or the threat of it. Versioning strategy and time-locked trial strategy were compared in the presence of piracy and it was fund that the latter is more applicable to enhancing profit. Reasons for these results and some managerial implications are given.

Key words: marketing strategy; pricing; software; piracy; time-locked trial

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0 Introduction

Piracy is a top concern the commercial software companies around the world because it does harm to the profit and the industry's development. The Internet has made piracy easy because the spread of P2P and illegal cracking lower the search cost and time cost of pirating^[1,2]. According to a report of the BSA (business software alliance)^[3], the global software piracy rate was 39% in 2015. In the last five years, this rate has stayed at around 40%, and no evidence indicates that it will go down. The BSA declared that unlicensed software has swallowed up 52242 million dollars worldwide in 2015. Of course, software companies would not allow piracy to win over users and damage the market without doing something. In fact, they have adopted many measures to combat piracy. These measures can be divided into three areas: law, technique, and marketing strategy^[4]. For example, governments legislate to protect intellectual property so companies can prosecute pirates^[5]. A company can use an activation code or key and detection means to raise the cost of piracy and exploit more effective anti-piracy methods [6,7]. Although such methods are sometimes useful, they are also time-consuming

and expensive. Unfortunately, in most circumstances, piracy is not likely to be eliminated completely. As a result, managers have to take piracy into account when making decisions on marketing strategies.

Free trial is one of the widely-used marketing strategies, especially in the software industry. In the physical goods context, firms can give away free samples to customers[8] and intuitively this generates a marginal cost. For information goods, e.g., software, a trial is also a useful tool but unlike a physical product, the marginal cost of a software free trial can be considered as zero or negligible. There are two main kinds of free trials in the context of software. One is a permanent but function-limited trial. Suppliers provide at least two versions of the software: a full-functionality version and a low-capability version with some functions missing. The other trial strategy is called timelocked trial. Users can experience the software with its entire functionality for a limited period of time and they do not have to pay for the software until the trial period expires. They try the software and then decide whether to pay to continue using it. Strategies that users pay the full price and the company only charges the price after a trial period if users do not cancel it are not included. We can observe many cases of software companies implementing these two trial strategies in practice. MathType is a graphical editor for mathematical equations. The supplier allows users to try it for 30 days and then users must pay \$97.00 for a permanent usage license. Microsoft Office 2007 offers a 60-day trial and Windows 7 provides an even longer trial of more than 180 days. As for functionlimited trial strategy, Foxit Reader can be used free of charge permanently but if the user wants more functions like editing PDFdocuments transforming PDF into other formats, the user must pay for it. The two trial strategies have proved to be useful and profit-enhancing in some circumstances.

However, customers who do not want to pay for the full-functionality software or when the trial period ends will choose to a pirated version. Therefore, this study focuses on the time-locked trial strategy in the context of piracy. The goal is to discuss the issue of how the company should respond to piracy and adapt its trial decisions to alleviate the loss piracy causes as much as possible. In this study we try to answer the following research questions:

Is offering a trial period always better than no trial? In other words, under what conditions should a software company provide trial period?

What is the optimal length of trial time and what is the associated price if the company offers a trial?

How do the optimal trial period and the optimal price vary with the piracy cost?

The answers are of significance for software companies worrying about the impact of piracy. To solve these problems, we build a model based on the work of Cheng and Liu^[9] and further introduce an environmental factor, piracy cost, into the previous model. Our main assumptions all follow those of prior research. We assume that customers are evenly distributed in their valuations and piracy cost is mostly dependent on the economic and legal environment. We also assume the users' belief about the true quality of the software increases linearly during the trial period. In our model, a monopolistic software company sells its product and determines whether to offer a free trial period and the price of the software at the beginning of the selling period. Users who try the software have to decide to purchase it, to pirate it or no longer use it when the trial period ends. We study the application range of time-locked trial strategy with different levels of piracy. We also discuss how the optimal trial period and the optimal price vary with the piracy cost and compare versioning and time-locked trial to find out which is more profit-enhancing and more applicable.

Our research provides some managerial implications into whether the software firm should offer a time-locked free trial. First, the combination of time-locked trial strategy and pricing can be a useful method most of the time to mitigate the harm of piracy as a complement to legal and technical means. When it is in the firm's best interest to introduce the time-locked free trial to the market when piracy exists, the software company should charge a higher price and offer shorter free trial time with the cost of piracy getting higher. The profit increases when piracy cost rises. Second, time-locked a free trial is not a cure-all. Under some conditions, a free trial is not necessary at all. Whether to offer a free trial depends on four factors: the cost of using pirated software, the quality of pirated software, consumers' prior perception, and unit increment of perception about functionalities during the free trial. In general, only when the positive effect of the trial that enhances the attractiveness of legal software dominates the negative effects of the trial that postpones the sales season should the free trial be offered. The detailed application range of free trial is given in our research. Finally, the time-locked trial is more applicable than versioning with piracy considered. In other words, the time-locked free trial strategy analyzed in our model outperforms the versioning strategy except under some strictly defined conditions, e.g., when the piracy cost is relatively small in the threat region.

The remainder of this paper is organized as follows. In Section 1, we review existing research related to our study. In Section 2, we explain a basic model to describe the issues we focus on. In Section 3, we provide the solutions of the model and show some helpful results. In Section 4, we compare the time-locked trial strategy and versioning strategy in the presence of piracy through numerical analysis. Section 5 concludes the paper and proposes managerial implications, after which we discuss some possible future research directions. Proofs for all propositions are given in the

Appendix.

1 Literature review

Previous literature related to our research can be divided into two parts: one is about piracy and the other is about trial strategy. The first part focuses on the effect of piracy and how the company reacts. Besides many reports and some empirical research indicates that piracy harms the profit of the company, some researches state that piracy does not always hurt profits and can even be profit-increasing under some strictly defined conditions[10,11], which is contrast to common intuition. Shy and Thisse^[12], Conner and Rumelt^[13] show that considering the positive network externality, piracy can be beneficial because it can expand the market. Besides, more users can bring into play the word-ofmouth effect more effectively. What we must note, however, is that the profit-increasing effect of piracy can be achieved only when there are few illegal users^[14]. Prasad and Mahajan^[15] also emphasize that the company should tolerate a little piracy just at the start of the sales season. Before the product has diffused over a half of the sales season, the company should strengthen its effort against piracy because piracy does some good in the expansion stage but does harm thereafter. In the presence of piracy, the software supplier can try to alleviate the harm of piracy and even take advantage of it by resorting to technical measures and lawsuits. The enforcement level of anti-piracy has been discussed in several previous researches. Some show how much investment a firm should put into piracy control as a complementary or substituted strategy of versioning^[16].

However, the two measures just mentioned are costly and time-consuming, so the company should consider adjusting its marketing strategies to fight piracy. A widely-used strategy is versioning: The company provides several versions of the software with different quality and correspondingly different prices to win over the users who are inclined to pirate^[17]. Versioning should be adopted under some conditions^[18]. Lahiri and Dey^[19] show that piracy itself can exclude the low-valuation part of the market, so the company can tolerate small-scale piracy to serve only the high-valuation part. When piracy gets more severe, then versioning should be used to combat the cannibalization effect of piracy. Wu and Chen^[20] provide a general model to give

guidance about how many versions to offer and, at what quality levels and prices. No matter whether the piracy cost is symmetric for different types of customers, versioning can remedy the profit loss due to piracy. If the cost is symmetric, when the piracy cost is relatively low for the high-type customers, then versioning can separate the two types through self-selection, although there must be a sufficient proportion of the high-type customers to guarantee versioning can enhance profit. If the piracy cost is asymmetric, piracy is done not only by the high-type customers but also by the low-type ones, and the company should adjust versioning strategy accordingly. Cho and Ahn^[21] further point out that when confronted with piracy, a company has an incentive to choose a lower quality for the high version but a higher quality for the low version.

The second part focuses on the prior literature concerns free trial strategies. One popularly adopted trial method is to provide function-limited software to users, who can then pay to get full service. Another one is to offer time-locked software provided with an option to pay to get permanent service. As for the former, Cheng and Liu^[9] provide an analytical model to design the functionlimited strategy. They find that when the positive effect of network externality dominates the negative cannibalization effect, a free trial version should be They also provide a solution for offered. determining how to set the quality of the trial version. There is relatively less research about the time-locked trial strategy. Heiman and Muller [22] perform related research in the context of motor vehicles and computer hardware. They discuss the optimal demonstration time before customers buy the product and found that the probability of purchase after demonstration goes up and then goes down as the demonstration time increases. Cheng and Liu^[9] further discuss the trial period strategy in the context of the software industry, which is relevant to our research. They elucidate when the company should offer a trial period and the corresponding optimal length of free trial time. They compare function-limited trials with timelocked trials to find under what conditions a strategy is more likely to maximize profit. However, the optimal trial strategy in an environment with piracy has not been fully revealed. Another relevant research made by Chellappa and Shivendu^[23] studies the optimal sampling strategy in vertically segmented markets. They model a two-stage process of users' decisions to buy after a trial or pirating. The quality of the software can be underestimated or overestimated by users during the trial period and these two kinds of software evaluations require setting different prices and sampling strategies. They find that piracy causes more severe damage to overestimated software than underestimated, so the company should invest more in piracy control if the software is overestimated. A sampling strategy is not always beneficial to suppliers; it is advantageous only under strictly defined conditions. Different from our study, Chellappa and Shivendu^[23] focus on the optimal sampling size namely the best function-limited strategy.

To the best of our knowledge, few researchers have taken piracy into consideration when discussing the time-locked trial strategies of software companies. In this study, we incorporate piracy in our analytical model and investigate the optimal time-locked trial strategy of the software company at different levels of piracy.

2 Model formulation

We consider a scenario where a monopolistic original software supplier sells its product at a price P considering the possibility of piracy. The supplier provides customers with a period of time t for free trial ($t \ge 0$). For tractability, when the solution of optimal trial period length is equal to 0 we treat it as a no-trial strategy. Before the trial, customers have a basic belief s about the quality (functionality) of the software. During the trial period, each customer's belief increases at the speed of δ . After the trial, the customer's belief about the quality of the software becomes $s+t\delta$. The real quality of the software after the whole span of life is s_0 and trial users may not perceive the total quality of the software after trial, so we have $s + t\delta \leqslant s_0$. We assume that the customers' goal is to maximize their surplus and there is no switching cost if users shift to the pirated version after the trial. At first all customers choose to experience the trial since it is the best choice for them. They would not pay or pirate until the trial ends. We set the lifespan of the software as 1 and the we have t < 1. The valuation of the perceived functionality is distributed uniformly in $(-\infty,1]$ among customers.

We assume that the pirated software is not identical to the original software. The disparities between the pirated version and the original one can be explained in three ways. First, software suppliers provide services to users like technical support and training services while the users of pirated software are excluded. Second, even though the Internet makes pirating easier, the pirated software still may be missing some important features because the original company makes efforts to protect its core intellectual property from being encroached upon. Third, pirated software cannot be upgraded over time. When the customers shift to the pirated version from the original version after the trial, they can only perceive a proportion β of the belief about the functionality of the original software, namely $\beta(s+t\delta)$. To avoid triviality we

assume that $\beta>\frac{2}{3}$. In the numerical analysis section, we assign a relatively high value to β following the assumption of Ref. [19]. The reason why this assumption is rational is that although the pirated version is worse than the legal version, it is good enough to appeal to many users because of the three reasons mentioned above. This assumption is provided to more accurately model practical conditions.

The cost of piracy is denoted as r, containing the cost of searching for the pirated software, the moral cost of users committing an illegal act, and the probable penalty if the piracy is detected and prosecuted. The enforcement degree against piracy can be reflected by the cost of piracy. Then in this study, we use r to describe both concepts interchangeably. We follow the assumption of Ref. [19] that r is basically determined by the economic and legal environment. Although the company also exerts effort to combat piracy, its impact is relatively tiny. There is a great deal of evidence supporting this assumption. In some countries, the laws on intellectual property are undeveloped and corresponding punishment for piracy is not forceful enough. Therefore, the cost of piracy in such environments is low although software suppliers exert the same effort to combat piracy as do their counterparts in countries where intellectual property laws are more developed.

The sequence of events in our model is shown in Fig. 1. At the beginning, the company decides on the trial time length t and the product price P.

Customers decide whether to try the software. When the free trial ends at time t, users have to decide to purchase or pirate the software, or stop using it.

The utility of customers using legal software, denoted U_1 , is

$$U_1 = \theta(s + t\delta) - P \tag{1}$$

The utility of customers using the pirated version, denoted U_2 , is

$$U_2 = \theta \beta(s + t\delta) - r \tag{2}$$

Customers decide to pay or to pirate based on individual rationality (IR) and incentive compatibility (IC) constraints. A customer purchases the legal software if and only if $U_1 > 0$ and $U_1 \geqslant U_2$. A customer pirates if and only if $U_2 > 0$ and $U_2 \geqslant U_1$. Then we have three indifferent points: the indifferent point between users purchasing the software and users pirating it is

$$\theta_1 = \frac{P - r}{(1 - \beta)(s + t\delta)} \tag{3}$$

the indifferent point between users purchasing the software and users who stop using it is

$$\theta_2 = \frac{P}{s + t\delta} \tag{4}$$

and the indifferent point between users pirating the software and users who do not try it at the beginning is

$$\theta_3 = \frac{r}{\beta(s+t\delta)} \tag{5}$$

We assume that the people at point θ_3 pirate and those at the point θ_1 purchase the legal version. To ensure that users who pirate exist, we have this condition: $\theta_3 < \theta_2 < \theta_1$. Then when $P > \frac{r}{\beta}$ the demand for original software is

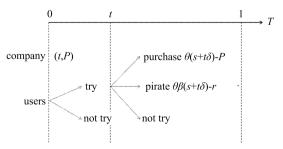


Fig. 1 The timeline of the company and customer decisions

$$D = \left(1 - \frac{P - r}{(1 - \beta)(s + t\delta)}\right) \tag{6}$$

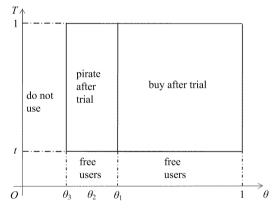
Otherwise, when $P\leqslant \frac{r}{\beta}$, customers find that the utility buying legal software is always better than that using the pirated version and no one will try to pirate. Then the demand for original software is

$$D = \left(1 - \frac{P}{s + t\delta}\right) (1 - t) \tag{7}$$

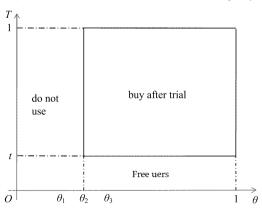
Fig. 2 shows the demand distribution during the lifespan of the software with trial.

Considering the character of software mentioned above, we assume that both the marginal cost of the software and the cost of the trial are negligible. Hence, we have the profit of the software supplier:

$$\Pi = \begin{cases}
P\left(1 - \frac{P - r}{(1 - \beta)(s + t\delta)}\right) (1 - t), P > \frac{r}{\beta} \\
P\left(1 - \frac{P}{s + t\delta}\right) (1 - t), P \leqslant \frac{r}{\beta}
\end{cases}$$
(8)



(a) demand distribution of customers when there is piracy



(b) demand distribution of customers when there is no piracy

Fig. 2 Demand distribution of customers

3 Model solution

We solve our model by working out the optimal price and then the optimal trial period length. According to the principle of profit maximization, the optimal price is

$$P^* = \begin{cases} \frac{r + (1 - \beta)(s + t\delta)}{2}, t > \frac{(2 - \beta)r - s(1 - \beta)\beta}{(1 - \beta)\beta\delta} \\ \frac{r}{\beta}, \frac{2r - \beta\delta}{\beta\delta} \leqslant t \leqslant \frac{(2 - \beta)r - s(1 - \beta)\beta}{(1 - \beta)\beta\delta} \\ \frac{s + t\delta}{2}, t < \frac{2r - \beta\delta}{\beta\delta} \end{cases}$$
(9)

Next, we base our discussions on three situations according to regions divided by the piracy cost: no-piracy region without threat, no-piracy region with threat and piracy region. Proposition 3. 1 details results of the no-piracy situation:

Proposition 3. 1 No-piracy region without threat:

When $r>\frac{s+\delta}{4\delta}$, no piracy exists. When $s<\delta$, the following four formulas hold: The optimal trial period is $\frac{-s+\delta}{2\delta}$, the corresponding price is $\frac{s+\delta}{4}$, the demand for the legal software is $\frac{s+\delta}{4\delta}$, and the profit is $\frac{(s+\delta)^2}{16\delta}$. When $s\geqslant \delta$, the software

company chooses not to offer a trial.

When the cost is high enough, no users will risk pirating, which is consistent with intuition. We find the company chooses to provide a free trial period when $s < \delta$. That means users' prior belief about the quality of the software is small. Users are not sure enough about the true quality of the software so their willingness to pay for it directly is low. They expect to have a chance to try it to enhance their perception of the software and eliminate the uncertainty about quality. When $s \ge \delta$, there is no need to offer a trial because prior valuations are already high enough to entice users to purchase the software. A trial strategy would not play a useful role but would only postpone the sales season, so the supplier chooses not to offer a trial.

When the piracy cost r is moderate and belongs to a certain range, the piracy does not exist but the threat of it does. The trial strategy under this situation is more complex and written more specifically:

Proposition 3.2 No-piracy region with threat:

When
$$r \in \left[\frac{(\beta-1)^2\beta(s+\delta)}{(\beta-2)^2}, \frac{\beta(s+\delta)}{4}\right]$$
,

there is no piracy but the threat of it exists. The company must set the price at $\frac{r}{\beta}$ to guarantee that piracy is prevented. When $s \in [0, (1-\beta)\delta]$, then trial should be offered in the whole threat region. In contrast, when $s \in [(1-\beta)\delta,\delta]$, the company does not use a free trial strategy unless $r \in \left[\frac{\beta s^2}{s+\delta},\frac{\beta(s+\delta)}{4}\right]$. The optimal trial period is $\frac{-\beta s\delta + \sqrt{\beta r s \delta^2 + \beta r \delta^3}}{\beta \delta^2}$ if it is offered and the corresponding profit is

$$\frac{r[r\delta+\beta\delta(s+\delta)-2\sqrt{\beta r\delta^2(s+\delta)}]}{\beta^2\delta^2}.$$

Although there is no piracy yet, the company should adopt very different strategy from the optimal in the no-piracy scenario mentioned in Proposition 3.1. The threat of piracy is always there and piracy will win over some proportion of users as long as the company sets an inappropriate price. Prior research has already established the existence of this condition. A precedent can be seen in Ref. [19]. The price has to equal the ratio of the piracy cost and the quality of the pirated version. Only in this way will users choose the legal version. Under these conditions, the optimal price is no longer relevant to the users' belief about the quality. Piracy is a de facto competitor of the company, although it is latent, and we can consider that the software company is in price competition with piracy.

It is not always better to offer a trial period in this scenario, either. This is the result of the combined action of the pushing effect of the piracy cost and the pulling effect of the trial strategy. To be specific, the cost of piracy pushes users away from pirated versions and the trial pulls them toward legal software. When prior valuation is relatively small, in the range $s \in [0, (1-\beta)\delta]$, a trial strategy can contribute in the whole threat region by enhancing total belief about the quality of the software. However, when users already have high basic confidence in the range $s \in [(1-\beta)\delta, \delta]$, in presence of a low cost of piracy, a trial may not be of significant use and, therefore, the company should avoid the trial strategy. With the same prior

valuation belief, when the cost of piracy is higher, which means the piracy threat is easier to mitigate, a trial strategy could help more in winning over users. High piracy cost pushes users away from the pirated version so that at the same time the introduction of a trial strategy becomes more attractive to them. Then these two effects combine to attract more users and benefit the ultimate profits of the legal software. Consistent with the conditions in the no-piracy region, when s is high enough to be over δ , in the whole threat region there is no need to offer a trial.

Piracy appears as the cost of using pirated software gets lower. In this circumstance, piracy can be seen as a "competitor" of the original software company. The details of the trial strategy in the piracy region are shown as follows:

Proposition 3.3 Piracy region:

When
$$r<\frac{(\beta-1)^2\beta(s+\delta)}{(\beta-2)^2}$$
, piracy encroaches upon the market. The optimal trial period is $\frac{1}{4}\left[1+\frac{-3s+A}{\delta}\right]$. The corresponding optimal price is $\frac{1}{8}[4r+s+\delta-\beta(s+\delta)-\sqrt{(\beta-1)(s+\delta)[8r-(1-\beta)(s+\delta)]}]$. When $s\in\left[(1-\beta)\delta,\frac{\beta}{4-3\beta}\right]$, a trial should be offered in the whole piracy region. When $s\in\left[0,(1-\beta)\delta\right]$ or $s\in\left[\frac{\beta\delta}{4-3\beta},\delta\right]$, the company provides a trial only when $r\in\left[0,\frac{s(\delta-s)(1-\beta)}{s+\delta}\right]$. When the trial is offered, the corresponding profit is $\Pi_p^*=\frac{[-3(s+\delta)+A][-4r-(1-\beta)(s+\delta)+(1-\beta)A]}{64(1-\beta)^2\delta[s+\delta+A]}$ where $A=\sqrt{\frac{(s+\delta)[8r-(1-\beta)(s+\delta)]}{\beta-1}}$.

When r is small enough, some users will choose to pirate. In other words, piracy cannibalizes the market. As we assumed above, since the quality gap is small enough, we may assume that $\beta > \frac{2}{3}$. When there is piracy, conditions get more complex. Identically, these are the results of the coaction of trial strategy and piracy cost. We find that when $s \in \left[(1-\beta)\delta, \frac{\beta}{4-3\beta} \right]$, the company ought to choose to offer a trial in the presence of piracy. When prior belief is moderate, the trial strategy is

helpful as long as there is piracy. When piracy is very small, i. e., $r \in \left[0, \frac{(\beta-1)^2\beta(s+\delta)}{(\beta-2)^2}\right]$, then a trial should be offered when the prior valuation belief satisfies $s \in \left[0, (1-\beta)\delta\right]$ or $s \in \left[\frac{\beta}{4-3\beta}, \delta\right]$.

On the contrary, in the presence of piracy but when piracy cost is a little bigger, it is better not to provide a trial to users with a relatively low or a high prior belief. Faced with an extremely low cost of piracy, as long as the prior belief is smaller than δ , a trial is an effective method to mitigate the impact of piracy and to enhance profit. However, in the presence of piracy but with a higher cost of piracy, a trial strategy is not always necessary: only when prior valuation is moderate is the trial strategy useful for increasing profit. Similar to the conditions in the no-piracy region and the threat region, once prior valuation is over the unit increment of quality perception, there is no need to offer a trial for users. To be clearer, we show the range of application of the trial strategy in the presence of different costs of piracy in Fig. 3.

Now we have the optimal length of the trial period under different situations, how does the optimal trial period length change with the level of enforcement? To investigate this issue, we verify the monotonicity of the function on optimal trial period length and the piracy cost. We state the result as Proposition 3.4:

Proposition 3.4 The optimal trial time length is decreasing with the piracy cost when there is piracy, whereas when there is only a threat of piracy, the optimal trial time length is increasing with the piracy cost.

The optimal trial period is decreasing with the

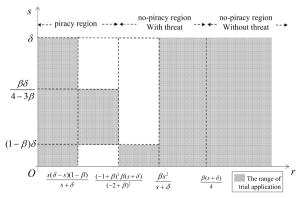


Fig. 3 The range of application of the trial strategy in the presence of different costs of piracy

piracy cost, which means the presence of piracy enhances the incentive for the company to provide a longer trial period. The users' belief about the quality gap between two versions will be bigger if they try the product for a longer time. As the cost of piracy decreases, the company has to offer a longer trial period to add more attraction to the original software. However, faced with the threat of piracy, as the piracy cost increases (which means the piracy threat is reduced because of other considerations) the company should offer a longer trial period can charge a higher price. As the piracy cost grows, the competitiveness of the pirated version decreases so the software company has more power to set a higher price. The monopoly position of the company gets firmer. At the same time, when a higher price is charged, users will not be satisfied unless they obtain more utility so the company offers a longer trial time. When r > $\frac{\beta(s+\delta)}{\delta}$, there is no piracy so it is obvious the optimal trial period is independent of r.

To be clearer, we plot how the optimal trial time varies with piracy cost in Fig. 4. We set appropriate values for related parameters: prior belief, unit increment, and the quality of pirated software, namely s=0.6, $\beta=0.75$, and $\delta=2$. (As mentioned above, β is satisfied with $\beta>\frac{2}{3}$. Prior belief s is moderate belonging to $\left[(1-\beta)\delta,\frac{\beta}{4-3\beta}\right]$ so that a trial period is offered in the whole piracy region. Although the trial strategy varies with different prior valuation beliefs, the trend (monotonicity) is the same, so we choose only the condition when the prior valuation is moderate.

The reason behind this impact of piracy on free

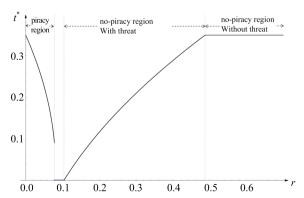


Fig. 4 Optimal trial period varies with the cost of piracy when $s = 0.6, \beta = 0.75, \delta = 2$

trial period is as follows: When confronted with the direct "competition" of the pirated software, the software company offers free trial period software to ensure users have a higher belief about the quality of the original software when they decide whether to purchase the software. As a consequence, when anti-piracy enforcement decreases, it is optimal for the software company to offer a longer free trial period. In other words, when faced with piracy, the lower the degree of piracy, the more incentive there is for the software company to offer a free trial to enhance customer valuation of the software.

To investigate how the optimal price changes with the level of enforcement, we verify the monotonicity of the function on optimal price and the piracy cost. We state the result in Proposition 3.5.

Proposition 3.5 If a trial strategy is adopted, then the corresponding price is increasing in the cost of piracy in the piracy region. In the threat region, the optimal price increases with the piracy cost.

This trend of the optimal price varying with the piracy cost is in line with our conventional wisdom. In the piracy region and the threat region, if the time-locked trial strategy is adopted, as the enforcement against piracy gets higher, the original software suppliers can weed out more piracy and regain portions of its monopoly power as well as pricing power. Therefore, as the cost of piracy gets higher, the software company has more power to charge a higher price.

4 Comparison with versioning strategy

Instead of offering a free trial period to users, the software company can also choose to provide two versions of the software with different qualities and prices aiming at different kinds of users. This versioning strategy is often adopted as the optimal strategy to alleviate the impact of piracy by the software company in many situations. Ref. [19] found that if the software company chooses versioning rather than free trials, then the optimal choice is as follows:

(I) In the no-piracy region without threat, the software company is a monopolist so there is no need to provide two versions. The utility of users is $\theta s-P$. Then the demand is $1-\frac{P}{s}$. Combining these, the profit is $I\!I=P\left(1-\frac{P}{s}\right)$. According to

the principle of profit maximization, we can get the optimal profit: $\Pi = \frac{s}{4}$.

(II) In the no-piracy region with threat, except for the higher version with quality at s_0 , the company provides a lower version with quality βs_0 which is identical to the pirated version to fight against piracy. The utility of higher users is then $\theta s - P$. The utility of lower users is $\theta \beta s - \frac{r}{\beta}$.

Then the demand of higher users is $1-\frac{P-\frac{r}{\beta}}{s(1-\beta)}$. Combining these, the profit is $\Pi=P\left(1-\frac{P-\frac{r}{\beta}}{s(1-\beta)}\right)+\frac{r}{\beta}\left(\frac{P-\frac{r}{\beta}}{s(1-\beta)}-\frac{r}{s\beta^2}\right)$. Then the

optimal profit is $\Pi = \frac{s(1-\beta)}{4} + \frac{r}{\beta} - \frac{r^2}{s\beta^3}$ according to the principle of profit maximization.

(II) In the piracy region, piracy wins over the lower-valuation customers, so it is better to provide only one version to serve the high-valuation customers. The utility of legal users is $\theta s - P$. The utility of pirate users is $\theta \beta s - r$. Then the demand of legal users is $1 - \frac{P - r}{s(1 - \beta)}$ and the profit is

 $I\!I = P(1-\frac{P-r}{s(1-\beta)})$. We can get the optimal profit $I\!I = \frac{[r+s(1-\beta)]^2}{4(1-\beta)s}$ according to the principle of profit maximization.

In Fig. 5, we compare the profit of the software supplier with versioning strategy and time-locked trial strategy. We set s=0.6, $\beta=0.75$ and $\delta=2$. In the piracy region, namely $[0,r_1)$, the software

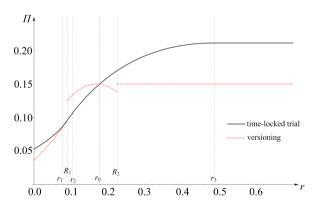


Fig. 5 Comparison of versioning strategy and time-locked trial strategy in the presence of piracy when $s=0.6,\beta=0.75,\delta=2$

company offers trials. In the threat region, the software company offers trials during $[r_1, r_2)$ and during $[r_2, r_3)$ the company does not provide free trials. When $r > r_3$, there is no piracy and the company offers trials. If the software adopt versioning strategy, r_1 will be the indifferent point between the piracy region and the threat region and r_2 will be the indifferent point between the threat region and the no-piracy region.

From Fig. 5, we can know when versioning is optimal and when to offer a trial period. A timelocked trial is not always better than versioning, although it is profit-increasing when the piracy cost is relatively too high or too low. When the cost of piracy is moderate in a particular range, the software company prefers providing two versions instead of offering a free trial. When the piracy cost is moderate, any trial strategy de facto postpones the sales season while versioning can expand demand in the low-valuation end of the market. Therefore, when the piracy cost is in a certain range, for example, when $r \in [R_1, r_0)$, versioning is better than the time-locked trial strategy. The reason for this situation is as follows: in this range, there would be more users if the trial strategy is adopted but the price cannot be higher than $\frac{r}{R}$. The total amount of users of the two versions is less than under the versioning strategy while the highquality version can be priced high enough to obtain more profit from the users and the low-quality version can prevent the piracy from encroaching upon the market.

5 Conclusion

The time-locked trial strategy is prevalently adopted as a trial method. Prior research efforts have established it as a useful strategy to enhance users' utility and ultimately the profit of the company under some specific conditions in a perfect, piracy-free environment. In reality, the market is flawed because of the existence of piracy. Therefore, it is of practical importance to explore how the time-locked trial strategy performs in settings with piracy considered.

We have made some important assumptions in the study. We assume there is no switching cost from the trial version to the pirated version, so at the beginning, all the users choose to try the free software until they have to decide whether to purchase or pirate. We also assume the cost of piracy is correlated to the level of enforcement and it is determined mostly by the economic and legal environment. Furthermore, in keeping with the characteristics of software, we treat the marginal cost and cost during the trial period as negligible. All of these assumptions are usually adopted by important prior research efforts.

Under these assumptions, we have some key We base our discussions on three situations according to regions divided by the piracy cost: piracy region, no-piracy region without threat and no-piracy region with threat. Piracy can be seen as a potential competitor of the original software company as it can threaten the monopoly power of the company to force the company to provide a sufficiently high quality product and not to charge too high. First, we find that when customers' prior belief about the functionality (quality) of the software is high enough, there is no need for the company to offer a free trial period, while when the prior belief is smaller than the unit increment of the perception, situations get more complex: (i) in the no-piracy region without threat, the company had better to offer a free trial; (ii) in the no-piracy region where its threat exists, when the customer's basic perception about the quality of the software is tiny, in the whole region it is better for the company to offer a trial, while when the basic belief is relatively high, the company does not need to provide a trial period unless the piracy cost is relatively high; (iii) in the piracy region, when the customers' basic perception about the quality of the software is moderate, in the whole region it is better for the company to offer a trial, while when the basic belief is relatively high or low, the company does not need to provide a trial period unless the piracy cost is relatively tiny. Second, we state that the optimal trial period length decreases with the piracy cost when there is piracy, while the trend is reversed when there is only a threat of piracy. Third, we also find that in the presence of piracy, the profit of original software using the versioning strategy is smaller than the profit using the time-locked trial strategy except in some strictly defined conditions.

For tractability and without loss of rationality, we have simplified some conditions in our model. We set the valuation of users to be uniform over $(-\infty,1]$ and we use a linear form to describe the

quality gap between the original software and pirated version. As for the belief about the quality of the software, we consider that it increases evenly during the trial period. Future research can relax these assumptions and establish a more general analytical model. In our model, we do not account for network externality so future research could reconsider the problems we discussed in the presence of network externality. More research should focus on the trial strategy issue considering piracy and models should be improved to be more true to reality. Meanwhile, more empirical research should be conducted to support the arguments of related theoretical research efforts.

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盗版影响下的软件产品定价和试用期策略研究

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摘要:首先研究了在盗版的影响下正版软件企业如何调整其定价和试用期策略;基于效用模型,通过决策确定了最优的定价与最优的试用期时长;同时利用算例分析比较了版本划分策略与试用期策略.研究表明,当面对盗版时,如果消费者的基本质量感知处于中等水平,则在整个有盗版的区间内都需要提供试用期;如果基本质量感知较小或者较大,则只有盗版的成本较小时才需要提供试用期.当盗版存在时,软件的最优试用期时长随着盗版成本的增加而减少;当只有盗版的威胁存在时,软件的最优试用期时长随着盗版成本的增加而增加.在有盗版或其威胁的区间内,软件的最优价格随着盗版成本的增加而增加.研究同时发现,除了在某些严格限定的范围内,与版本划分策略相比,试用期策略更适用于正版软件企业提高利润.

关键词:市场策略;定价;软件产品;盗版;试用期策略

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Appendix

Proof of Proposition 3.1

We solve our model by working out the optimal price first. When $P<\frac{r}{\beta}$, the profit is $II=P\left(1-\frac{P}{s+t\delta}\right)(1-t)$.

The first order condition:

$$\frac{\partial \Pi}{\partial P} = \frac{(1-t)(-2P+s+t\delta)}{s+t\delta} \tag{A1}$$

The second order condition:

$$\frac{\partial^2 \Pi}{\partial P^2} = \frac{2(t-1)}{s+t\delta} \tag{A2}$$

Since t < 1, then (A2) is less than 0. Equating A1 to 0 we obtain the optimal price:

$$P^* = \frac{s + t\delta}{2} \tag{A3}$$

Substituting (A3) into $P < \frac{r}{\beta}$, then

$$t < \frac{2r - \beta \delta}{\beta \delta} \tag{A4}$$

We substitute (A3) into $II = P\left(1 - \frac{P}{s + t\delta}\right) (1 - t)$ to get

$$\Pi = \frac{(t-1)(s+t\delta)}{4} \tag{A5}$$

Then, we can get the optimal trial period from

$$\Pi'(t) = \frac{-s + \delta - 2t\delta}{4} = 0 \tag{A6}$$

since

$$\Pi''(t) = -\frac{\delta}{2} < 0 \tag{A7}$$

The optimal trial period is

$$t_n^* = \frac{-s + \delta}{2\delta} \tag{A8}$$

When t>0, we have $\delta>s$. When $t\leqslant 0$, it is better not to offer a trial.

Combining (A8) and $t < \frac{2r - \beta \delta}{\beta \delta}$, we have

$$r > \frac{\beta(s+\delta)}{4} \tag{A9}$$

In this situation, piracy will not occur.

We substitute (A8) into $II = \frac{(t-1)(s+t\delta)}{4}$ to get the corresponding optimal profit is

$$\Pi_n^* = \frac{(s+\delta)^2}{16\delta} \tag{A10}$$

We substitute (A8) into (A3) to get the corresponding optimal price is

$$P_n^* = \frac{s+\delta}{4} \tag{A11}$$

Proof of Proposition 3.2

When $P>\frac{r}{\beta}$, the profit is $I\!I=P\left(1-\frac{P-r}{(1-\beta)(s+t\delta)}\right)(1-t)$.

We can get the optimal price from

$$\frac{\partial \Pi}{\partial P} = 1 - t - \frac{2(P - r)(t - 1)}{(\beta - 1)(s + t\delta)} = 0$$
 (A12)

Since t < 1 and $\beta < 1$, we have

$$\frac{\partial^2 \Pi}{\partial P^2} = \frac{2(t-1)}{(1-\beta)(s+t\delta)} < 0 \tag{A13}$$

Therefore, we have the optimal price when there is piracy:

$$P^* = \frac{r + (1 - \beta)(s + t\delta)}{2}$$
 (A14)

We substitute (A14) into $P>rac{r}{eta}$, then

$$t > \frac{(2-\beta)r - s(1-\beta)\beta}{(1-\beta)\beta\delta}$$
(A15)

The remaining values of t are

$$\frac{2r - \beta \delta}{\beta \delta} \leqslant t \leqslant \frac{(2 - \beta)r - s(1 - \beta)\beta}{(1 - \beta)\beta \delta} \tag{A16}$$

and under these conditions, we have $P = \frac{r}{\beta}$ and it means there is a threat of piracy.

We substitute $P = \frac{r}{\beta}$ into $\Pi = P\left(1 - \frac{P}{s + t\delta}\right) (1 - t)$ to get $\Pi = \frac{r(t - 1)[r - \beta(s + t\delta)]}{\beta^2(s + t\delta)}$ (A17)

We can get the optimal trial period from

$$\Pi'(t) = \frac{\left[r + (1-\beta)(s+t\delta)\right] \left[r(s+\delta) + (1-\beta)(s+t\delta)(s+2t\delta-\delta)\right]}{4(\beta-1)(s+t\delta)^2} = 0$$
(A18)

since

$$\Pi''(t) = -\frac{2r^2\delta(s+\delta)}{\beta^2(s+t\delta)^3} < 0$$
(A19)

The results are

$$t_{1} = \frac{-\beta s \delta + \sqrt{\beta r s \delta^{2} + \beta r \delta^{3}}}{\beta \delta^{2}}$$

$$t_{2} = \frac{-\beta s \delta - \sqrt{\beta r s \delta^{2} + \beta r \delta^{3}}}{\beta \delta^{2}}$$
(A20)

It is obvious that $t_2 < 0$ so it is not a practical solution.

We take $t = \frac{-\beta s\delta + \sqrt{\beta r s\delta^2 + \beta r\delta^3}}{\beta\delta^2}$ into (A16) to get the range when there is no piracy but the threat exists:

$$\frac{(\beta-1)^2\beta(s+\delta)}{(\beta-2)^2} \leqslant r \leqslant \frac{\beta(s+\delta)}{4} \tag{A21}$$

We combine $t_1 > 0$ and $\frac{(\beta - 1)^2 \beta(s + \delta)}{(\beta - 2)^2} \leqslant r \leqslant \frac{\beta(s + \delta)}{4}$ to get the conditions that t is positive meaning it is better to offer a trial and otherwise t is negative and it is better not to offer a free trial.

The result where t is positive is as follows:

$$\frac{(\beta-1)^{2}\beta(s+\delta)}{(\beta-2)^{2}} \leqslant r \leqslant \frac{\beta(s+\delta)}{4}, s \in [0, (1-\beta)\delta]$$

$$\frac{\beta s^{2}}{s+\delta} \leqslant r \leqslant \frac{\beta(s+\delta)}{4}, s \in [(1-\beta)\delta, \delta]$$
(A22)

Therefore, the optimal trial period length is

$$t_t^* = \frac{-\beta s\delta + \sqrt{\beta r s\delta^2 + \beta r\delta^3}}{\beta \delta^2}$$
 (A23)

We substitute (A20) into (A17) to get the corresponding optimal profit

$$\Pi_{t}^{*} = \frac{r[r\delta + \beta\delta(s+\delta) - 2\sqrt{\beta r\delta^{2}(s+\delta)}]}{\beta^{2}\delta^{2}}$$
(A24)

Proof of Proposition 3.3

is

From the Proof of Proposition 3. 1 and Proof of Proposition 3. 2 we can get the range where piracy exists

 $r < \frac{(\beta - 1)^2 \beta(s + \delta)}{(\beta - 2)^2} \tag{A25}$

We substitute (A14) into $\Pi = P\left(1 - \frac{P - r}{(1 - \beta)(s + t\delta)}\right)(1 - t)$ to get $\Pi = \frac{(1 - t)[r + (1 - \beta)(s + t\delta)]^2}{4(1 - \beta)(s + t\delta)}$ (A26)

We can get the optimal trial period from

$$\Pi'(t) = \frac{\left[r + (1-\beta)(s+t\delta)\right] \left[r(s+\delta) + (1-\beta)(s+t\delta)(s+2t\delta-\delta)\right]}{4(\beta-1)(s+t\delta)^2} = 0$$
(A27)

The results are as follows:

$$t_{1} = \frac{r+s-s\beta}{\delta(\beta-1)}$$

$$t_{2} = \frac{1}{4} \left[1 + \frac{-3s+A}{\delta} \right]$$

$$t_{3} = \frac{1}{4} \left[1 + \frac{-3s-A}{\delta} \right]$$
(A28)

Note that $A = \sqrt{\frac{(s+\delta)[8r-(1-\beta)(s+\delta)]}{\beta-1}}$ and $r < \frac{(1-\beta)(s+\delta)}{8}$ is assured when $r < \frac{(1-\beta)^2\beta(s+\delta)}{8}$

$$\frac{(\beta-1)^2\beta(s+\delta)}{(\beta-2)^2}.$$

Obviously, $t_1 < 0$ so it can be excluded.

We put t_2 and t_3 into

$$\Pi''(t) = \frac{\delta[(1-\beta)^2(s+t\delta)^3 - r^2(s+\delta)]}{2(\beta-1)(s+t\delta)^3}$$
(A29)

to verify whether the second derivation is negative.

We solve the set of inequalities

$$\left.\frac{II''(t_3)>0}{r<\frac{(\beta-1)^2\beta(s+\delta)}{(\beta-2)^2}}\right\}$$

and the result is $(3\beta - 2)^2 \ge 0$. This result means that when there is piracy, $II''(t_3)$ is positive permanently and is not the optimal solution.

Similarly, we solve

$$\left. \frac{\Pi''(t_2) < 0}{r < \frac{(\beta - 1)^2 \beta(s + \delta)}{(\beta - 2)^2} \right\}$$

and then we have $(3\beta-2)^2 \geqslant 0$, so when there is piracy $\Pi''(t_2)$ is negative permanently and therefore we can be sure that t_2 is a local maximum. Next, we have to make sure that it is a unique maximum when $t_2 > 0$. $\Pi''(t_2)$ is decreasing in t_2 , so when $t > t_2$, $\Pi''(t_2)$ is negative too. In the other words, at all values of t exceeding t_2 , the first derivative is strictly negative. Hence, t_2 must also be the unique real maximum. Therefore, the optimal trial period length is

$$t_p^* = \frac{1}{4} \left[1 + \frac{-3s + A}{\delta} \right] \tag{A30}$$

We reduce this set of inequalities

$$r < \frac{(\beta - 1)^2 \beta(s + \delta)}{(\beta - 2)^2}$$

to get the range that time-locked trial should be offered and the result is

$$r < \frac{(\beta - 1)^{2}\beta(s + \delta)}{(\beta - 2)^{2}}, s \in \left[(1 - \beta)\delta, \frac{\beta\delta}{4 - 3\beta} \right]$$

$$0 \le r < \frac{s(\delta - s)(1 - \beta)}{s + \delta}, s \in \left[0, (1 - \beta)\delta \right] \cup s \in \left[\frac{\beta\delta}{4 - 3\beta}, \delta \right]$$
(A31)

We substitute $t = \frac{1}{4} \left[1 + \frac{-3s + A}{\delta} \right]$ into $P = \frac{r + (1 - \beta)(s + t\delta)}{2}$ to get the corresponding optimal price $P_{\rho}^* = \frac{1}{8} \left[4r + s + \delta - \beta(s + \delta) - B \right] \tag{A32}$

where $B = \sqrt{(\beta - 1)[s + \delta][8r - (1 - \beta)(s + \delta)]}$.

We substitute $t = \frac{1}{4} \left[1 + \frac{-3s + A}{\delta} \right]$ into $\Pi = \frac{(1-t)[r + (1-\beta)(s+t\delta)]^2}{4(1-\beta)(s+t\delta)}$ to get the corresponding optimal profit

$$\Pi_{\rho}^{*} = \frac{\left[-3(s+\delta) + A\right] \left[-4r - (1-\beta)(s+\delta) + (1-\beta)A\right]}{64(1-\beta)^{2}\delta\left[s+\delta + A\right]}$$
(A33)

Proof of Proposition 3.4

From the Proof of Proposition 3. 2 and Proof of Proposition 3. 3, we have the optimal trial period length t_t^* in the threat region and the optimal trial period t_t^* in the piracy region. We have

$$\frac{\partial t_i^*}{\partial r} = \frac{\beta s \delta^2 + \beta \delta^3}{2\beta \delta^2 \sqrt{\beta r s \delta^2 + \beta r \delta^3}} > 0 \tag{A34}$$

$$\frac{\partial t_{p}^{*}}{\partial r} = \frac{\sqrt{s+\delta}}{(\beta-1)\delta A} < 0 \tag{A35}$$

then the monotonicity is proved.

Proof of Proposition 3.5

From the Proof of Proposition 3. 2 and Proof of Proposition 3. 3, we have the optimal price P_t^* in the threat region and the optimal trial period P_p^* in the piracy region. We have

$$\frac{\partial P_i^*}{\partial r} = \frac{1}{\beta} > 0 \tag{A36}$$

$$\frac{\partial P_{p}^{*}}{\partial r} = \frac{1}{2} \left(1 + \frac{(1-\beta)(s+\delta)}{B} \right) > 0$$
 (A37)

then the monotonicity is proved.