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Dual credit channels for supply chain financing with a guarantee

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Abstract: A supply chain financing system when the retailer is capital constrained under demand uncertainty was studied. Considering that the manufacturer provides part of the trade credit, the retailer has to access the loan from the bank(bank credit). Different from extant literatures, the manufacturer offers a credit guarantee for the loan the retailer borrows from the bank (trade credit). Their strategic interaction was modelled as a Stackelberg game with the manufacturer acting as the leader. And the manufacturer decides the credit proportion. The dual credit channels create a higher value for the manufacturer than the single credit channel under some circumstances. This is well demonstrated by numerical examples. An optimal credit proportion may not always equal 0 or 1, which implies that an integration of the two credits can benefit the supply chain effectively.

Key words: supply chain financing; credit guarantee; credit proportion; trade credit; bank credit

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担保下的双信贷模式对供应链金融的策略

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摘要:研究了资金短缺的零售商在需求不确定下的一种供应链金融模式.在制造商提供部分贸易信贷情况下,零售商可以从银行直接获得贷款(银行信贷).以往的文献设定制造商为零售商从银行贷款提供一个信用担保(贸易信贷).在这个Stackelberg博弈中,设定制造商为领导者,并决定两种信贷模式比例.对制造商来说在某些情况下,双信贷模式比单一信贷模式更有利.并且给出了数例证明.最优信贷比例并不总是0或者1,即联合两种信贷可以更有效提高供应链绩效.

关键词:供应链金融;信用担保;信贷比例;贸易信贷;银行信贷

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0 Introduction

With the increasingly operational and market uncertainties in recent years, middle and small-sized enterprises (SME) have limited credit to loans from the bank because of a high possibility of loan defaults and bad loans. Acting as a substitute for bank credit, trade credit offers opportunities for small corporations to finance from a partner in a supply chain. In the absence of bank credit, offering retailers a permissible delay in payments is an effective way for suppliers to stimulate sales and reduce inventory as it represents a form of price reduction. According to *the Financial Times*, in 2007, 90% of world commodity trade had trade credit sizes of about $25 trillion. When checking private enterprises, we find that, for instance, Walmart amounted to 75 percent of its total inventory ($34.5 billion) on the balance sheet on January 31, 2009. Because of its wide usage, supplier financing has been realized as a promising field by researchers and practitioners.

In practice, the retailer has the incentive to incorporate multiple financing strategies to take further advantages. Trade credit and bank credit simultaneously constitute the main sources of the fund for SME's normal operations. For example, according to the China Household Electrical Appliances Assn, trade credit and bank credit are two popular financing strategies adopted by China's appliance industry and its franchisers finance. Besides trade credit, the appliance manufacturers offer credit guarantee for downstream dealers to ease the access of loans from banks.

Since there are alternatives for the retailer to loan from intermediary financing institutions and suppliers, tremendous research has been devoted to investigating the advantages of both supplier financing and bank financing. Cai et al.[1] compared bank credit with trade credit under moral risks and find that the two credits are complementary or substitutable as different internal capitals. Yan and Sun[2] analyzed credit line and interactions between the operational and financial decisions, and discussed the optimal credit line for the commercial bank. Given that extant research separates trade credit from bank credit, little is understood about the incorporation of trade credit and bank credit in literature. This paper intends to fill the gap.

Here we consider a stylized newsvendor setting, in which the retailer has limited initial capital and has to loan from the bank. Different from previous research, the manufacturer offers a guaranteed credit to the retailer contingent on the possibility of bankruptcy. The bank does not participate in the decision-making but provides loans. We focus on homogeneous products with intense competition, such as electrical appliances, and assume that the wholesale price is exogenously given, which is in line with Refs. [3-4]. We demonstrate that the design of the mechanism by incorporating bank and supplier credit is meaningful if there exists an equilibrium credit proportion between the two credits.

The remainder of this article is organized as follows. We review the extant literature in Section 1, and Section 2 presents the basic modeling of supply chain financing (SCF), including the model, notations, and assumptions. Section 3 analyzes optimal decisions for the retailer. Section 4 formulates a Stackelberg game and analyzes optimal decisions for larger bank credit rate. We analytically and numerically explore the impacts of financial decisions on operational decisions, and conduct numerical studies to demonstrate our results. In Section 5, we further analyze larger trade credit rate, and give numerical examples. We conclude the article in Section 6. All proofs are relegated to the Appendix.

1 Literature review

Our paper extends the existing literatures on trade credit and bank credit by incorporating credit guarantees into supply chain financing problems. Our following review is from perspectives of supply chain finance, the interface of operations and finance, and trade credit financing, the comparative advantage between trade credit financing and bank financing.

The stream of research relative to supply chain finance is to deal with the financial difficulty of the small and medium-sized enterprises. Berger and Udell[5] proposed a complete conceptual framework for the analysis of SMEs credit availability issues. Pfohl and Gomm[6] reviewed the state-of-the-art research regarding financial flows in supply chains, and proposed a conceptual framework for mathematically modeling "supply chain finance". Gomm[7] introduced a framework for investigating the financial issues in logistics and supply chain management (SCM) and showed that introducing a supply chain perspective on financial issues offers great opportunities for SCM professionals.

As an additional research stream, joint operational and financial decisions have recently received fast-growing attention. Buzacott and Zhang[8] discussed the situation in which the capital-constrained retailer finances from a bank that maximizes the profit by setting the interest rate and loan limit. Zhang[9] examined a multi-product newsvendor problem with both supplier quantity discounts and a budget constraint. By extending the forecasting-based discount dividend pricing method into an optimization-based valuation framework, Xu and Birge[10] proposed an integrated corporate planning model to make production and financial decisions simultaneously for a firm. Other works on the interface of operations and finance are referred to Refs. [11-13].

Recently, trade credit is viewed as an external financing strategy, which is largely extended in finance and economics literature. Empirical studies investigated the increasing impact of trade credit and the interaction with other financing strategies[4,14-16]. In addition, effort has been devoted to explicitly discussing the effect of trade credit in coordinating a buyer-vendor supply chain under different settings. Jing et al.[17] suggested that the production cost plays a determinant role in the choice between supplier financing and bank financing. Zhang et al.[18] investigated the impact of trade credit and its risk on supply chain coordination.

Moreover, various research has been devoted to investigating the advantage between supplier financing and bank financing. Zhou and Groenevelt[19] compared supplier financing and bank financing from the perspective of asset-based financing. They showed that with a fairly priced bank loan the overall supply chain and the retailer prefer bank credit whereas trade credit is preferable for the manufacturer. Kouvelis and Zhao[20] studied the retailer's financing strategy in a newsvendor setting, and found that the retailer prefers trade credit to bank financing. Cai et al.[1] demonstrate that trade credit and bank credit are complementary if the retailer's internal capital is substantially low but becomes substitutable as the internal capital grows. Given the opposite view on the optimal choice between trade credit and bank credit, we study the retailer's financing strategy in the presence of availability of both trade credit and bank credit, and examine a mixed equilibrium of financing strategy for the retailer.

Probably the work most closely related to ours is Ref. [17]. This paper considers a distribution channel that consists of a manufacturer with a deep pocket and a retailer with no wealth endowment. The optimal interest rate for trade credit can be greater than the risk-free rate. Our paper introduces manufacturer's guarantee and coexistence of both trade credit and bank credit. In addition, the wholesale price is assumed to be exogenously given.

2 The model

2.1 Model description

We consider a single-period newsvendor SCF system in which three parties are included: a capital-constrained retailer ("he" in this study), an upstream manufacturer, and a commercial bank. The retailer, who has limited liability, orders quantity q from the manufacturer and then sells it to customers. The market demand D for the product is unknown until the end of the period. The retailer is capital constrained in the procurement process. To ease the financial conditions, he borrows a loan from the manufacturer and the bank. The system framework is shown in Fig. 1.

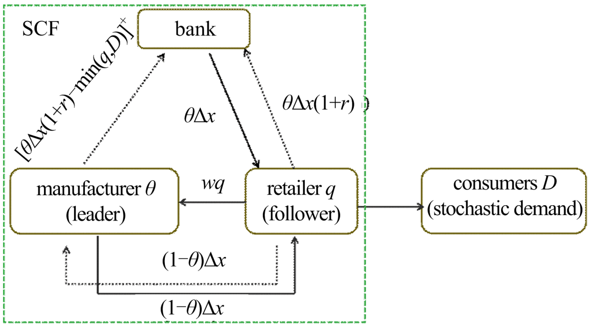

Fig. 1 Framework of a supply chain financing system

The sequence of the events is as follows:

At the beginning of the period, the demand is not observed. The retailer decides an order quantity q and pays the amount wq to the manufacturer. If necessary, the retailer borrows a loan from the supplier and/or the bank. Then, the supplier and the bank engage in a negotiation for the credit proportion to the loan size. The wholesale price w is assumed to be exogenously given.

At the end of the period, the demand is resolved. Upon receiving the product from the supplier, the retailer realizes his revenue in the commodity market. If the retailer's liquid assets can cover his loan obligations, he repays the loan; if not, the retailer declares bankruptcy, and the bank gains control of the defaulting of the retailer's remaining wealth. In addition, if the retailer can't repay the debt obligations to the bank, the supplier undertakes the rest of the retailers in arrears.

2.2 Notation and assumptions

In this section, we illustrate the notations and assumptions (Tab. 1) to formulate the models in this study.

Tab. 1 Summarization of notations

q	retailer's order quantity (retailer's decision variable)
θ	credit proportion to the loan size (manufacturer's decision variable)
p	unit retail price (normalized to 1)
w	unit wholesale price
c	unit production cost
B	retailer's initial capital
ΔX	retailer's capital gap $\Delta X = wq - B$
r_b	bank's exogenous interest rate
r_t	manufacturer's exogenous interest rate
$\Pi(\cdot)$	expected profit for each decision maker

Demand D is a nonnegative random variable with a cumulative distribution function and is not realized until the end of the period. The ex-ante demand distribution function $F(D)$ satisfies the following properties: ① $F(D)$ is absolutely continuous with density $f(D)>0$ on (a,b) for $0\leqslant a\leqslant b\leqslant\infty$; ② $F(D)$ has a finite mean and its hazard rate $h(D)=\dfrac{f(D)}{\bar{F}(D)}$ is increasing in D, where $\bar{F}(D)=1-F(D)$. Namely we focus on demand distributions of increasing failure rate (IFR); ③ The generalized failure rate (GFR) of the demand D is $H(D)=Dh(D)$. Then, $H(D)$ is monotonically increasing in D.

We propose the following assumptions to describe and analyze our models clearly.

Assumption 2.1 The bank is assumed to face no risk; namely, the bank can take back all the loan principals and interests. However, due to existing labor cost and administration cost, the bank interest rate is not equal to zero.

Assumption 2.2 The bank, retailer, manufacturer, are risk neutral.

Assumption 2.3 All information is symmetric. The retailer's capital condition and demand information are common knowledge among the three parties.

Assumption 2.4 The goodwill loss for unmet demand is ignored. All leftover stock salvages at price $s=0$.

Assumption 2.5 The retailer has no moral hazard and default possibility, but may face bankruptcy risk depending on whether final wealth is no less than the loan principal and interest.

Assumption 2.6 Conditional on the increasingly global competition, many manufacturers have no (less) pricing power. As a result, the wholesale price is assumed to be exogenously given. To avoid trivial cases, let $p \geqslant w(1+r_b) \geqslant c(1+r_b)$. For ease of exposition, we assume the supplier's and retailer's opportunity cost of capital $r_f=0$.

Assumption 2.7 The retailer is capital constrained. The bank and the manufacturer have unlimited capital.

3 Retailer's optimal decision

In this section, we consider the retailer's optimal decision, mainly focusing on the retailer's optimal quantity and its properties. Conditional on wholesale price w and the retailer's internal capital B, the retailer has a capital gap of ΔX, where $\Delta X = wq - B$. $(1-\theta)\Delta X$ is the credit size the manufacturer may offer to the retailer. It implies that the bank credit size is $\theta\Delta X$.

The retailer's objective is to maximize the profit by ordering $q(\theta)$, which can be expressed as

$$\max_q \pi_{\mathrm{R}}(q;\theta) = E\{[\min(q,D) - \theta\Delta X(1+r_b) - (1-\theta)\Delta X(1+r_t)]^+\} \quad (1)$$

We define $k(\theta) = \theta\Delta X(1+r_b) + (1-\theta)\Delta X(1+r_t)$.

At the end of the period, the retailer collects revenue $\min(q,D)$, which is used to repay his debt

$$k(\theta) = \theta\Delta X(1+r_b) - (1-\theta)\Delta X(1+r_t) = (1+\theta r_b + (1-\theta)r_t)\Delta X.$$

$k(\theta)$ is the retailer's bankruptcy threshold, the minimal demand level that the retailer can repay the loan obligation. If the realized demand is too low, the retailer is not able to repay the bank loans and has to face bankruptcy risk. To some degree, such risk of bankruptcy may transfer to the manufacturer.

Proposition 3.1 For IFR demand distributions,

(i) the capital-constrained retailer's optimal order quantity q^* is given by $\overline{F}(q^*) = \Omega\overline{F}(k(q^*))$, where $\Omega = (1+\theta r_b + (1-\theta)r_t)w$;

(ii) q^* is decreasing in θ, i.e.,

$$\frac{\mathrm{d}q^*}{d\theta} < 0;$$

(iii) $k(q^*)$ is decreasing in θ, i.e.,

$$\frac{\mathrm{d}k(q^*)}{\mathrm{d}\theta} < 0.$$

See the Appendix for its proof. The superscript "$*$" represents the optimum of decision variable.

Proposition 3.1(i) shows that the optimal order quantity q^* is closely related to the financing size of credit and cost parameters. That is, operational and financial decisions cannot be decoupled. In addition, q^* is dependent on bank interest and credit line, as it states capital cost and bankruptcy risk.

Proposition 3.1(ii) and (iii) demonstrate that the manufacturer intends to choose a lower credit proportion to acquire the higher order quantity. Given the initial capital, the fewer the credit proportion, the lower the bankrupt threshold the retailer will face. On the other hand, a higher credit proportion conducts a larger interest payment. Therefore, a rational manufacturer is willing to choose an appropriate credit proportion to control the bankrupt risk. As a consequence, the equilibrium interest payment maximizes the profit.

4 Manufacturer's optimal decision when $r_t < r_b$

Recall that in this study the manufacturer is assumed to have no capital constraints and have unlimited capacity. Under the symmetric information setting, the manufacturer can anticipate the retailer's order quantity $q^*(\theta)$ and acts as a Stackelberg leader. The optimal credit proportion is achieved by solving

$$\pi_M(\theta,q^*(\theta))=\int_0^{\theta\Delta X(1+r_b)}-[\theta\Delta X(1+r_b)-\min(q^*(\theta),D)]f(D)dD+$$
$$\int_{\theta\Delta X(1+r_b)}^{+\infty}\min\{\min(q^*(\theta),D)-\theta\Delta X(1+r_b),(1-\theta)\Delta X(1+r_t)\}f(D)dD+B+\theta\Delta X-cq^*(\theta)=$$
$$\int_0^{k(\theta)}[\min(q^*(\theta),D)-\theta\Delta X(1+r_b)]f(D)dD+\int_{k(\theta)}^{+\infty}(1-\theta)\Delta X(1+r_t)f(D)dD+B+\theta\Delta X-cq^*(\theta) \quad (2)$$

The manufacturer's revenue is made up of two components. The first one $\min\{\min(q^*(\theta),D)-\theta\Delta X(1+r_b),(1-\theta)\Delta X(1+r_t)\}f(D)dD+B+\theta\Delta X-cq^*(\theta)$, represents the payoff for production given bank credit and trade credit, and the second one, $[\theta\Delta X(1+r_b)-\min(q^*(\theta),D)]^+$ addresses the risk of guarantee for bank credit.

4.1 Trade credit rate $r_t>0$

We first analyze the situation in which the trade credit rate $r_t\neq 0$. Before we obtain the manufacturer's optimal decision, we have Lemma 4.1 as follows.

Lemma 4.1 Given initial capital and bank rate, there exist production cost thresholds c_1 and c_2, where c_2 satisfies

$$[r_b\Delta X+(w+\theta wr)\frac{dq^*(\theta)}{d\theta}](\overline{F}(k(\theta))-1)+(w-c_2)\frac{dq^*(\theta)}{d\theta}=0,$$

and c_1 satisfies

$$r_b\Delta X[f(q^*)-\Omega^2 f(k(q^*))]+(c_1+\theta wr_b)wr[k(q^*)f(k(q^*))-\overline{F}(k(q^*))]=0.$$

By setting (2) equal to zero, and after rearrangement, we obtain the manufacturer's optimal decision.

Proposition 4.1 For $r_t<r_b$, the manufacturer optimizes credit rate θ as

(i) $c_1<0,c_2<0$.

$\forall c\in(0,w)$, $\theta^*=1$.

(ii) $c_1>0,c_2>0$.

① $c\in(0,c_1]$, $\theta^*=0$.

② $c\in(c_1,c_2]$, the optimal θ^* satisfies

$$\overline{F}(k(\theta^*))=\frac{\Delta Xr_b+(\theta wr_b+c)\frac{dq^*(\theta)}{d\theta}}{\Delta X(r_b-r_t)+[1+\theta r_b+(1-\theta)r_t]w\frac{dq^*(\theta)}{d\theta}}.$$

③ $c\in(c_2,w)$, $\theta^*=1$.

(iii) $c_1<0,c_2>0$.

① $c\in(0,c_2]$, the optimal θ^* satisfies

$$\overline{F}(k(\theta^*))=\frac{\Delta Xr_b+(\theta wr_b+c)\frac{dq^*(\theta)}{d\theta}}{\Delta X(r_b-r_t)+[1+\theta r_b+(1-\theta)r_t]w\frac{dq^*(\theta)}{d\theta}}.$$

② $c\in(c_2,w)$, $\theta^*=1$.

Recall that the expected revenue

$$\int_0^{k(\theta)}[\min(q^*(\theta),D)]f(D)dD+\int_{k(\theta)}^{+\infty}[1+(1-q)r_t]\Delta Xf(D)dD+B$$

decreases with bankruptcy risk $k(\theta)$, and $\frac{dk(\theta)}{d\theta}<0$. Then, $\int_0^{k(\theta)}[\min(q^*(\theta),D)]f(D)dD+\int_{k(\theta)}^{+\infty}[1+(1-\theta)r_t]\Delta Xf(D)dD$ increases with θ. The manufacturer's expected repayment is $\int_0^{k(\theta)}\theta\Delta Xr_bf(D)dD+cq^*(\theta)$. Given appropriate variables, we can find that the optimal θ^* is derived from equilibrium between expected marginal revenue and expected marginal repayment, as Proposition 4.1 (ii) shows. If expected marginal revenue is larger than expected marginal repayment, $\theta^*=1$. Conversely, if expected marginal revenue is less than expected marginal repayment, $\theta^*=0$. In this case, the manufacturer's expected repayment also increases with θ.

Proposition 4.1 intuits the manufacturer to make financing decisions contingent on different production situations, defined by the thresholds c_1 and c_2. Based on the relationship between unit production cost and the thresholds, the manufacturer can identify various production scenarios and effectively make the optimal financing decision.

Corollary 4.1 Given initial capital and bank rate, the manufacturer optimal profit is $\max(\pi_M(0, q^*(0)), \pi_M(\theta^*, q^*(\theta^*)), \pi_M(1, q^*(1)))$, where the optimal θ^* satisfies $\theta^*=0$; $\overline{F}(k(\theta^*))=$

$$\frac{\Delta X r_b+(\theta w r_b+c)\dfrac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}}{\Delta X(r_b-r_t)+[1+\theta r_b+(1-\theta)r_t]w\dfrac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}}$$

and $\theta^*=1$ respectively.

We use the uniform distribution with tail function $\overline{F}(x)=1-\dfrac{x}{100}(x\in(0,100))$ to illustrate the impact of the credit proportion on manufacturer's revenues as a function, as shown in Fig. 2. We adopt a market setting of $w=0.8$, $B=10$, $r_b=0.06$, $r_t=0.01$. Given $0<c_1<c_2$, if $c=0.71<c_1$, the manufacturer's payoff is decreasing in credit proportion, and is better off under $\theta^*=0$. Under this condition, the manufacturer prefers trade credit. If $c_1<c=0.72<c_2$, the manufacturer is better off under $\theta^*=0.5$. If $c=0.74>c_2$, the manufacturer prefers bank credit. It clearly illustrates the result of Proposition 4.1(ii).

For an exogenous whole price, we can obtain the optimal manufacturer's payoff under different product costs. Since a lower product cost induces a lower whole price, we focus on the product cost just for ease of exposition. In reality, it is not easy for the manufacturer to reduce cost, instead of increasing the wholesale price when possible. Therefore, without considering Assumption 2.6, the manufacturer sets the whole price as large as possible and prefers full trade credit, i.e. a lower c and $\theta^*=0$ in Fig. 2.

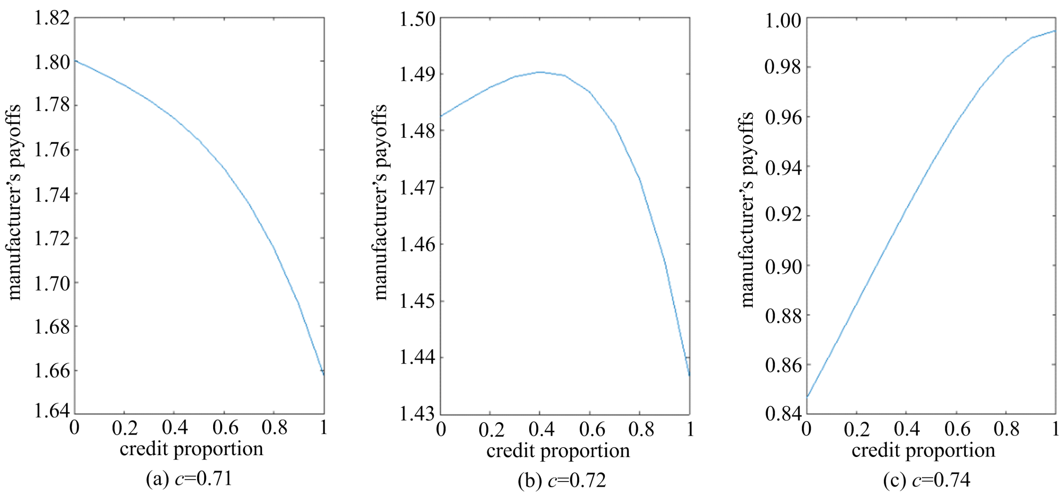

Fig. 2 For $r_t<r_b$, the manufacturer's payoffs as a function of the credit proportion under different c

In some cases where $r_t<r_b$, there exists an optimal credit rate θ satisfying $0<\theta^*<1$, which implies a corporation of both trade credit and bank credit. Meanwhile, under the same conditions, the manufacturer has a boost in payoff compared with Proposition 4.1. It indicates that the manufacture is induced to set a trade credit rate as large as possible.

4.2 Trade credit rate $r_t=0$

To better understand the production cost thresholds $c_1, c_2\in(0,w)$ and Proposition 4.1, we study a special case, in which trade credit rate $r_t=0$. The retailer's objective is to maximize his profit by ordering the quantity $q(\theta)$, which can be expressed as

$$\max_q \pi_R(q;\theta)=E\{[\min(q,D)-$$

$$\theta\Delta X(1+r_b)-(1-\theta)\Delta X]^+\} \quad (3)$$

The capital-constrained retailer's optimal order quantity q^* is given by $\bar{F}(q^*)=\Omega\bar{F}(k(q^*))$, where $\Omega=(1+\theta r_b)w$.

The manufacturer acts as a Stackelberg leader, and chooses an optimal credit proportion by solving

$$\max_{\theta}\pi_M(\theta,(q^*(\theta))= E\{\min\{[\min(q^*(\theta),D)-\theta\Delta X(1+r_b)]^+,(1-\theta)\Delta X\}-[\theta\Delta X(1+r_b)-\min(q^*(\theta),D)]^+ + B+\theta\Delta X-cq^*(\theta)\} \quad (4)$$

There exist $c_1,c_2\in(-\infty,w)$, $c_1<c_2$, where c_1 and c_2 satisfy

$$c_1=w-\Omega\frac{1-\dfrac{\Delta X}{wq^*(\theta)}H(q^*(\theta))}{1-H(k(\theta))}$$

and

$$c_2=c_1+\Omega\bar{F}(k(\theta))\frac{1-\dfrac{\Delta X}{wq^*(\theta)}H(q^*(\theta))}{1-H(k(\theta))}$$

respectively.

We find that $c_1,c_2\in(-\infty,w)$ depends on demand distribution, initial capital and bank rate. It implies jointly operational and financial decisions for the manufacture. For instance, if $c_1>0,c_2>0$ and $c\in(c_1,c_2]$, the manufacturer would set the optimal credit rate θ satisfying

$$\bar{F}(k(\theta^*))=1-\frac{(w-c)[1-H(k(\theta))]}{\Omega[1-\dfrac{\Delta X}{wq^*(\theta)}H(q^*(\theta))]}.$$

5 Manufacturer's optimal decision when $r_t>r_b$

So far, we have studied the case in which trade credit rate is less than bank credit rate. In reality, when the retailers such as household appliances dealers don't have a credit record, the trade credit contract may charge a relatively high cost in some situations. That is, the manufacturer with trade credit rate surpasses bank credit rate. Therefore, it is non-trivial for us to study an inverse scenario where $r_t>r_b$. Let $w=0.8, B=10$, and Tab. 2 presents the results from uniform distribution with the parameter (0,100).

Tab. 2 The manufacturer's payoff when $r_b=0.03$ and $r_t=0.07$

θ	c					
	0.5	0.56	0.62	0.68	0.74	0.8
0	6.096	4.919	3.743	2.566	1.389	0.213*
0.1	6.361	5.123	3.884	2.646	1.408*	0.169
0.2	6.597	5.300	4.002	2.704	1.406	0.108
0.3	6.806	5.451	4.096	2.741	1.386	0.031
0.4	6.990	5.580	4.170	2.760	1.349	−0.061
0.5	7.151	5.687	4.224	2.761*	1.298	−0.166
0.6	7.290	5.775	4.261	2.746	1.231	−0.283
0.7	7.409	5.845	4.281	2.717	1.152	−0.412
0.8	7.509	5.897	4.285*	2.673	1.061	−0.551
0.9	7.592	5.934	4.276	2.617	0.959	−0.699
1.0	7.659*	5.956*	4.253	2.550	0.846	−0.857

[Note] Figures with * is the maximum profit.

Given $r_b=0.03$ and $r_t=0.07$, Tab. 2 shows that the manufacturer's payoff under different product costs and credit proportions. We can obtain the optimal payoff for each product cost. When c is lower, the manufacturer's optimal credit proportion satisfies $\theta^*=1$. When c is larger, the manufacturer sets $\theta^*=0$ to get the optimal payoff. Therefore, we propose the manufacturer's optimal decision as follows.

Proposition 5.1 In case where $r_t>r_b$, there exists $c_1,c_2\in(0,w)$, such that the manufacturer chooses the optimal credit rate θ to satisfy

① $c\in(0,c_1]$, $\theta^*=0$;

② $c\in(c_1,c_2]$, the optimal θ^* satisfies $\theta^*\in(0,1)$;

③ $c\in(c_2,w]$, $\theta^*=1$.

From the above numerical examples, we find that, when c is larger, the manufacturer sets optimal credit proportion $\theta^*\in(0,1)$, which is different from Proposition 4.1. We present Tab. 3 from $w=0.8, B=10$ uniform distribution with the parameter (0,100). Different from Proposition 5.1, when c is larger, the manufacturer sets optimal credit proportion $\theta^*\in(0,1)$.

Tab. 3 The manufacturer's payoff when $r_b = 0.03$ and $r_t = 0.08$

θ	c					
	0.5	0.56	0.62	0.68	0.74	0.8
0	5.259	4.249	3.238	2.228	1.218	0.207
0.1	5.687	4.592	3.496	2.400	1.305	0.209*
0.2	6.062	4.885	3.708	2.532	1.355	0.179
0.3	6.388	5.134	3.881	2.627	1.374*	0.120
0.4	6.671	5.344	4.017	2.690	1.364	0.037
0.5	6.914	5.518	4.121	2.724	1.328	−0.069
0.6	7.122	5.659	4.196	2.732*	1.269	−0.194
0.7	7.298	5.771	4.244	2.717	1.190	−0.338
0.8	7.445	5.856	4.268	2.680	1.092	−0.497
0.9	7.564	5.917	4.270*	2.623	0.977	−0.670
1.0	7.659*	5.956*	4.253	2.550	0.846	−0.857

[Note] Figures with * is the maximum profit.

Corollary 5.1 If r_t is relatively larger than r_b, there exists $c_2 > w$. For $r_t > r_b$, there exist $c_1 \in (0, w)$. The manufacturer would set the optimal credit rate θ satisfying

① $c \in (0, c_1]$, $\theta^* = 0$;

② $c \in (c_1, w]$, the optimal θ^* satisfies $\theta^* \in (0,1)$.

6 Conclusion

This paper investigates an incorporation of bank and trade credits in a supply chain where a capital constrained retailer faces demand uncertainty. The manufacturer has the incentive to offer a credit for the retailer's loan from the bank. We first theoretically characterize the optimal credit proportion in two scenarios where the trade credit rate is larger than bank credit rate, or vice versa. We then use numerical examples to hypothetically test the correctness of two cases.

Our paper contributes to the literature in several ways. First, we propose a risk sharing mechanism in Stackelberg games where the manufacturer teams up with the bank to provide the loan which equals the capital gap to the retailer. The manufacturer provides the credit guarantee for retailers if bankruptcy occurs. We complement the present research by combining bank financing and supplier financing. Second, we show that as a supply chain mechanism, a credit proportion between bank credit and trade credit optimizes supply chain efficiency. Third, our model shows the equilibrium credit proportion when the wholesale price is assumed to be exogenously given.

We have found several results to extend the related literature. First, although prior research has been devoted to identifying the conditional advantages of two distinct financing policies, trade credit and bank credit, the retailer with a financial constraint and the manufacturer may have the incentive to achieve advantage by incorporating the two policies simultaneously.

Second, conditional on a trade credit contract, we propose a risk sharing mechanism for the supplier to skim the financial risk to the bank. Intuitively, the supplier may face the risk of the retailer's bankruptcy, and has a possibility not to obtain the account receivable from the retailer. In this perspective, the supplier intends to share the financial risk with the bank. A credit proportion is suggested to maximize his profit in the presence of demand uncertainty.

Third, this paper offers the manufacturer an intuition to make financing decisions under different demand distributions, initial capital and bank rate. We introduce two thresholds of production cost c_1 and c_2 which reflect different production situations. Based on the values of c_1 and c_2, the manufacturer can effectively decide the optimal financing strategies. In this article, the wholesale price w is assumed to be exogenously given. If there exists an endogenous wholesale price w, the optimal decision is smaller than credit rate with an exogenous wholesale price.

This paper can be extended along several possible orientations. First, the wholesale price w is assumed to be exogenously given. The assumption is well adopted in the existing related literature. Nevertheless, it is a future research territory to make the wholesale price endogenous and then utilize it to explore additional conclusions. Second, competition among multiple suppliers and multiple retailers could be feasible. Finally, we assume that all the institutions in the monetary market are risk-neutral. Apparently, different assumptions on the attitude to risk may result in different optimal decisions.

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Appendix

A1 Proof of Proposition 3.1

Part 1 Let $k(\theta)=\theta\Delta X(1+r_b)+(1-\theta)\Delta X(1+r_t)$, then

$$\pi_R(q;\theta)=E\{[\min(q,D)-\theta\Delta X(1+r_b)-(1-\theta)\Delta X(1+r_t)]^+\}=\int_{k(\theta)}^{+\infty}[\min(q,D)-k(\theta)]f(D)\mathrm{d}D.$$

Differentiating $\pi_R(q;\theta)$ with respect to q and collecting terms, we have

$$\frac{\mathrm{d}\pi_R(q;\theta)}{\mathrm{d}q}=\overline{F}(q)-(1+\theta r_b+(1-\theta)r_t)w\overline{F}(k).$$

The retailer's optimal ordering level q^* must satisfy the first-order condition:

$$\overline{F}(q^*)=(1+\theta r_b+(1-\theta)r_t)w\overline{F}(k(q^*)).$$

Let $\Omega=(1+\theta r_b+(1-\theta)r_t)w$, then $\overline{F}(q^*)=\Omega\overline{F}(k(q^*))$. We further have

$$\left.\frac{\mathrm{d}^2\pi_R(q;\theta)}{\mathrm{d}(q)^2}\right|_{q=q^*}=-f(q^*)+\Omega^2 f(k(q^*))=-\overline{F}(q^*)h(q^*)+W^2\overline{F}(k(q^*))h(k(q^*))=$$

$$-\overline{F}(q^*)[h(q^*)-\Omega h(k(q^*))]<0.$$

Since $h(x)$ is an increasing function of assumption, we have $h(q^*)>h(k(q^*))>\Omega h(k(q^*))$.

Therefore, $\left.\frac{\mathrm{d}^2\pi_R(q;\theta)}{\mathrm{d}(q)^2}\right|_{q=q^*}<0$ and q^* is thus unique.

Part 2 Inspection of $\overline{F}(q^*)=\Omega\overline{F}(k(q^*))$ readily reveals that

$$\frac{\mathrm{d}q^*}{\mathrm{d}\theta}=-\frac{-[w(r_b-r_t)\overline{F}(k(q^*))-\Omega(r_b-r_t)\Delta Xf(k(q^*))]}{-f(q^*)+\Omega^2 f(k(q^*))}=$$

$$\frac{w(r_b-r_t)[k(q^*)f(k(q^*))-\overline{F}(k(q^*))]}{f(q^*)-\Omega^2 f(k(q^*))}.$$

First, we would prove $xf(x)<\overline{F}(x)$.

Suppose $xf(x)<\overline{F}(x)$, let $g(x)=x\overline{F}(x)$, then

$$g'(x)=\overline{F}(x)-xf(x)<0, k(q^*)\overline{F}(k(q^*))>q^*\overline{F}(q^*)=\Omega q^*\overline{F}(k(q^*)).$$

We have $k(q^*)>wq^*(1+\theta r_b+(1-\theta)r_t)$ a contradiction to the inequality above.

Therefore, we must have

$$xf(x)<\overline{F}(x), k(q^*)f(k(q^*))<\overline{F}(k(q^*)).$$

Moreover, we have proved $f(q^*)-\Omega^2 f(k(q^*))>0$ in Part 1.

So, we get $\frac{\mathrm{d}q^*}{\mathrm{d}\theta}<0$.

Part 3 $\frac{\mathrm{d}k(q^*)}{\mathrm{d}\theta}=(r_b-r_t)\Delta X+(w+\theta wr_b+(1-\theta)wr_t)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}$.

We will prove $(r_b-r_t)\Delta X+(w+\theta w\mathrm{r}_b+(1-\theta)wr_t)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}<0$.

Because of $f(q^*(\theta))-\Omega^2 f(k(q^*(\theta)))>0$, we just prove

$$(r_b-r_t)\Delta X[f(q^*(\theta))-\Omega^2 f(k(q^*(\theta)))]+$$

$$\Omega w(r_b-r_t)[k(q^*(\theta))f(k(q^*(\theta)))-\overline{F}(k(q^*(\theta)))]<0.$$

The following results are showed.

$$(r_b-r_t)\Delta X[f(q^*(\theta))-\Omega^2 f(k(q^*(\theta)))]+$$

$$\Omega w(r_b-r_t)[k(q^*(\theta))f(k(q^*(\theta)))-\overline{F}(k(q^*(\theta)))]=$$

$$(r_b-r_t)\Delta Xf(q^*(\theta))-w(r_b-r_t)\Omega\overline{F}(k(q^*(\theta)))=$$

$$\frac{(r_b - r_t)}{q^*(\theta)}[(wq^*(\theta) - B)q^*(\theta)f(q^*(\theta)) - wq^*(\theta)\overline{F}(q^*(\theta))] < 0.$$

Thus, $\frac{\mathrm{d}k(q^*)}{\mathrm{d}\theta} < 0$.

A2 Proof of Proposition 4.1

We can show that

$$\pi_{\mathrm{M}}(\theta, q^*(\theta)) = \int_0^{\theta\Delta X(1+r_b)} -[\theta\Delta X(1+r_b) - \min(q^*(\theta), D)]f(D)\mathrm{d}D +$$

$$\int_{\theta\Delta X(1+r_b)}^{+\infty} \min\{\min(q^*(\theta), D) - \theta\Delta X(1+r_b), (1-\theta)\Delta X(1+r_t)\} f(D)\mathrm{d}D + B + \theta\Delta X - cq^*(\theta) =$$

$$\int_0^{k(\theta)} [\min(q^*(\theta), D) - \theta\Delta X(1+r_b)]f(D)\mathrm{d}D +$$

$$\int_{k(\theta)}^{+\infty} (1-\theta)\Delta X(1+r_t)f(D)\mathrm{d}D + B + \theta\Delta X - cq^*(\theta),$$

$$\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta, q^*(\theta))}{\mathrm{d}\theta} = [\Delta X(r_b - r_t) + [1 + \theta r_b + (1-\theta)r_t]w\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}] \cdot$$

$$[\overline{F}(k(\theta)) - \frac{\Delta X r_b + (\theta w r_b + c)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}}{\Delta X(r_b - r_t) + [1 + \theta r_b + (1-\theta)r_t]w\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}}].$$

We can have

$$[\Delta X r_b + (\theta w r_b + c)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}] - [\Delta X(r_b - r_t) + [1 + \theta r_b + (1-\theta)r_t]w\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}] =$$

$$\Delta X r_t + [c - w - wr_t(1-\theta)]\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta} > 0,$$

$$\Delta X(r_b - r_t) + [1 + \theta r_b + (1-\theta)r_t]w\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta} =$$

$$\frac{r_b - r_t}{q^*(\theta)[f(q^*(\theta)) - \Omega^2 f(k(\theta))]}[\Delta X q^*(\theta)f(q^*(\theta)) - wq^*(\theta)\overline{F}(q^*(\theta))] < 0.$$

Since $\Delta X < wq^*(\theta)$ and $q^*(\theta)f(q^*(\theta)) < \overline{F}(q^*(\theta))$.

For $\Delta X r_b + (\theta w r_b + c)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}$, we can't judge its value. Therefore, we assume that exist $c_1(c_1 \in (-\infty, w))$ satisfies $\Delta X r_b + (\theta w r_b + c_1)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta} = 0$.

Let $G(A) = \Delta X r_b + (\theta w r_b + A)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}$, we can have

$$\frac{\mathrm{d}G(A)}{\mathrm{d}A} = \frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta} < 0.$$

Case 1 $c_1 > 0$.

(a) When $c \in (0, c_1]$,

$$G(c) = \Delta X r_b + (\theta w r_b + c)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta} > 0.$$

There exist $\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta, q^*(\theta))}{\mathrm{d}\theta} < 0$, $\theta^* = 0$.

(b) When $c \in (c_1, w)$, $G(c) = \Delta X r_b + (\theta w r_b + c)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta} < 0$.

In this way, we can get

$$\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}\theta}=[\Delta X(r_b-r_t)+[1+\theta r_b+(1-\theta)r_t]w\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}](\overline{F}(k(\theta))-1)+(w-c)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}-$$

$$[r_t\Delta X-(1-\theta)wr_t\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}].$$

Let

$$K(A)=[\Delta X(r_b-r_t)+[1+\theta r_b+(1-\theta)r_t]w\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}](\overline{F}(k(\theta))-1)+(w-A)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}-$$

$$[r_t\Delta X-(1-\theta)wr_t\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}],$$

we can have $\frac{\mathrm{d}K(A)}{\mathrm{d}A}=-\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}>0$.

Therefore, there exist $c_2(c_2\in(c_1,w))$ satisfies

$$\forall\theta\in(0,1),K(c_2)=0\left(\because K(c_2)=0,K(c_1)<0,\frac{\mathrm{d}K(A)}{\mathrm{d}A}>0,\therefore c_1<c_2\right).$$

(b_1) When $c\in(c_2,w)$, $\forall\theta\in[0,1]$,

$$\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta)}{\mathrm{d}\theta}=K(c)>K(c_2)=0.$$

The optimal θ^* satisfies $\theta^*=1$.

(b_2) When $c\in(c_1,c_2]$, it is possibly that

$$\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}\theta}=0.$$

The optimal θ^* satisfies

$$\overline{F}(k(\theta^*))=\frac{\Delta Xr_b+(\theta wr_b+c)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}}{\Delta X(r_b-r_t)+[1+\theta r_b+(1-\theta)r_t]w\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}}.$$

It is assumed that $\frac{\mathrm{d}^2\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}(\theta)^2}<0$.

Case 2 $c_1<0$.

When $c\in(0,w)$, $\Delta Xr_b+(\theta wr_b+c_1)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}<0$.

Case 2.1 $c_2>0$.

(a) When $c\in(c_2,w)$, $\forall\theta\in[0,1]$,

$$\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}\theta}=K(c)>K(c_2)=0.$$

The optimal θ^* satisfies $\theta^*=1$.

(b) When $c\in(0,c_2]$, it is possibly that

$$\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}\theta}=0.$$

The optimal θ^* satisfies

$$\overline{F}(k(\theta^*))=\frac{\Delta Xr_b+(\theta wr_b+c)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}}{\Delta X(r_b-r_t)+[1+\theta r_b+(1-\theta)r_t]w\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}}.$$

Case 2.2 $c_2<0$.

(b_1) When $c\in(0,w)$, $\forall\theta\in[0,1]$,

$$\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}\theta}=K(c)>K(c_2)=0.$$

The optimal θ^* satisfies $\theta^*=1$.

A3 Proof of Corollary 5.1

$$\pi_{\mathrm{M}}(\theta,q^*(\theta))=\int_0^{\mathrm{k}(\theta)}[\min(q^*(\theta),D)-\theta\Delta X(1+r_b)]f(D)\mathrm{d}D+$$
$$\int_{k(\theta)}^{+\infty}(1-\theta)\Delta Xf(D)\mathrm{d}D+B+\theta\Delta X-cq^*(\theta)\ \frac{\mathrm{d}\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}\theta}=$$
$$[r_b\Delta X+(w+\theta wr_b)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}](\overline{F}(k(\theta))-1)+(w-c)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}.$$

Let

$$K(A)=[r_b\Delta X+(w+\theta wr_b)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}](\overline{F}(k(\theta))-1)+(w-A)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}.$$

We can have $\forall\theta\in[0,1],K(w)>0$,

$$\frac{\mathrm{d}K(A)}{\mathrm{d}A}=-\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}>0.$$

Therefore, there exist $c_2(c_2\in(-\infty,w))$ satisfies $\forall\theta\in[0,1]$,

$$K(c_2)=[r_b\Delta X+(w+\theta wr_b)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}](\overline{F}(k(\theta))-1)+(w-c_2)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}=0.$$

Namely,

$$\Omega(\overline{F}(k(\theta))-1)[\frac{\Delta X}{wq^*(\theta)}H(q^*(\theta))-1]+(w-c_2)[H(k(\theta))-1]=0,$$

$$c_2=w-\Omega(1-\overline{F}(k(\theta)))\frac{1-\frac{\Delta X}{wq^*(\theta)}H(q^*(\theta))}{1-H(k(\theta))}.$$

Case 1 $c_2<0$.

When $c\in(0,w)$, $\forall\theta\in[0,1]$,

$$\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}\theta}=K(c)>0.$$

The optimal θ^* satisfies $\theta^*=1$.

Case 2 $c_2>0$.

(a) $c\in(c_2,w)$, $\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}\theta}>0$, the optimal θ^* satisfies $\theta^*=1$.

(b) When $c\in(0,c_2]$, it is possibly that $\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}\theta}=0$.

$$\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}\theta}=[r_b\Delta X+(w+\theta wr_b)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}][\overline{F}(k(\theta))-\frac{r_b\Delta X+(c+\theta wr_b)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}}{r_b\Delta X+(w+\theta wr_b)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}}].$$

For $r_b\Delta X+(c+\theta wr_b)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}$, we can't judge its value. It depends on r and B and demand distribution.

Let

$$G(A)=r_b\Delta X+(c+\theta wr_b)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}.$$

We can have

$$G(w)<0,\frac{\mathrm{d}G(A)}{\mathrm{d}A}=\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}<0.$$

Therefore, there exist $c_1(c_1\in(-\infty,c_2))$ satisfies

$$G(c_1)=r_b\Delta X+(c_1+\theta wr_b)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}=0\left(\because K(c_2)=0,K(c_1)<0,\frac{\mathrm{d}K(A)}{\mathrm{d}A}>0,\therefore c_1<c_2\right)$$

$$c_1=w-\frac{\frac{\Delta X}{wq^*(\theta)}q^*(\theta)f(q^*(\theta))-\overline{F}(q^*(\theta))}{kf(k(\theta))-\overline{F}(k(\theta))}=w-\Omega\frac{1-\frac{\Delta X}{wq^*(\theta)}H(q^*(\theta))}{1-H(k(\theta))},$$

$$c_2=c_1+\Omega\overline{F}(k(\theta))\frac{1-\frac{\Delta X}{wq^*(\theta)}H(q^*(\theta))}{1-H(k(\theta))}.$$

Case 2.1 $c_1>0$.

(b_1) When $c\in(0,c_1]$, $\forall\theta\in[0,1]$, we have

$$r\Delta X+(c+\theta wr)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}>r\Delta X+(w+\theta wr)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}>0.$$

There must exist

$$\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}\theta}<0.$$

The optimal θ^* satisfies $\theta^*=0$.

(b_2) When $c\in(c_1,c_2]$,

$$r_b\Delta X+(c+\theta wr_b)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}\leqslant 0,\ \frac{r_b\Delta X+(c+\theta wr_b)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}}{r_b\Delta X+(w+\theta wr_b)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}}<1.$$

There exist

$$\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}\theta}=0.$$

The optimal θ^* satisfies

$$\overline{F}(k(\theta^*))=1-\frac{(w-c)[1-H(k(\theta))]}{\Omega[1-\frac{\Delta X}{wq^*(\theta)}H(q^*(\theta))]}.$$

Case 2.2 $c_1<0$.

When $c\in(0,c_2]$,

$$r_b\Delta X+(c+\theta wr_b)\frac{\mathrm{d}q^*(\theta)}{\mathrm{d}\theta}<0.$$

There exist $\frac{\mathrm{d}\pi_{\mathrm{M}}(\theta,q^*(\theta))}{\mathrm{d}\theta}=0$.

The optimal θ^* satisfies

$$\overline{F}(k(\theta^*))=1-\frac{(w-c)[1-H(k(\theta))]}{\Omega[1-\frac{\Delta X}{wq^*(\theta)}H(q^*(\theta))]}.$$