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广义(3+1)维浅水波方程的相互作用解

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摘要:通过 Hirota 双线性导数法,并借助于符号计算软件 Maple,得到广义(3+1)维浅水波方程的 lump 解和呼吸波解.同时结合图像研究了 lump 型孤子的动力学性质(位置、高度、深度、运动速度和运动轨迹).最后特别讨论了不同类型解之间的相互作用,显示了 lump 型孤子被扭结孤立波吞噬的现象.

关键词:Hirota 双线性导数法;lump 解;动力学性质;呼吸波解;相互作用现象

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The interaction solution to generalized (3+1)-dimensional shallow wave equation

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Abstract: By using the Hirota bilinear derivative method and the symbolic computation software Maple, the lump solution and the respiratory wave solution to the generalized (3+1)-dimensional shallow water wave equation were obtained. In combination with images, the dynamic properties (position, height, depth, velocity, and trajectory) of the lump-type soliton were studied. Finally, the interaction between different types of solutions were discussed, and it was found that lump-type solitons were phagocytosed by kink waves.

Key words: Hirota bilinear derivative method; lump solution; dynamic properties; breather solution; interaction phenomenon

0 引言

孤立子作为非线性科学三大分支之一,指的是一大类具有特殊性质的非线性偏微分方程的解。多年以来研究者为了求出该类型的解做了许多的工作,并且已经取得了一批杰出而又重要的成果。比如反散射方法[1]、李群与非经典李群法[2-3]、达布变换[4-5]、齐次平衡法[6-8]、函数展开法[9-12]等一系列求解方法应运而生.

lump 解[13-17]作为一种在空间各个方向都局域的有理函数解,在近些年越来越受到了研究者们的关注[18-22].与此同时,Hirota 提出了一种获得非线

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性偏微分方程孤立子解的简单而直接的方法,即双线性导数法[23-25]. 本文就是基于 Hirota 双线性方法研究了研究广义(3+1)维浅水波方程

$$u_{xxxy}+3u_{xx}u_y+3u_xu_{xy}-u_{yt}-u_{xz}=0 \quad (1)$$

的 lump 解,并且求出了 lump 解的位置、高度、运动轨迹等等动力学性质. 然后给出了方程的呼吸波解,最后研究了不同类型孤立子之间的相互作用. 此外,由于孤立子之间的碰撞方式有两种:一种是弹性碰撞[26-28],即孤子在碰撞前后除了相位发生变化之外,其大小、形状、速度均没有发生明显改变;另一种为非弹性碰撞[29-31],也就是孤子在碰撞之后,不仅相位发生了变化,而且孤子的动能、动量、形状也都发生了变化. 在本文中研究的是 lump 孤子与扭结孤立子之间的完全非弹性碰撞(融合成为新的孤子,不再分开),并且解释了 lump 孤子被扭结孤立子吞噬消失的原因.

1　方程的 lump 解

对于式(1),先使用 Hopf-Cole 变换

$$u=2(\ln f)_x \quad (2)$$

得到方程的双线性形式

$$(D_x^3D_y-D_yD_t-D_xD_z)f\cdot f=0 \quad (3)$$

式中,D 算子为 Hirota 双线性算符.

再对式(3)进行约化,令 $z=x$,得到

$$(D_x^3D_y-D_yD_t-D_x^2)f\cdot f=0 \quad (4)$$

再设

$$f=g^2+h^2+a_9 \quad (5)$$

$$g=a_1x+a_2y+a_3t+a_4 \quad (6)$$

$$h=a_5x+a_6y+a_7t+a_8 \quad (7)$$

式中,$a_i(i=1,\cdots,9)$为待定常系数.

然后将式(5)～(7)代入式(4),再将式(4)整理化简为 x,y,t 的幂次多项式,并使 x,y,t 幂次前的系数为零,得到一组代数方程组,最后求解该代数方程组得到

$$\left.\begin{aligned}&a_3=\frac{a_2a_5^2-2a_1a_5a_6-a_1^2a_2}{a_2^2+a_6^2},\\&a_7=\frac{a_1^2a_6-2a_1a_2a_5-a_5^2a_6}{a_2^2+a_6^2},\\&a_9=\frac{3(a_2^2+a_6^2)(a_1^2+a_5^2)(a_1a_2+a_5a_6)}{(a_1a_6-a_2a_5)^2},\\&a_i=a_i(i=1,2,4,5,6,8)\end{aligned}\right\} \quad (8)$$

将式(8)代入式(5)～(7),再利用式(2),就能得到约化的方程式(1)的 lump 解. 同时选择参数

$$a_1=2,a_2=2,a_4=1,a_5=-1,a_6=1,a_8=1 \quad (9)$$

再由式(8)得

$$a_3=-\frac{2}{5},a_7=\frac{11}{5},a_9=\frac{225}{16} \quad (10)$$

最后经过一系列的运算得到

$$u=\frac{320(5x+3y-3t+1)}{400x^2+400y^2+400t^2+480xy-480xt+224yt+160x+480y+288t+1285} \quad (11)$$

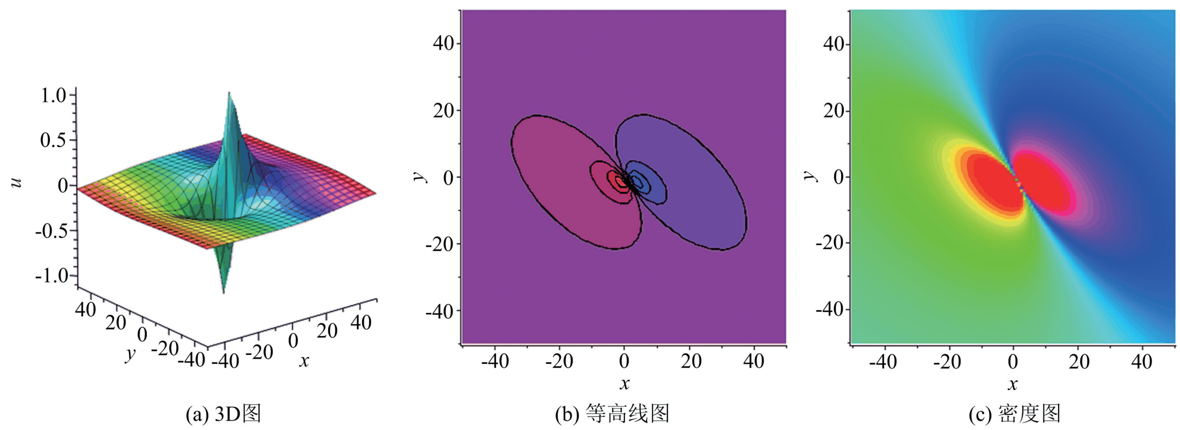


图 1　$t=0$ 时 lump 孤子

Fig. 1　Lump soliton when $t=0$

图 1 为 $t=0$ 时的 lump 型孤子的图像. 图 1 显示该方程的 lump 型孤子由一个高耸的波峰与一个低垂的波谷组成,并且被拘束在有限范围之内. 与此同时,对于约化的方程式(1)的 lump 解

$$u=2(\ln f)_x=4\frac{a_1g+a_5h}{f} \tag{12}$$

假定 t 是常数,然后分别求出 u 对 x,y 的偏导数,并令偏导数为 0,可以得到 2 个驻点的坐标:

$$x_1=2\frac{(a_1a_2+a_5a_6)t}{a_2^2+a_6^2}+\frac{a_2a_8-a_4a_6+\sqrt{3(a_2^2+a_6^2)(a_1a_2+a_5a_6)}}{a_1a_6-a_2a_5} \tag{13}$$

$$y_1=\frac{(a_1^2+a_5^2)t}{a_2^2+a_6^2}+\frac{a_1a_8-a_4a_5}{a_1a_6-a_2a_5} \tag{14}$$

$$x_2=2\frac{(a_1a_2+a_5a_6)t}{a_2^2+a_6^2}+\frac{a_2a_8-a_4a_6-\sqrt{3(a_2^2+a_6^2)(a_1a_2+a_5a_6)}}{a_1a_6-a_2a_5} \tag{15}$$

$$y_2=\frac{(a_1^2+a_5^2)t}{a_2^2+a_6^2}+\frac{a_1a_8-a_4a_5}{a_1a_6-a_2a_5} \tag{16}$$

通过计算可得,在驻点(x_1y_1)处的 Hessian 矩阵

$$\Delta=\begin{vmatrix}\frac{\partial^2u(xy)}{\partial x^2} & \frac{\partial^2u(xy)}{\partial x\partial y}\\ \frac{\partial^2u(xy)}{\partial x\partial y} & \frac{\partial^2u(xy)}{\partial y^2}\end{vmatrix}_{(x_1,y_1)}=\frac{4(a_1a_6-a_2a_5)^8}{27(a_1a_2+a_5a_6)^3(a_2^2+a_6^2)^3(a_1^2+a_5^2)^2} \tag{17}$$

和

$$u_{xx}(x_1,y_1)=-\frac{2(a_1a_6-a_2a_5)^3}{(3(a_2^2+a_6^2)(a_1a_2+a_5a_6))^{3/2}} \tag{18}$$

所以当 $a_1a_6-a_2a_5>0$, $a_1a_2+a_5a_6>0$ 时,$\Delta>0$, $u_{xx}(x_1,y_1)<0$,u 在驻点(x_1,y_1)取极大值也是最大值,即波峰高度为

$$u_{\max}=\frac{2(a_1a_6-a_2a_5)}{\sqrt{3(a_2^2+a_6^2)(a_1a_2+a_5a_6)}} \tag{19}$$

同理,在 $a_1a_6-a_2a_5>0$, $a_1a_2+a_5a_6>0$ 时,在驻点(x_2,y_2) 处的 Hessian 矩阵

$$\Delta=\begin{vmatrix}\frac{\partial^2u(xy)}{\partial x^2} & \frac{\partial^2u(xy)}{\partial x\partial y}\\ \frac{\partial^2u(xy)}{\partial x\partial y} & \frac{\partial^2u(xy)}{\partial y^2}\end{vmatrix}_{(x_2y_2)}=\frac{4(a_1a_6-a_2a_5)^8}{27(a_1a_2+a_5a_6)^3(a_2^2+a_6^2)^3(a_1^2+a_5^2)^2}>0 \tag{20}$$

$$u_{xx}(x_2,y_2)=\frac{2(a_1a_6-a_2a_5)^3}{(3(a_2^2+a_6^2)(a_1a_2+a_5a_6))^{3/2}}>0 \tag{21}$$

因此,u 在驻点(x_2y_2)取极小值也是最小值,即波谷深度为

$$u_{\min}=-\frac{2(a_1a_6-a_2a_5)}{\sqrt{3(a_2^2+a_6^2)(a_1a_2+a_5a_6)}} \tag{22}$$

在式(13)～(16)中,分别对 t 求导,还可以得到波峰或者波谷的运动速度:

$$v_x=\frac{2a_1^2a_2a_6-2a_1a_2^2a_5+2a_1a_5a_6^2-2a_2a_5^2a_6}{(a_1a_6-a_2a_5)(a_2^2+a_6^2)} \tag{23}$$

$$v_y=-\frac{a_1^3a_6-a_1^2a_2a_5+a_1a_5^2a_6-a_2a_5^3}{a_1a_2^2a_6+a_1a_6^3-a_2^3a_5-a_2a_5a_6^2} \tag{24}$$

$$v=\sqrt{v_x^2+v_y^2}=\frac{\sqrt{a_1^4+4a_1^2a_2^2+2a_1^2a_5^2+8a_1a_2a_5a_6+a_5^4+4a_5^2a_6^2}}{a_2^2+a_6^2} \tag{25}$$

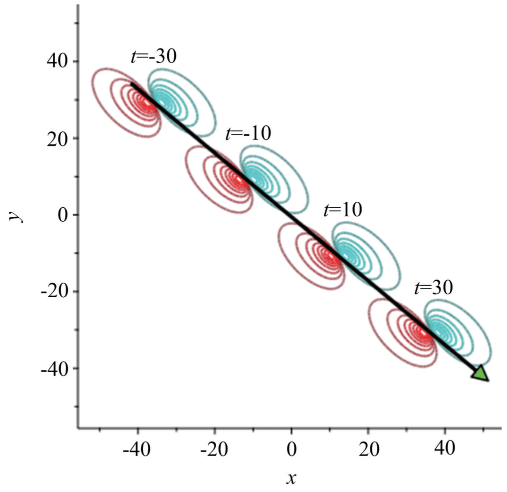
并且在(x_1,y_1)和(x_2,y_2)的坐标表达式(13)～(16)中消去时间 t,还可以得到波峰与波谷的运动轨迹方程:

$$y_1=-\frac{(a_1^2+a_5^2)x_1}{2(a_1a_2+a_5a_6)}+\frac{2a_1a_5(a_2a_4-a_6a_8)-(a_1-a_5)(a_1+a_5)(a_2a_8+a_4a_6)}{2(a_1a_2+a_5a_6)(a_1a_6-a_2a_5)}+\frac{\sqrt{3(a_2^2+a_6^2)(a_1a_2+a_5a_6)}(a_1^2+a_5^2)}{2(a_1a_2+a_5a_6)(a_1a_6-a_2a_5)} \tag{26}$$

$$y_2=-\frac{(a_1^2+a_5^2)x_2}{2(a_1a_2+a_5a_6)}+\frac{2a_1a_5(a_2a_4-a_6a_8)-(a_1-a_5)(a_1+a_5)(a_2a_8+a_4a_6)}{2(a_1a_2+a_5a_6)(a_1a_6-a_2a_5)}-\frac{\sqrt{3(a_2^2+a_6^2)(a_1a_2+a_5a_6)}(a_1^2+a_5^2)}{2(a_1a_2+a_5a_6)(a_1a_6-a_2a_5)} \tag{27}$$

再将式(26)～(27)相加后除以 2,得到 lump 型孤子中心的运动轨迹方程:

$$y_C = -\frac{(a_1^2 + a_5^2)x_C}{2(a_1a_2 + a_5a_6)} + \frac{2a_1a_5(a_2a_4 - a_6a_8) - (a_1 - a_5)(a_1 + a_5)(a_2a_8 + a_4a_6)}{2(a_1a_2 + a_5a_6)(a_1a_6 - a_2a_5)} \tag{28}$$



红色表示波谷,蓝色表示波峰

图 2　Lump 型孤子的运动轨迹

Fig. 2　Motion trajectory lump-type soliton

然后再选择与式(9)相同的参数,得到 lump 型孤子运动图像(见图 2).从图 2 中能够观察到 lump 型孤子沿着一条直线前进,并且从不同时刻孤子的位置可以看出 lump 孤子在做匀速运动,与式(25)相符合.

2　方程的呼吸解

在式(4)中设 f 的表达式为

$$f = b_1 e^{px+qy+wt} + b_2\cos(kx + ly + vt) + b_3 e^{-(px+qy+wt)} \tag{29}$$

式中,$b_1, b_2, b_3, p, q, w, k, l, v$ 为待定常系数.然后将式(29)代入式(4),整理化简为含有 $e^{px+qy+wt}$,$\sin(kx+ly+vt)$,$\cos(kx+ly+vt)$的各次幂多项式,并令其各次幂的系数为 0,得到一组代数方程组,求解该方程组得到满足呼吸波形式的解

$$\left.\begin{aligned} & k = -\frac{1}{3l}, l = l, p = p, q = \frac{1}{3p}, v = \frac{1}{27l^3}, \\ & w = p^3, b_i = b_i,\ i = 1,2,3 \end{aligned}\right\} \tag{30}$$

再将式(30)代入式(29),并且利用式(2)可得方程的呼吸波解为

$$u = \frac{2\left(3b_1 pl e^{\frac{3p^4t+3p^2x+y}{3p}} + b_2\sin\left(\frac{27l^4y - 9l^2x + t}{27l^3}\right) - 3b_3 pl e^{-\frac{3p^4t+3p^2x+y}{3p}}\right)}{3l\left(b_1 e^{\frac{3p^4t+3p^2x+y}{3p}} + b_2\cos\left(\frac{27l^4y - 9l^2x + t}{27l^3}\right) + b_3 e^{-\frac{3p^4t+3p^2x+y}{3p}}\right)} \tag{31}$$

同时选择参数

$$p = 1, l = 1, b_1 = 1, b_2 = 2, b_3 = 1 \tag{32}$$

然后代入式(31)得

$$u = \frac{2\left(3e^{x+\frac{y}{3}+t} + 2\sin\left(-\frac{x}{3} + y + \frac{t}{27}\right) - 3e^{-(x+\frac{y}{3}+t)}\right)}{3\left(e^{x+\frac{y}{3}+t} + 2\cos\left(-\frac{x}{3} + y + \frac{t}{27}\right) + e^{-(x+\frac{y}{3}+t)}\right)} \tag{33}$$

它在 $t=0$ 时的图像如图 3 所示.从图 3 可以看出该解一方面沿着某个方向做周期性的上下运动,符合呼吸波特征,同时还在与该方向相垂直的方向上做类似于扭结孤子一样的水平移动.

3　方程的相互作用解

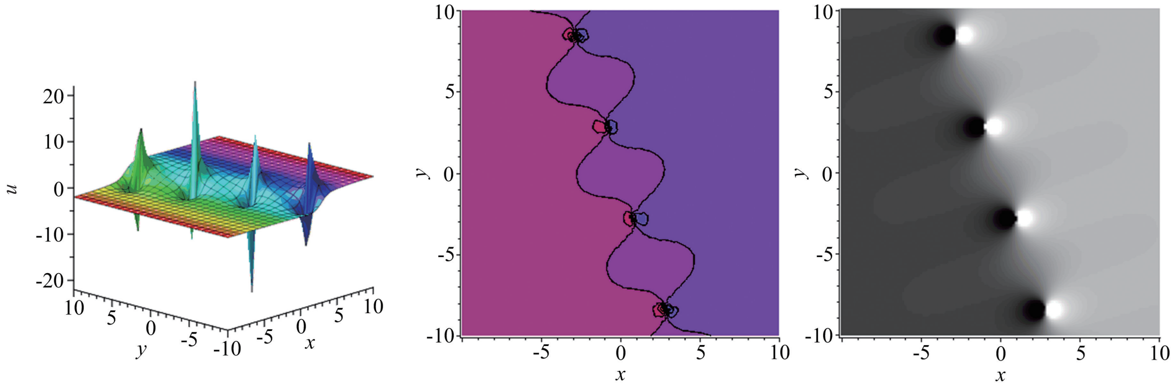
为了研究解之间的相互作用,将式(5)～(7)和式(29)进行组合叠加:

$$f = g^2 + h^2 + a_9 + b_1 e^{\xi} + b_2\cos(\eta) + b_3 e^{-\xi} \tag{34}$$

$$\xi = px + qy + wt \tag{35}$$

$$\eta = kx + ly + vt \tag{36}$$

式中,g,h 由式(6)～(7)给出.然后将式(34)～(36)代入式(4),按照求解 lump 解(式(5)～(8))和呼吸波解(式(29)～(30))类似的过程,得到 3 组解:



(a) 3D图　(b) 等高线图　(c) 密度图

图 3　不同角度下呼吸波的形状

Fig. 3　The shape of the breather-wave from different perspectives

第一组解：

$$\left.\begin{aligned}&k=k,l=l,p=p,q=\frac{12p^3a_6^2}{9p^4a_6^2+4a_1^2},v=v,w=\frac{3p^4a_6^2-4a_1^2}{12pa_6^2},a_2=0,\\&a_3=\frac{a_1(9p^4a_6^2-4a_1^2)}{6p^2a_6^2},a_5=\frac{4a_1^2-9p^4a_6^2}{12p^2a_6},a_7=\frac{216p^4a_1^2a_6^2-81p^8-a_6^4-16a_1^4}{144p^4a_6^3},\\&a_9=-\frac{(9p^4a_6^2-4a_1^2)(81p^8a_6^4+72p^4a_1^2a_6^2+16a_1^4)}{576p^6a_6^2a_1^2},\\&b_1=b_2=0,b_3=b_3,a_i=a_i,\ i=1,4,6,8\end{aligned}\right\}\tag{37}$$

第二组解：

$$\left.\begin{aligned}&k=k,l=l,p=\frac{2}{3q},q=q,v=v,w=-\frac{4}{27q^3},a_1=\pm\frac{2a_6}{3q^2},a_7=\frac{4a_6}{9q^4},\\&a_2=a_3=a_5=a_9=b_1=b_2=0,b_3=b_3,a_i=a_i,\ i=4,6,8\end{aligned}\right\}\tag{38}$$

第三组解：

$$\left.\begin{aligned}&k=k,l=l,p=p,q=\frac{2}{3p},v=v,w=-\frac{1}{2p^3},a_1=\mp\frac{3p^2a_6}{2},a_2=\pm\frac{2a_5}{3p^2},\\&a_3=\pm\frac{3p^2a_5}{2},a_7=\frac{9a_6q^4}{4},a_9=b_1=b_2=0,b_3=b_3,a_i=a_i,\ i=4,5,6,8\end{aligned}\right\}\tag{39}$$

再选择参数

$$k=1,l=1,p=1,v=1,a_4=1,a_5=1,a_6=1,a_8=1,b_3=50\tag{40}$$

代入第三组解中的

$$\left.\begin{aligned}&k=k,l=l,p=p,q=\frac{2}{3p},v=v,w=-\frac{1}{2p^3},a_1=-\frac{3p^2a_6}{2},a_2=\frac{2a_5}{3p^2},\\&a_3=\frac{3p^2a_5}{2},a_7=\frac{9a_6q^4}{4},a_9=b_1=b_2=0,b_3=b_3,a_i=a_i,\ i=4,5,6,8\end{aligned}\right\}\tag{41}$$

并利用式(2)和式(34)～(36)得到

$$u=\frac{144(13x-2-100\mathrm{e}^{\frac{t}{2}-x-\frac{2y}{3}})}{468x^2+208y^2+1053t^2+936ty-144x+480y+1080t+288+7200\mathrm{e}^{\frac{t}{2}-x-\frac{2y}{3}}}\tag{42}$$

然后画出不同时刻的相互作用解的图像(图 4). 从图 4 中可以看出，随着 lump 型孤子与扭结孤立波相互靠近，lump 型孤子的高度与深度逐渐变小，最后 lump 型孤子消失在扭结孤立波之中.

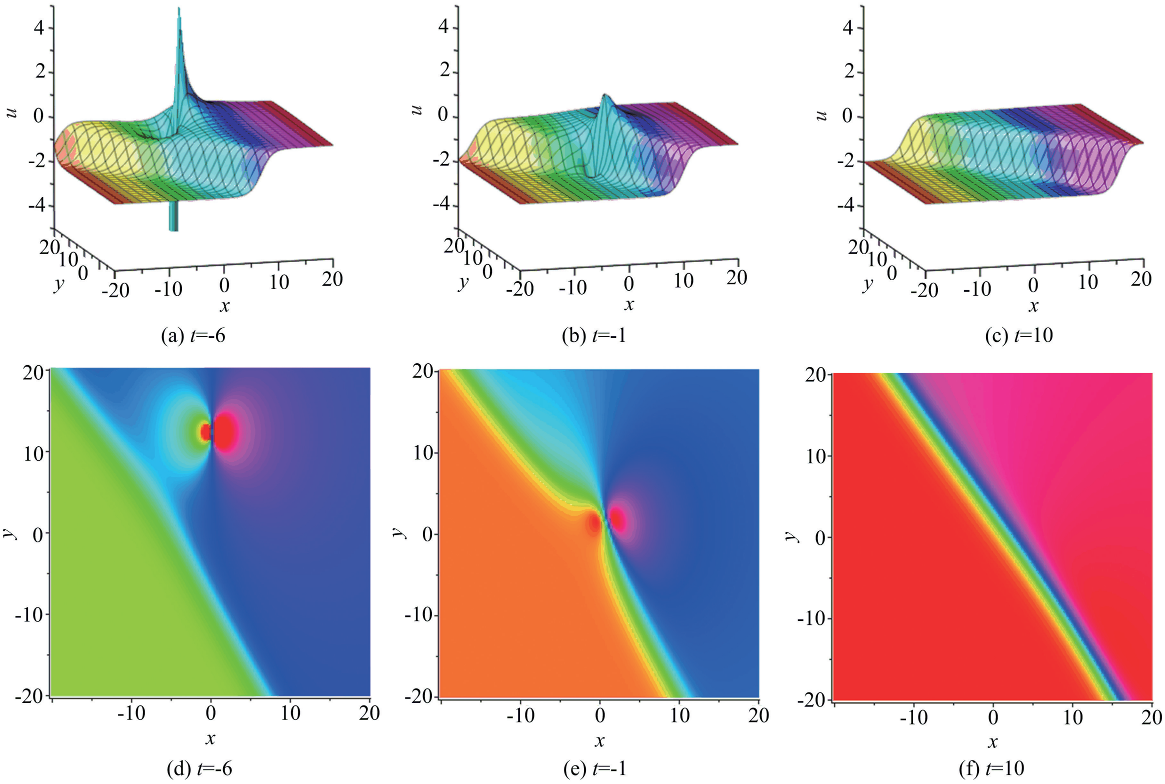


图 4　不同时刻相互作用解的 3D 图((a)～(c))与密度图((d)～(f))

Fig. 4　3D diagram ((a)～(c)) and density map ((d)～(f)) of interaction solutions at different time

对于上述 lump 消失现象,我们可以做如下解释:将式(39)代入式(34)～(36)得

$$f=g^2+h^2+b_3\mathrm{e}^{-\xi} \tag{43}$$

$$g=-\frac{3p^2a_6x}{2}+\frac{2a_5y}{3p^2}+\frac{3p^2a_5t}{2}+a_4 \tag{44}$$

$$h=a_5x+a_6y+\frac{9p^4a_6t}{4}+a_8 \tag{45}$$

从 f 的表达式可以看出,g^2,h^2 为代数部分(代数 lump 孤子),$\mathrm{e}^{-\xi}$ 为指数部分(孤立波部分),在 $\xi>0$ 的区域里,g^2,h^2 趋于无穷大,成为 f 的主要部分,于是整个解表现出 lump 型孤子形状,但是当 lump 型孤子运动到 $\xi<0$ 的区域时,指数部分 $\mathrm{e}^{-\xi}$ 趋于无穷大,所以 $\mathrm{e}^{-\xi}$ 替换先前的 g^2,h^2 成为 f 的主要部分,因此,lump 型孤子在靠近 $\xi<0$ 的区域过程中高度与深度逐渐变小至消失.

4　结论

本文利用 Hirota 双线性导数法,得到了约化的广义(3+1)维浅水波方程双线性形式,然后通过预先设解的形式,求出了方程的有理函数解(lump 解),并制作出 lump 型孤子的图像以及计算出其高度、深度、运动轨迹方程等特征量;再通过相同方法求出了该浅水波方程的呼吸波解;最后还研究了 lump 解与扭结波解的相互作用,解释 lump 孤子消失在扭结孤立子中的原因,即 lump 型孤子解与扭结孤子解在不同区域主次地位的变化.这些结果对于理解(3+1)维浅水波方程所描述的物理现象具有一定的理论指导作用.同时,运用该方法也可以探究其他形式的解的组合以及它们之间的相互作用,并能丰富方程解的形式.值得指出的是本文没有计算出 lump 孤子消失被扭结孤子吞噬的时间,也没有研究其他类型解之间的相互作用,所以在今后的工作中将对这两个方面进行详细具体全面的研究.

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