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Principal eigenvectors of nonnegative tensors and hypergraphs

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Abstract: Let T be a nonnegative weakly irreducible tensor, and Q(G) be the signless Laplacian tensor of a connected uniform hypergraph G. Some lower bounds of the principal ratio and some bounds on the entries for the principal eigenvector of T and Q(G) were given, respectively.

Key words: nonnegative weakly irreducible tensor; uniform hypergraph; signless Laplacian tensor; principal eigenvector

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非负张量和超图的主特征向量

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摘要:令T是一个非负不可约张量,Q(G)是一个连通一致超图的无符号拉普拉斯向量.分别给出了T和Q(G)主比率的一些界以及主特征向量元素的一些界.

关键词:非负弱不可约张量;一致超图;无符号拉普拉斯张量;主特征向量

0 Introduction

For a positive integer n, let $[n]=\{1,2,\cdots,n\}$. An order m dimension n complex tensor $\mathrm{T}=(t_{i_1\cdots i_m})$ consists of n^m entries: $t_{i_1\cdots i_m}\in\mathbb{C}$, where $i_j\in[n], j\in[m]$. If every entry $t_{i_1\cdots i_m}\geqslant 0$, then T is called nonnegative. Let $\mathbb{C}^{[m,n]}$ ($\mathbb{R}^{[m,n]}$) be the set of order m dimension n complex (real) tensors, and $\mathbb{C}^n$ be the set of n-vectors over the complex field $\mathbb{C}$.

In 2005, Qi[1] and Lim[2] proposed the concept of eigenvalues of tensors, independently. For $\mathrm{T}=(t_{i_1 i_2\cdots i_m})\in\mathbb{C}^{[m,n]}$, if there exists a number $\lambda\in\mathbb{C}$ and a nonzero vector $x=(x_1,\cdots,x_n)^{\mathrm{T}}\in\mathbb{C}^n$ such that

$$\mathrm{T}x^{m-1}=\lambda x^{[m-1]},$$

then λ is called an eigenvalue of T, x is called an eigenvector of T corresponding to λ, where $\mathrm{T}x^{m-1}$ and $x^{[m-1]}$ are vectors whose i-th component are

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$$(Tx^{m-1})_i=\sum_{i_2,i_3,\cdots,i_m=1}^{n} t_{ii_2\cdots i_m}x_{i_2}x_{i_3}\cdots x_{i_m}$$

and

$$(x^{[m-1]})_i=x_i^{m-1},$$

respectively. Let $\sigma(T)$ denote the set of all eigenvalues of T, the spectral radius $\rho(T)=\max\{|\lambda| \mid \lambda\in\sigma(T)\}$.

For a tensor $T=(t_{i_1i_2\cdots i_m})\in\mathbb{C}^{[m,n]}$, if there exists a nonempty proper index subset $I\subset\{1,\cdots,n\}$ such that

$$t_{i_1\cdots i_m}=0,\ \forall i_1\in I,\ \exists\ i_j\notin I,\ j=2,\cdots,m,$$

then T is called weakly reducible; if T is not weakly reducible, then T is weakly irreducible[3-4].

For a nonnegative weakly irreducible tensor $T\in\mathbb{R}^{[m,n]}$, by Perron-Frobenius theorem[4-5], we know that $\rho(T)$ is an eigenvalue of T and there exists a unique positive eigenvector $y=(y_1,\cdots,y_n)^T$ corresponding to $\rho(T)$ with $\sum_{i=1}^{n}y_i^m=1$. Such y is called the principal eigenvector of T, the maximum and minimum entries of y are denoted by $y_{\max}$ and $y_{\min}$, respectively. $\gamma=\frac{y_{\max}}{y_{\min}}$ is called the principal ratio of T.

Let $G=(V(G),E(G))$ be a hypergraph with vertex set $V(G)=[n]$ and edge set $E(G)=\{e_1,e_2,\cdots,e_m\}$. If each edge of G contains exactly r distinct vertices, then G is called r-uniform. In particular, 2-uniform hypergraphs are ordinary graphs. For $i\in[n]$, $E_i(G)$ denotes the set of edges of G containing i, and the degree of a vertex i is $d_i=|E_i(G)|$. We denote by $\Delta=\max\{d_i\}$ and $\delta=\min\{d_i\}$ the maximum and minimum degrees of the vertices of G, respectively. If $\Delta=\delta$, then G is called a regular hypergraph. The adjacency tensor[6] A_G of an r-uniform hypergraph G is an order r dimension n nonnegative tensor with entries

$$a_{i_1i_2\cdots i_r}=\begin{cases}\frac{1}{(r-1)!}, & \text{if } \{i_1,i_2,\cdots,i_r\}\in E(G);\\ 0, & \text{otherwise.}\end{cases}$$

The degree tensor D_G of G is an order r dimension n diagonal tensor, with diagonal entry $d_{i\cdots i}=d_i$, for all $i\in[n]$, $Q_G=D_G+A_G$ is the signless Laplacian tensor of G[7]. The spectral radius of Q_G is called the signless Laplacian spectral radius of G, denoted by $\mu(G)$.

In recent years, the study of the principal eigenvectors of tensors and hypergraphs has attracted extensive attention. Some bounds for the principal ratio of nonnegative tensors were given in Refs.[8-10]. For a connected uniform hypergraph G, some bounds on entries for the principal eigenvector of G were presented in Refs.[11-13].

This paper mainly studies the principal eigenvectors of nonnegative and hypergraphs. It is organized as follows. In Section 1, we give some bounds for the principal ratio, the maximum and minimum entries in the principal eigenvector of nonnegative weakly irreducible tensors. In Section 2, we obtain some bounds for the principal ratio, the entries in the principal eigenvector of the signless Laplacian tensors for connected uniform hypergraphs.

1 Principal eigenvectors of nonnegative weakly irreducible tensors

For a nonnegative tensor $T=(t_{i_1\cdots i_m})\in\mathbb{R}^{[m,n]}$, let

$$r_i(T)=\sum_{i_2,\cdots,i_m=1}^{n}t_{ii_2\cdots i_m},$$

$$r_i'(T)=\sum_{(i_2,\cdots,i_m)\neq(i,\cdots,i)}t_{ii_2\cdots i_m},$$

$$R'(T)=\max_i r_i'(T),\ r'(T)=\min_i r_i'(T).$$

In the following, the lower bound for the principal ratio of a nonnegative weakly irreducible tensor is presented.

Theorem 1.1 For a nonnegative weakly irreducible tensor $T=(t_{i_1\cdots i_m})\in\mathbb{R}^{[m,n]}$, let γ and $y=(y_1,\cdots,y_n)^T$ be the principal ratio and principal eigenvector of T, respectively. Let $r_i'(T)=R'(T)$, $r_j'(T)=r'(T)$, for $i,j\in[n]$. Then

$$\gamma\geqslant\left(\frac{(\rho(T)-t_{j\cdots j})R'(T)}{(\rho(T)-t_{i\cdots i})r'(T)}\right)^{\frac{1}{2(m-1)}}\tag{1}$$

Proof By $Ty^{m-1}=\rho(T)y^{[m-1]}$, we have

$$\rho(T)y_i^{m-1}=\sum_{i_2,\cdots,i_m=1}^{n}t_{ii_2\cdots i_m}y_{i_2}y_{i_3}\cdots y_{i_m},$$

then

$$(\rho(T)-t_{i\cdots i})y_i^{m-1}=\sum_{(i_2,\cdots,i_m)\neq(i,\cdots,i)}t_{ii_2\cdots i_m}y_{i_2}y_{i_3}\cdots y_{i_m}\geqslant\sum_{(i_2,\cdots,i_m)\neq(i,\cdots,i)}t_{ii_2\cdots i_m}y_{\min}^{m-1}=R'(T)y_{\min}^{m-1},$$

i.e.

$$(\rho(\mathrm{T})-t_{i\cdots i})y_i^{m-1}\geqslant R'(\mathrm{T})y_{\min}^{m-1} \quad (2)$$

Similarly, we can obtain

$$(\rho(\mathrm{T})-t_{j\cdots j})y_j^{m-1}\leqslant r'(\mathrm{T})y_{\max}^{m-1} \quad (3)$$

Combining (2) and (3) together gives

$$(\rho(\mathrm{T})-t_{i\cdots i})y_i^{m-1}\cdot r'(\mathrm{T})y_{\max}^{m-1}\geqslant (\rho(\mathrm{T})-t_{j\cdots j})y_j^{m-1}\cdot R'(\mathrm{T})y_{\min}^{m-1},$$

$$\left(\frac{y_{\max}}{y_{\min}}\right)^{m-1}\cdot\left(\frac{y_i}{y_j}\right)^{m-1}\geqslant\frac{(\rho(\mathrm{T})-t_{j\cdots j})R'(\mathrm{T})}{(\rho(\mathrm{T})-t_{i\cdots i})r'(\mathrm{T})},$$

$$\gamma^{2(m-1)}=\left(\frac{y_{\max}}{y_{\min}}\right)^{2(m-1)}\geqslant \left(\frac{y_{\max}}{y_{\min}}\right)^{m-1}\cdot\left(\frac{y_i}{y_j}\right)^{m-1}\geqslant \frac{(\rho(\mathrm{T})-t_{j\cdots j})R'(\mathrm{T})}{(\rho(\mathrm{T})-t_{i\cdots i})r'(\mathrm{T})} \quad (4)$$

which implies that

$$\gamma\geqslant\left(\frac{(\rho(\mathrm{T})-t_{j\cdots j})R'(\mathrm{T})}{(\rho(\mathrm{T})-t_{i\cdots i})r'(\mathrm{T})}\right)^{\frac{1}{2(m-1)}}.$$

Remark 1.1 If we take $\mathrm{T}=\mathrm{A}_G$ in Theorem 1.1, the result is given by Ref.[4, Theorem 2.1].

Applying the lower bound for the principal ratio γ of T, we can obtain the following result.

Theorem 1. 2 For a nonnegative weakly irreducible tensor $\mathrm{T}=(t_{i_1\cdots i_m})\in\mathbb{R}^{[m,n]}$ with the principal eigenvector $y=(y_1,\cdots,y_n)^{\mathrm{T}}$, let

$$r_i'(\mathrm{T})=R'(\mathrm{T}),\ r_j'(\mathrm{T})=r'(\mathrm{T}),$$

for $i,j\in[n]$. Then

① $y_{\max}\geqslant\left(\left(\frac{(\rho(\mathrm{T})-t_{i\cdots i})r'(\mathrm{T})}{(\rho(\mathrm{T})-t_{j\cdots j})R'(\mathrm{T})}\right)^{\frac{m}{2(m-1)}}+n-1\right)^{-\frac{1}{m}}$;

② $y_{\min}\leqslant\left(\left(\frac{(\rho(\mathrm{T})-t_{j\cdots j})R'(\mathrm{T})}{(\rho(\mathrm{T})-t_{i\cdots i})r'(\mathrm{T})}\right)^{\frac{m}{2(m-1)}}+n-1\right)^{-\frac{1}{m}}$.

Proof ① Let γ be the principal ratio of T. By Theorem 1.1, it easy to see

$$1=\sum_{l=1}^{n}y_l^m\leqslant(n-1)y_{\max}^m+y_{\min}^m= y_{\max}^m(n-1+\gamma^{-m})\leqslant y_{\max}^m\left(n-1+\left(\frac{(\rho(\mathrm{T})-t_{i\cdots i})r'(\mathrm{T})}{(\rho(\mathrm{T})-t_{j\cdots j})R'(\mathrm{T})}\right)^{\frac{m}{2(m-1)}}\right),$$

then we have

$$y_{\max}\geqslant\left(\left(\frac{(\rho(\mathrm{T})-t_{i\cdots i})r'(\mathrm{T})}{(\rho(\mathrm{T})-t_{j\cdots j})R'(\mathrm{T})}\right)^{\frac{m}{2(m-1)}}+n-1\right)^{-\frac{1}{m}}.$$

② Similar to the proof in ①, we obtain that ② holds.

Remark 1.2 If we take $\mathrm{T}=\mathrm{A}_G$ in Theorem 1.2, the result was given by Ref.[4, Theorem 2.2].

Next, we present another lower bound for $y_{\max}$ in terms of $\rho(\mathrm{T})$ and $r_i(\mathrm{T})$ of T.

Theorem 1.3 Let $\mathrm{T}\in\mathbb{R}^{[m,n]}$ be a nonnegative weakly irreducible tensor, and $y=(y_1,\cdots,y_n)^{\mathrm{T}}$ be the principal eigenvector of T. Then

$$y_{\max}\geqslant\frac{\rho(\mathrm{T})^{\frac{1}{m-1}}}{\left(\sum_{i=1}^{n}r_i(\mathrm{T})^{\frac{m}{m-1}}\right)^{\frac{1}{m}}}.$$

Equality holds if and only if $r_1(\mathrm{T})=\cdots=r_n(\mathrm{T})$.

Proof By $\mathrm{T}y^{m-1}=\rho(\mathrm{T})y^{[m-1]}$, for all $i\in[n]$, we have

$$\rho(\mathrm{T})y_i^{m-1}=\sum_{i_2,\cdots,i_m=1}^{n}t_{ii_2\cdots i_m}y_{i_2}y_{i_3}\cdots y_{i_m}\leqslant \sum_{i_2,\cdots,i_m=1}^{n}t_{ii_2\cdots i_m}y_{\max}^{m-1}=r_i(\mathrm{T})y_{\max}^{m-1}.$$

So we obtain

$$\rho(\mathrm{T})^{\frac{m}{m-1}}y_i^m\leqslant r_i(\mathrm{T})^{\frac{m}{m-1}}y_{\max}^m,$$

$$\rho(\mathrm{T})^{\frac{m}{m-1}}\sum_{i=1}^{n}y_i^m\leqslant\sum_{i=1}^{n}r_i(\mathrm{T})^{\frac{m}{m-1}}y_{\max}^m.$$

Since $\sum_{i=1}^{n}y_i^m=1$, we have

$$y_{\max}^m\geqslant\frac{\rho(\mathrm{T})^{\frac{m}{m-1}}}{\sum_{i=1}^{n}r_i(\mathrm{T})^{\frac{m}{m-1}}},$$

i.e.

$$y_{\max}\geqslant\frac{\rho(\mathrm{T})^{\frac{1}{m-1}}}{\left(\sum_{i=1}^{n}r_i(\mathrm{T})^{\frac{m}{m-1}}\right)^{\frac{1}{m}}}.$$

Equality holds if and only if

$$\rho(\mathrm{T})y_i^{m-1}=\sum_{i_2,\cdots,i_m=1}^{n}t_{ii_2\cdots i_m}y_{\max}^{m-1},$$

which implies that $y_{i_2}=y_{i_3}=\cdots=y_{i_m}=y_{\max}$, for all $i\in[n]$, if $t_{ii_2i_3\cdots i_m}\neq 0$. Since T is weakly irreducible, we have $y_1=y_2=\cdots=y_n=y_{\max}$, i.e. $r_1(\mathrm{T})=\cdots=r_n(\mathrm{T})$.

2 Principal eigenvectors of the signless Laplacian tensors

Let G be an r-uniform hypergraph, for $u,v\in V(G)$, a walk from u to v in G is defined to be a sequence of vertices and edges $v_0,e_1,v_1,\cdots,v_{p-1},e_p,v_p$ with $v_0=u$ and $v_p=v$ such that edge e_i contains vertices v_{i-1} and v_i, $v_{i-1}\neq v_i$ for $i\in[p]$. A hypergraph is connected if there is a walk connecting every pair of different vertices. If G is connected, then Q_G is a nonnegative weakly irreducible tensor[7]. In this section, we use γ_Q and $x=(x_1,\cdots,x_n)^T$ to denote the principal ratio and principal eigenvector of Q_G, respectively.

If we take $T=Q_G$ in Theorem 1.1, we obtain the following result.

Theorem 2.1 Let G be a connected r-uniform hypergraph with maximum degree Δ and minimum degree δ. Then

$$\gamma_Q \geqslant \left(\frac{(\mu(G)-\delta)\Delta}{(\mu(G)-\Delta)\delta}\right)^{\frac{1}{2(r-1)}}.$$

Remark 2.1 Since $\mu(G)\leqslant 2\Delta$, we have

$$\frac{(\mu(G)-\delta)\Delta}{(\mu(G)-\Delta)\delta}=\frac{\Delta}{\delta}\left(1+\frac{\Delta-\delta}{\mu(G)-\Delta}\right)\geqslant \frac{\Delta}{\delta}\left(1+\frac{\Delta-\delta}{\Delta}\right)=\frac{2\Delta}{\delta}-1 \tag{5}$$

i.e.

$$\gamma_Q \geqslant \left(\frac{(\mu(G)-\delta)\Delta}{(\mu(G)-\Delta)\delta}\right)^{\frac{1}{2(r-1)}} \geqslant \left(\frac{2\Delta}{\delta}-1\right)^{\frac{1}{2(r-1)}} \tag{6}$$

When equality in (5) holds, we have $\mu(G)=2\Delta$.

By the proof in Theorem 1.2 and (6), we can obtain the bound for the maximum and minimum entries of the signless Laplacian principal eigenvector of G.

Theorem 2.2 For a connected r-uniform hypergraph G on n vertices, let $x=(x_1,\cdots,x_n)^T$ be the signless Laplacian principal eigenvector of G. Then

① $x_{\max} \geqslant \left(\left(\frac{\delta}{2\Delta-\delta}\right)^{\frac{r}{2(r-1)}}+n-1\right)^{-\frac{1}{r}}$;

② $x_{\min} \leqslant \left(\left(\frac{2\Delta}{\delta}-1\right)^{\frac{r}{2(r-1)}}+n-1\right)^{-\frac{1}{r}}$.

If we take $T=Q_G$ in Theorem 1.3, we can get the following result.

Theorem 2.3 Let G be a connected r-uniform hypergraph with vertex degrees $d_1, d_2, \cdots, d_n$, $x=(x_1,\cdots,x_n)^T$ be the signless Laplacian principal eigenvector of G. Then

$$x_{\max} \geqslant \frac{\mu(G)^{\frac{1}{r-1}}}{2^{\frac{1}{r-1}}\left(\sum_{i=1}^{n} d_i^{\frac{r}{r-1}}\right)^{\frac{1}{r}}},$$

with equality if and only if G is regular.

Lemma 2.1 For a connected r-uniform hypergraph G with vertex degrees $d_1, d_2, \cdots, d_n$, let $x=(x_1,\cdots,x_n)^T$ and $\mu(G)$ be the signless Laplacian principal eigenvector and the signless Laplacian spectral radius of G, respectively. Then

$$\mu(G)\leqslant 2\sum_{i=1}^{n} d_i x_i^r,$$

with equality if and only if G is regular.

Proof Let $Q_G=(q_{i_1\cdots i_r})$ be the signless Laplacian tensor of G. Then

$$\mu(G)=x^T(Q_G x^{r-1})=\sum_{i_1,\cdots,i_r=1}^{n} q_{i_1\cdots i_r} x_{i_1}\cdots x_{i_r}=\sum_{i=1}^{n} d_i x_i^r + r\sum_{e\in E(G)} x^e,$$

where $x^e=x_{i_1}x_{i_2}\cdots x_{i_r}$, $\{i_1,i_2,\cdots,i_r\}=e$ (see Refs.[6-7]).

$$2\sum_{i=1}^{n} d_i x_i^r-\mu(G)=2\sum_{i=1}^{n} d_i x_i^r-\left(\sum_{i=1}^{n} d_i x_i^r+r\sum_{e\in E(G)} x^e\right)=\sum_{i=1}^{n} d_i x_i^r-r\sum_{e\in E(G)} x^e=\sum_{\{i_1,\cdots,i_r\}=e\in E(G)}(x_{i_1}^r+\cdots+x_{i_r}^r-r x_{i_1}\cdots x_{i_r})\geqslant 0.$$

Equality holds if and only if $x_1=x_2=\cdots=x_n$, which implies G is regular.

In the following, a simple lower bound on $x_{\max}$ is presented.

Theorem 2.4 Let G be a connected r-uniform hypergraph with n vertices and m edges, $x=(x_1,\cdots,x_n)^T$ be the signless Laplacian principal eigenvector of G. Then

$$x_{\max} \geqslant \left(\frac{\mu(G)}{2rm}\right)^{\frac{1}{r}},$$

with equality if and only if G is regular.

Proof Let Q_G be the signless Laplacian tensor of G. By Lemma 2.1, we have

$$\mu(G)\leqslant 2\sum_{i=1}^{n} d_i x_i^r \leqslant 2\sum_{i=1}^{n} d_i x_{\max}^r = 2rm x_{\max}^r.$$

Then

$$x_{\max} \geqslant \left(\frac{\mu(G)}{2rm}\right)^{\frac{1}{r}}.$$

Equality holds if and only if $x_1=x_2=\cdots=x_n$, it follows that G is regular.

We give the upper bound for ith entry of the signless Laplacian principal eigenvector in the following theorem.

Theorem 2.5 Let G be a connected r-uniform hypergraph with vertex degrees $d_1, d_2, \cdots, d_n$, $x=(x_1,\cdots,x_n)^{\mathrm{T}}$ being the signless Laplacian principal eigenvector of G. Then for all $i\in[n]$, we have

$$x_i \leqslant \left(\frac{\mu(G)}{4(\mu(G)-d_i)}\right)^{\frac{1}{r}}.$$

Equality holds if and only if G is a hypergraph with one edge.

Proof Since $\mu(G)x^{[r-1]}=Q_G x^{r-1}$, for all $i\in[n]$, we know that

$$\mu(G)x_i^{r-1}=(Q_G x^{r-1})_i=d_i x_i^{r-1}+\sum_{e\in E_i(G)} x^{e\setminus\{i\}} \tag{7}$$

where $x^{e\setminus\{i\}}=x_{i_2}x_{i_3}\cdots x_{i_r}$, $\{i, i_2,\cdots,i_r\}=e$. So we have

$$\mu(G)x_i^r=d_i x_i^r+\sum_{e\in E_i(G)} x^e \tag{8}$$

By Eqs.(7) and (8) we have

$$2\mu(G)x_i^r-\mu(G)=2\mu(G)x_i^r-\mu(G)\sum_{j=1}^n x_j^r=$$

$$2\Big(d_i x_i^r+\sum_{e\in E_i(G)} x^e\Big)-\sum_{j=1}^n\Big(d_j x_j^r+\sum_{e\in E_j(G)} x^e\Big)=$$

$$2d_i x_i^r-\sum_{j=1}^n d_j x_j^r-\Big(\sum_{j=1}^n\sum_{e\in E_j(G)} x^e-2\sum_{e\in E_i(G)} x^e\Big)=$$

$$2d_i x_i^r-\sum_{j=1}^n d_j x_j^r-\Big(\sum_{j\neq i}\sum_{e\in E_j(G)} x^e-\sum_{e\in E_i(G)} x^e\Big),$$

clearly, $\sum_{j\neq i}\sum_{e\in E_j(G)} x^e-\sum_{e\in E_i(G)} x^e\geqslant 0$. Then we get

$$2\mu(G)x_i^r-\mu(G)\leqslant 2d_i x_i^r-\sum_{j=1}^n d_j x_j^r.$$

By Lemma 2.1, it follows that

$$2\mu(G)x_i^r-\mu(G)\leqslant 2d_i x_i^r-\frac{\mu(G)}{2},$$

and hence we have

$$x_i \leqslant \left(\frac{\mu(G)}{4(\mu(G)-d_i)}\right)^{\frac{1}{r}}.$$

Equality holds if and only if

$$\sum_{j\neq i}\sum_{e\in E_j(G)} x^e-\sum_{e\in E_i(G)} x^e=0$$

and G is regular. $\sum_{j\neq i}\sum_{e\in E_j(G)} x^e-\sum_{e\in E_i(G)} x^e=0$, implies all edges of G contain the vertex i, so G is a hypergragh with one edge.

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