第46卷第6期 中国科学技术大学学报 Vol. 46, No. 6
2016年6月 JOURNAL OF UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA Jun. 2016

Article ID: 0253-2778(2016)06-0494-08

Combined study of the η and η' mesons: Phenomenology, chiral extrapolation of lattice QCD and effective field theory

GUO Zhihui[1,2]

(1. Department of Physics, Hebei Normal University, Shijiazhuang 050024, China)
(2. State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, CAS, Beijing 100190, China)

Abstract: A comprehensive phenomenological study is carried out of the decay processes with the η or η' in the initial/final states within the effective field theory approach. Two primary types of processes are analyzed: The ones only with light-flavor hadrons and those involving the J/ψ. The couplings from the effective Lagrangian, together with the η-η' mixing parameters from the two-mixing-angle scheme, are fitted to a large number of experimental data, including the various decay widths and the form factors. With the phenomenological mixing parameters and the lattice simulation data of the masses and decay constants of the light pseudoscalar mesons as inputs, a next-to-next-to-leading order study is performed of the η-η' mixing system in $U(3)$ chiral perturbation theory. Updated values of the relevant low energy constants are obtained.

Key words: chiral Lagrangian; η and η' mixing

CLC number: O572.3 **Document code**: A doi: 10.3969/j.issn.0253-2778.2016.06.008

Citation: GUO Zhihui. Combined study of the η and η' mesons: Phenomenology, chiral extrapolation of lattice QCD and effective field theory[J]. Journal of University of Science and Technology of China, 2016, 46(6): 494-501.
郭志辉. 物理的唯象学和格点手征延拓研究[J]. 中国科学技术大学学报,2016,46(6): 494-501.

物理的唯象学和格点手征延拓研究

郭志辉[1,2]

(1. 河北师范大学物理科学与信息工程学院, 河北石家庄, 050024)
(2. 中国科学院理论物理所,理论物理国家重点实验室, 北京, 100190)

摘要:首先在有效场论框架下对 η 和 η' 唯象学进行了全面的研究,主要包括两种类型的衰变过程:一种是只含有轻味强子的过程;另一种是有 J/ψ 参与的反应过程.通过拟合大量相关的衰变宽度和形状因子的实验数据,我们确定了有效拉氏量中自由参数的取值,同时也给出了双混合角机制下的 η-η' 混合参数.结合通过实验数据定出的混合参数以及格点量子色动力学数值模拟给出的轻赝标介子质量和衰变常数,我们在 $U(3)$ 手征微扰理论框架下对 η-η' 混合进行了次次领头阶的分析,并给出了更新的低能耦合常数的数值.

关键词:手征有效场论; η-η' 混合

Received: 2015-11-30; **Revised**: 2016-04-20
Foundation item: Supported by National Natural Science Foundation of China (U1232105), Ministry of Science and Technology (2015CB856700).
Biography: GUO Zhihui (corresponding author), Professor/PhD. Research field: high energy physics. E-mail: zhguo@mail.hebtu.edu.cn

0 Introduction

The light-flavor η and η' mesons provide a valuable window to study many important nonperturbative properties of Quantum Chromodynamics (QCD), including the spontaneously chiral symmetry breaking, the mechanism of explicit breaking of $SU(3)$-flavor symmetry and the $U(1)_A$ anomaly of strong interactions.

There are many experimental collaborations that have measured or planned to measure the physical processes with the η or η' mesons, with high precision and high statistics, such as BES Ⅲ[1-2], Jefferson Lab[3-4], KLOE[5], CELSIUS/WASA[6], CBELSA/TAPS[7] and CMD-2[8]. On the other hand, lattice QCD simulations have greatly progressed on the η and η' mesons and many precise simulation data have been released[9-13].

We have performed a thorough analysis of the radiative decay processes involving the η or η' and light-flavor vector resonances within the framework of resonance chiral theory ($R\chi T$) in Ref.[14]. In the following sections, we extend the discussions to study the decay processes of $J/\psi \to VP$, $P\gamma^{(*)}$, with V denoting the light-flavor vectors and P the light pseudoscalar mesons[15]. The two-mixing-angle scheme for the η-η' system is used in these phenomenological discussions, and precise values of the four mixing parameters are extracted from the experimental data. Together with the phenomenological determinations of mixing parameters and the lattice simulation data as inputs, we have then carried out the next-to-next-to-leading order (NNLO) study of the η-η' mixing within the $U(3)$ chiral perturbation theory (χPT). The phenomenological inputs and lattice simulations are successfully reproduced, with reasonable values of low energy constants (LECs)[16]. In this paper, we briefly review the works in Refs. [14-16].

1 Theoretical formalism for the decay processes

1.1 Radiative processes with light-flavor hadrons

To describe the dynamics between the light psuedoscalar mesons and the light-flavor vector resonances, we use the relevant chiral Lagrangian from $R\chi T$ to calculate the decay widths and form factors. We simply elaborate the pertinent chiral Lagrangians in the following. The kinetic terms for the light vector resonances read

$$\mathscr{L}_{\rm kin}(V) = -\frac{1}{2}(\nabla^{\lambda}V_{\lambda\mu}\nabla_{\nu}V^{\nu\mu} - \frac{M_V^2}{2}V_{\mu\nu}V^{\mu\nu}) \quad (1)$$

with $V^{\nu\mu}$ denoting the vector octet plus singlet described in the anti-symmetric tensor formalism[17]. The transitions between the vectors and the photon are governed by the operator[17].

$$\mathscr{L}_2(V) = -\frac{F_V}{2\sqrt{2}}(V_{\mu\nu} - f_+^{\mu\nu}) \quad (2)$$

where $f_+^{\mu\nu}$ contain the external source fields. We are only interested in the photon field in this work. The VJP operators with one vector field, one external source and one light pseudoscalar, including the singlet η_0 state, read[14,18].

$$\begin{aligned}\mathscr{L}_{\rm VJP} = & \frac{\tilde{c}_1}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, \tilde{f}_+^{\rho\alpha}\}\nabla_\alpha\tilde{u}^\sigma\rangle + \\ & \frac{\tilde{c}_2}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\alpha}, \tilde{f}_+^{\rho\sigma}\}\nabla_\alpha\tilde{u}^\nu\rangle + \\ & \frac{i\tilde{c}_3}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, \tilde{f}_+^{\rho\sigma}\}\tilde{\chi}_-\rangle + \\ & \frac{i\tilde{c}_4}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle V^{\mu\nu}[\tilde{f}_-^{\rho\sigma}, \tilde{\chi}_+]\rangle + \\ & \frac{\tilde{c}_5}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu}, \tilde{f}_+^{\rho\alpha}\}\tilde{u}^\sigma\rangle + \\ & \frac{\tilde{c}_6}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\alpha}, \tilde{f}_+^{\rho\sigma}\}\tilde{u}^\nu\rangle + \\ & \frac{\tilde{c}_7}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, \tilde{f}_+^{\rho\alpha}\}\tilde{u}^\alpha\rangle - \\ & i\tilde{c}_8 M_V\sqrt{\frac{2}{3}}\varepsilon_{\mu\nu\rho\sigma}\langle V^{\mu\nu}\tilde{f}_+^{\rho\sigma}\rangle\ln(\det\tilde{u}) \end{aligned} \quad (3)$$

where the light pseudoscalar multiplet is incorporated in the $\tilde{u}_\mu$, $\tilde{u}$ and $\chi_\pm$ fields. The VVP types of effective Lagrangian involving η_0 read[14,18].

$$\begin{aligned}\mathscr{L}_{\rm VVP} = & \tilde{d}_1\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha\tilde{u}^\sigma\rangle + \\ & i\tilde{d}_2\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\tilde{\chi}_-\rangle + \\ & \tilde{d}_3\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu}, V^{\rho\alpha}\}\tilde{u}^\sigma\rangle + \\ & \tilde{d}_4\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, V^{\rho\alpha}\}\tilde{u}^\alpha\rangle - \\ & i\tilde{d}_5 M_V^2\sqrt{\frac{2}{3}}\varepsilon_{\mu\nu\rho\sigma}\langle V^{\mu\nu}, V^{\rho\alpha}\rangle\ln(\det\tilde{u}) \end{aligned} \quad (4)$$

In addition, the relevant part of the Wess-Zumino-Witten Lagrangian is

$$\mathscr{L}_{WZW} = \frac{\sqrt{2}N_C}{8\pi^2 F}\varepsilon_{\mu\nu\rho\sigma}(\Phi\partial^\mu v^\nu \partial^\rho v^\sigma) \tag{5}$$

We refer to Refs. [14,17-18] for further details about the previous Lagrangians. We then calculate the transition amplitudes, depicted by the Feynman diagrams shown in Figs. 1 and 2. With these transition amplitudes, it is straightforward to get the experimentally observed form factors and decay widths[14].

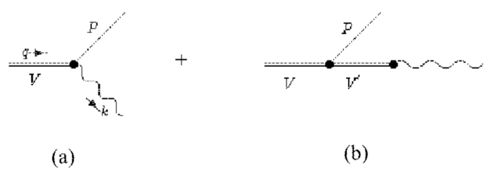

Fig. 1 Feynman diagrams for the $VP\gamma^{(*)}$ types of processes

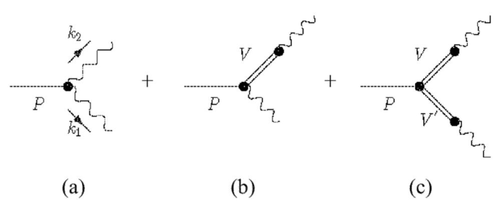

Fig. 2 Feynman diagrams for the $P\gamma\gamma^{(*)}$ types of processes

Before ending this section, we elaborate one more detail about the two-mixing-angle scheme to treat the η and η' mesons. The light pseudoscalar octet plus singlet mesons are incorporated in the $\tilde{u}$ field in the previous Lagrangians and we take the following two-mixing-angle formalism when calculating the amplitudes with η or η' states

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix} = \frac{1}{F}\begin{pmatrix}F_8\cos\theta_8 & -F_0\sin\theta_0\\ F_8\sin\theta_8 & F_0\cos\theta_0\end{pmatrix}\begin{pmatrix}\eta_s\\ \eta_0\end{pmatrix} \tag{6}$$

where η_0 and η_s stand for the $SU(3)$-flavor singlet and octet states, respectively, and η, η' denote the physical states. The four mixing parameters F_0, F_8, θ_0 and θ_8 will be fitted to experimental data.

1.2 Calculation of the $J/\psi \to VP$, $P\gamma^{(*)}$ amplitudes

In this part, we introduce the effective Lagrangian describing the dynamics of the $J/\psi \to VP$ and $P\gamma^{(*)}$ decays. Both the strong and the electromagnetic (EM) interactions will enter into these decays. We include three terms to describe the strong interactions in the $J/\psi \to VP$ decays

$$\begin{aligned}\mathscr{L}_{\psi VP} = {} & M_\psi h_1 \varepsilon_{\mu\nu\rho\sigma}\psi^\mu\langle \tilde{u}^\nu V^{\rho\sigma}\rangle + \\ & \frac{1}{M_\psi}h_2 \varepsilon_{\mu\nu\rho\sigma}\psi^\mu\langle \{\tilde{u}^\nu, V^{\rho\sigma}\}\tilde{\chi}_+\rangle + \\ & M_\psi h_3 \varepsilon_{\mu\nu\rho\sigma}\psi^\mu\langle \tilde{u}^\nu\rangle\langle V^{\rho\sigma}\rangle\end{aligned} \tag{7}$$

Notice that proper M_ψ factors are introduced in the previous equation in order to make the couplings $h_{i=1,2,3}$ dimensionless.

Two effective operators are constructed to describe the $J/\psi P\gamma^{(*)}$ interaction

$$\begin{aligned}\mathscr{L}_{\psi P\gamma} = {} & g_1 \varepsilon_{\mu\nu\rho\sigma}\psi^\mu\langle \tilde{u}^\nu \tilde{f}_+^{\rho\sigma}\rangle + \\ & \frac{1}{M_\psi^2}g_2 \varepsilon_{\mu\nu\rho\sigma}\psi^\mu\langle \{\tilde{u}^\nu, \tilde{f}_+^{\rho\sigma}\}\tilde{\chi}_+\rangle\end{aligned} \tag{8}$$

The transition strength between the J/ψ and the photon field reads

$$\mathscr{L}_2^\psi = \frac{-1}{2\sqrt{2}}\frac{f_\psi}{M_\psi}\langle \hat{\psi}_{\mu\nu}\tilde{f}_+^{\mu\nu}\rangle \tag{9}$$

with $\hat{\psi}_{\mu\nu} = \partial_\mu \psi^\nu - \partial_\nu \psi^\mu$.

Together with these effective Lagrangians and also the ones given in Sect. 1.1, we can calculate the $J/\psi \to P\gamma^{(*)}$ and VP amplitudes. The pertinent Feynman diagrams are depicted in Figs. 3 and 4.

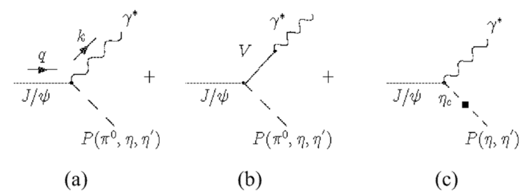

Fig. 3 Relevant Feynman diagrams for the $J/\psi \to P\gamma^*$ decays

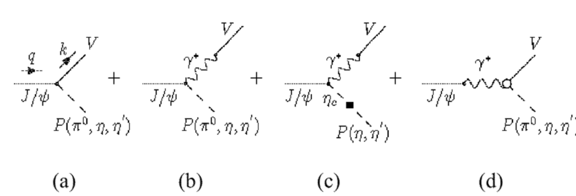

Fig. 4 Relevant Feynman diagrams for the $J/\psi \to VP$ decays

In order to reasonably reproduce the experimental results of the $J/\psi \to \eta'\gamma$ decay widths, it is necessary to include the mechanism depicted by the Fig. 3(c), i.e. to consider the contribution from the charmonium η_c and also the mixing between η_c and η'[19]. The decay widths and form factors can be easily obtained with the transition amplitudes corresponding to Figs. 3 and 4. We refer to Ref. [15] for further details.

2 Phenomenological discussions

We consider a large amount of experimental data in our work, including the available decay widths of $P\to V\gamma$, $V\to P\gamma$, $P\to\gamma\gamma$, $P\to\gamma l^+l^-$, $V\to Pl^+l^-$, $J/\psi\to P\gamma$ and $J/\psi\to VP$[20], with $P=\pi, K, \eta, \eta'$ and $V=\rho, K^*, \omega, \varphi$. In addition, the form factors of $J/\psi\to\eta'\gamma^*$, $\eta\to\gamma\gamma^*$, $\eta'\to\gamma\gamma^*$, $\varphi\to\eta\gamma^*$ will be also taken into account. We will make a global fit by including all of these data.

Before presenting our fit results, we point out that with the theoretical formalism in Sect. 1 alone it is impossible for us to reasonably reproduce the $J/\psi\to\omega\pi^0$ decay width. We simply include the excited vector ρ' in this channel to perform the fits[15].

A rather good reproduction of the experimental data in our global fit is achieved, with the χ^2/d.o.f close to one. The final results for the form factors of $J/\psi\to\eta'\gamma^*$, $\eta\to\gamma\gamma^*$ and $\eta'\to\gamma\gamma^*$ are given in Figs. 5, 6 and 7, respectively. Due to the large experimental error bars, the $\varphi\to\eta'\gamma^*$ form factor barely plays any important role in the fits and we do not explicitly show the result for this channel[14-15].

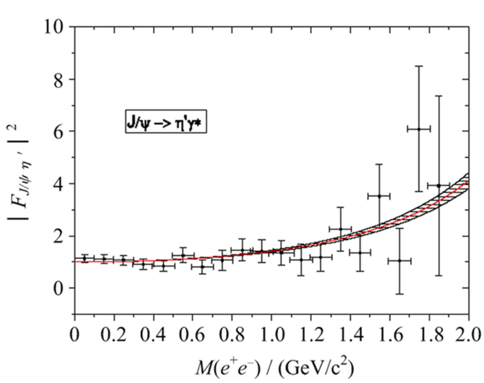

Fig. 5 The $J/\psi\to\eta'\gamma^*$ form factors

The solid (middle) line denotes our central results and the shaded areas represent the error bands at one-sigma level. The data are taken from Ref. [21].

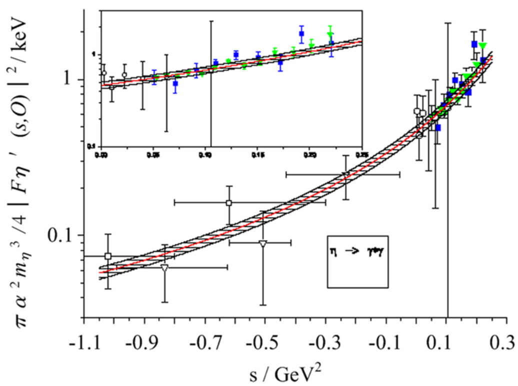

Fig. 6 The $\eta\to\gamma\gamma^*$ form factors

The solid (middle) line denotes our central values and the shaded areas represent the error bands at one-sigma level. The experimental data are taken from Refs. [22-27]. We clearly show the curve in the timelike region of $s>0$ in the framed figure.

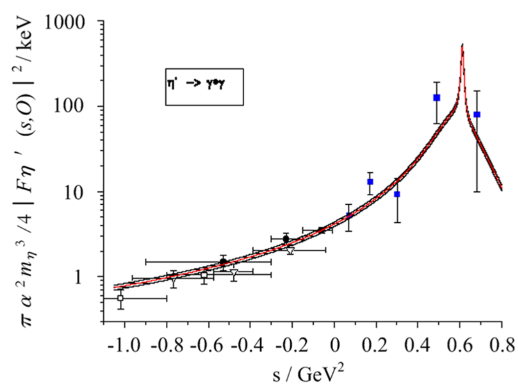

Fig. 7 The $\eta'\to\gamma\gamma^*$ form factors

The solid (middle) line denotes our central results and the shaded areas represent the error bands at one-sigma level. The experimental data are taken from Refs. [24-28].

In Tab. 1, we give the fitted values for the four mixing parameters defined in Eq. 6. In order to highlight the influence of the J/ψ data on the determination of the η-η' mixing, we explicitly show two different fit results. In the global fit, we include all of the previously mentioned experimental data, while in the partial fit situation only the data involving the light-flavor hadrons are taken into account.

Although the values of the mixing parameters from the two fits are compatible, the error bars after including the J/ψ data are clearly smaller than those with only the light-flavor data. Therefore one can conclude that the $J/\psi \to VP$, $P\ \gamma^*$ decays are important for constraining the η-η' mixing. For the remaining fitted parameters, we refer to Ref. [15] for details.

Tab. 1 Mixing parameters from the fits

parameter	global fit	partial fit
F_8/MeV	133.7 ± 3.7	126.3 ± 6.5
F_0/MeV	118.0 ± 5.5	109.7 ± 16.6
θ_8	$(-26.7\pm1.8)^\circ$	$(-21.1\pm6.0)^\circ$
θ_0	$(-11.0\pm1.0)^\circ$	$(-2.5\pm8.2)^\circ$

Next we analyze the different mechanisms that contribute to the $J/\psi \to Pl^+l^-$ processes. According to the Feynman diagrams in Fig. 3, there are three different kinds of contributions: The contact interacting vertex, the light-vector-resonance exchanges and the η_c-η' mixing. The effects from the intermediate vectors like $J/\psi \to \rho^0 P$, ωP and φP, with ρ^0, ω and φ decaying into the lepton pairs, have been removed when doing experimental analyses for the $J/\psi \to P\,e^+e^-$ decays in Ref. [21]. In order to be consistent with the experimental setups, we also drop the Fig. 3 (b) when fitting to the data. Nevertheless, we point out that it is a priori not justified to neglect the contributions from the intermediate light vectors in the $J/\psi \to Pl^+l^-$ decays. We have made a rough estimate that the light vectors can contribute around 30% in the $J/\psi \to \pi^0 e^+e^-$ decay[13], which qualitatively agrees with the findings in Refs. [29-31]. In our case, large destructive interference between the ρ^0 exchange and other mechanisms in the $J/\psi \to \pi^0\gamma$ are observed. As a result, a larger value of the branching ratio of the $J/\psi \to \pi^0 e^+e^-$ is predicted after neglecting the contributions from the intermediate ρ^0 resonance. So it is meaningful and important to make a revised experimental analysis on the $J/\psi \to \pi^0 e^+e^-$ decays by keeping all of the contributions, instead of removing parts of them. In contrast, the contributions from the intermediate light vectors turn out to be negligible in the $J/\psi \to \eta'\gamma$ decay processes. The branching ratios for the $J/\psi \to P\,l^+l^-$ are summarized in Tab. 2.

Tab. 2 Branching ratios ($\times10^{-5}$) for the $J/\psi \to Pl^+l^-$ decays

type	Exp	our results
$\psi \to \pi^0 e^+e^-$	0.0756 ± 0.0141	0.1191 ± 0.0138
$\psi \to \eta\, e^+e^-$	1.16 ± 0.09	1.16 ± 0.08
$\psi \to \eta' e^+e^-$	5.81 ± 0.35	5.76 ± 0.16
$\psi \to \pi^0 \mu^+\mu^-$	–	0.0280 ± 0.0032
$\psi \to \eta\, \mu^+\mu^-$	–	0.32 ± 0.02
$\psi \to \eta' \mu^+\mu^-$	–	1.46 ± 0.04

Another interesting subject is to analyze the roles of the strong and EM interactions in the $J/\psi \to VP$ decays. In our theoretical formalism, the strong interaction is depicted by the Fig. 4 (a), while the other diagrams correspond to the EM interactions. We confirm that the EM interactions play the dominant roles in the isospin violated decay channels, such as $J/\psi \to \rho^0\eta'$, and $\omega\pi^0$, and the strong interactions dominate in the isospin conserved channels, such as $J/\psi \to \rho\pi$, $\omega\eta'$, $\varphi\eta'$, $K^{*+}K^-$ and $K^{*0}\bar{K}^0$.

3 Chiral extrapolation of the η and η' masses

Previously when addressing the η-η' mixing, we simply adopted the two-mixing-angle scheme in Eq. (6) and do not give any further explanation to derive the formalism. Next we shall use the $U(3)\chi PT$ as an underlying theory to calculate the η-η' mixing pattern. In order to establish a consistent power counting, the simultaneous expansions on the momentum squared, light-quark masses and $1/N_C$, which will be denoted as δ expansion, need to be introduced into $U(3)\chi PT$. Up to NNLO in δ expansion, the pertinent chiral Lagrangians read

$$\mathscr{L}^{(\delta^0)} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle\chi_+\rangle + \frac{F^2}{12}M_0^2X^2 \tag{10}$$

$$\mathscr{L}^{\delta} = L_5\langle u^\mu u_\mu \chi_+\rangle + \frac{L_8}{2}\langle\chi_+\chi_+ + \chi_-\chi_-\rangle + \frac{F^2\Lambda_1}{12}D^\mu X D_\mu X - \frac{F^2\Lambda_2}{12}X\langle\chi_-\rangle \tag{11}$$

$$\mathscr{L}^{(\delta^2)} = \frac{F^2 v_2^{(2)}}{4}X^2\langle\chi_+\rangle + L_4\langle u^\mu u_\mu\rangle\langle\chi_+\rangle + L_6\langle\chi_+\rangle\langle\chi_+\rangle + L_7\langle\chi_-\rangle\langle\chi_-\rangle + L_{18}\langle u_\mu\rangle\langle u^\mu\chi_+\rangle + L_{25}X\langle\chi_+\chi_-\rangle + C_{12}\langle h_{\mu\nu}h^{\mu\nu}\chi_+\rangle + C_{14}\langle u_\mu u^\mu\chi_+\chi_+\rangle + C_{17}\langle u_\mu\chi_+ u^\mu\chi_+\rangle + C_{19}\langle\chi_+\chi_+\chi_+\rangle + C_{31}\langle\chi_-\chi_-\chi_+\rangle \tag{12}$$

With these chiral Lagrangians, one can then calculate the η-η' mixing pattern and express the mixing parameters in Eq. (6) in terms of the chiral LECs in Eqs. (10), (11) and (12). Due to the lengthy formulas, we refer to Ref. [16] for further details about the relations between the mixing parameters and LECs.

One of the biggest challenges when calculating the η-η' mixing pattern in $U(3)\chi PT$ is to determine the many unknown LECs. The precise lattice simulations of the light pseudoscalar mesons are valuable to constrain the values of the unknown LECs. We shall include in our fits the m_π dependence of the η and η' masses[9-13], the kaon masses[32-33], the π, K decay constants[32-33] and their ratios[34]. The previously determined phenomenological results of the η-η' mixing parameters will be also used to constrain these LECs.

It is interesting to mention that even at leading order the $U(3)\chi PT$, which only has one free parameter, can reasonably reproduce the lattice simulation data, as shown in Fig. 8 (The lattice simulation data are from Refs. [9-13]). While in order to simultaneously describe the lattice simulations on the light pseudoscalar mesons, specially the pion and kaon decay constants, it is essential to include the NNLO contributions. Among the NNLO LECs in Eq. (12), we fix $v_2^{(2)}$, L_{18}, L_{25} to zero, due to their marginal effects in our present discussion. While for the poorly known $\mathcal{O}(p^6)$ LECs C_i, we multiply the values of C_i from Refs. [35-36] by a common factor α in our fits. We find that two different sets of values from Refs. [35-36] lead to more or less similar results. Therefore, we only present the NNLO fit results by taking the values of C_i from Ref. [36] in Tab. 3. The values obtained here are compatible with the recent two-loop determinations of the next-to-leading order LECs[37]. The NNLO reproduction of the lattice simulation data is quite successful[16] and therefore the $U(3)$ chiral perturbation theory can be considered as a useful tool to perform the chiral extrapolation of the lattice data for the light pseudoscalars.

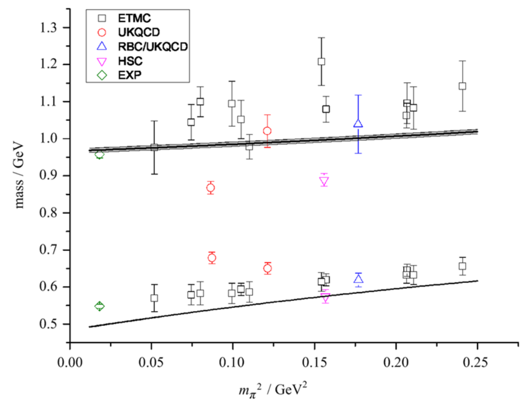

Fig. 8　The LO fit of the η and η' masses

Tab. 3　The values of the parameters from the NNLO fits by taking the $\mathcal{O}(p^6)$ LECs from Ref. [36]

parameter	numerical value
F_8/MeV	81.7 ±1.5 ±5.3
$10^3 \times L_5$	0.60 ±0.11 ±0.52
$10^3 \times L_8$	0.25 ±0.07 ±0.31
Λ_1	−0.003 ±0.060 ±0.19
Λ_2	0.08 ±0.11 ±0.20
$10^3 \times L_4$	−0.12 ±0.06 ±0.19
$10^3 \times L_6$	−0.05 ±0.04 ±0.02
$10^3 \times L_7$	0.26 ±0.05 ±0.06
α	0.59 ±0.09 ±0.18

4　Conclution

With the effective Lagrangian approach, we have made a comprehensive study of the processes involving η or η' mesons, including the radiative decays with light-flavor hadrons, such as the types of $VP\gamma^*$, $P\gamma\gamma^*$, and also the $J/\psi \to P\gamma^*$ and VP decays, with $P=\pi, K, \eta, \eta'$ and $V=\rho, K^*, \omega, \varphi$. The modern recipe of the two-mixing-angle scheme was used to describe the η and η' mesons. We made a global fit by considering a large amount of the experimental data. Reliable values of the η-η' mixing parameters, together with the resonance couplings from the effective Lagrangian, have been determined. We pointed out that the contributions from the intermediate light vectors in the $J/\psi \to \pi^0\gamma$ and $J/\psi \to \pi^0 l^+ l^-$ decays, with $l=e, \mu$, are important. Therefore a future revised experimental measurement is crucial to the verification of this mechanism.

We further calculate the η-η' mixing pattern from the underlying $U(3)$ chiral perturbation theory. And the four mixing parameters were expressed in terms of the chiral low energy constants. By including the lattice simulation data of the masses of η, η' and kaon, and the pion and kaon decay constants, we carried out a next-to-next-to-leading order study and determine the relevant chiral low energy constants, which are consisten with the recent two-loop results. We conclude that the $U(3)$ chiral perturbation theory can provide a useful tool to perform the chiral extrapolations of the lattice QCD data for the light pseudoscalar mesons.

References

[1] LI H B. η and η' Physics at BES-Ⅲ[J]. Journal of Physics, 2009, 36(8): 85009-85017.

[2] ZHANG Z Y, QIN L Q, FANG S S. Event generators for η/η' rare decays into $\pi^+\pi^-l^+l^-$ and $e^+e^-\mu^+\mu^-$ [J]. Chinese Physics C, 2012, 36(10): 926-931.

[3] FANGS S. η and η' Physics at BESⅢ[C]// The 7th International Workshop on Chiral Dynamics. Virginia, 2012: Pos (CD12) 036(1-6).

[4] GAN L P, GASPARIAN A. Search of new physics via eta rare decays[C]// Proceedings of the 6th International Workshop on Chiral Dynamics. Bern, Switzerland. 2009: 048(PoS CD 09).

[5] AMELINO-CAMELIAG, ARCHILLI F, BADONI D, et al. Physics with the KLOE-2 experiment at the upgraded DAφNE[J]. European Physics Journal C, 2010, 68(3-4): 619-681.

[6] BERLOWSKI M, BARGHOLTZ C, BASHKANOV M, et al. Measurement of eta meson decays into lepton-antilepton pairs[J]. Arxiv Cornell University Library, 2007, 77(3): 591-609.

[7] CREDE V, METAG V, SARANTSEV A V, et al. Photoproduction of η and η' mesons off protons[J]. Physical Review C, 2009, 80: 055202.

[8] AKHMETSHIN R R, ANASKIN E V, ARPAGAUS M, et al. Study of conversion decays $\varphi\to\eta e^+e^-$, $\eta\to e^+e^-\gamma$ and $\eta\to\pi^+\pi^-e^+e^-$ at CMD-2[J]. Physics Letter B, 2001, 501(3): 191.

[9] DUDEK J J, EDWARDS R G, JOO B, et al. Isoscalar meson spectroscopy from lattice QCD[J] Physical Review D, 2011, 83(11): 2437-2459.

[10] CHRIST N H, DAWSON C, IZUBUCHI T, et al. The η and η' mesons from lattice QCD[J]. Physical Review Letters, 2010, 105(1): 7-12.

[11] GREGORYE B, IRVING A C, RICHARDS C M, et al. A study of the η and η' mesons with improved staggered fermions[J]. Physical Review D, 2012, 86(1): 014504 (1-9).

[12] OTTNAL K, URBACH C, MICHAEL C. η and η′ masses and decay constants from lattice QCD with $N_f=2+1+1$ quark flavours [C]// Proceedings of 31st International Symposium on Lattice Field Theory. Mainz, Germany, 2013: 253-266.

[13] MICHAEL C, OTTNAL K, URBACH C, et al. η and η' mixing from lattice QCD[J]. Physical Review Letters, 2013, 111(18): 181602.

[14] CHEN Y H, GUO Z H, ZHENG H Q. Study of η-η' mixing from radiative decay processes [J]. Physical Review D, 2012, 85(5): 054018.

[15] CHEN Y H, GUO Z H, ZOU B S. Unified study of $J/\psi\to PV$, $P\gamma^*$ and light hadron radiative processes[J]. Physical Review D, 2015, 91(1): 014010.

[16] GUO X K, GUO Z H, Oller J A, et al. Scrutinizing the η-η' mixing, masses and pseudoscalar decay constants in the framework of $U(3)$ chiral effective field theory[J]. Journal of High Energy Physics, 2015, 6(2): 197-204.

[17] ECKER G, GASSER J, PICH A, et al. The role of resonances in chiral perturbation theory[J]. Nuclear Physics B, 1989, 321(12): 311-342.

[18] RUIZ-FEMENÍA P D, PICH A, PORTOLÉS J. Odd intrinsic parity processes within the resonance effective theory of QCD[J]. Journal of High Energy Physics, 2003, (7): 003(1-18).

[19] CHAO K T. Mixing of η, η' with *c* overline *c*, *b* overline *b* states and their radiative decays[J]. Nuclear Physics B, 1990, 335(1): 101-114.

[20] KLEIN S R, SARKAR S. Review of particle physics[J]. Chinese Physics C, 2014, 38(1): 594-602.

[21] ABLIKIM M, ACHASOV M N, AI X C, et al. Observation of electromagnetic Dalitz decays $J/\psi\to Pe^+e^-$ [J]. Physical Review D, 2014, 89(9): 092008(1-10).

[22] ARNALDI R, BANICZ K, CASTOR J, et al. Study of the electromagnetic transition form-factors in $\eta\to\mu^+\mu^-\gamma$ and $\omega\to\mu^+\mu^-\pi^0$ decays with NA60[J]. Physics Letters B, 2009, 677(5): 260-266.

[23] ACHASOV M N, AULCHENKO V M, BELOBORODOV K I, et al. Study of conversion decays $\varphi\to\eta e^+e^-$ *and* $\eta\to\gamma e^+e^-$ in the experiment with SND detector at the VEPP-2M collider[J]. Physics Letters B, 2001, 504(4): 275-281.

[24] DZHELYADIN R I, GOLOVKIN S V, KACHANOV V A, et al. Investigation of the electromagnetic structure of

the η meson in the decay $\eta \to \mu^+ \mu^- \gamma$[J]. Physics Letters B, 1980, 94(4): 548-550.

[25] DZHELYADIN R I, GOLOVKIN S V, GRITZK M V, et al. Observation of $\eta' \to \mu^+ \mu^- \gamma$ decay[J]. Physics Letters B, 1979, 88(3-4): 379-380.

[26] AIHARA H, ALSTONGARNJOST M, AVERY R E, et al. Investigation of the electromagnetic structure of η and η' mesons by two photon interactions [J]. Physical Review Letters, 1990, 64(2): 172-175.

[27] BEHREND H J, CRIEGEE L, FIELD J H, et al. A measurement of the π^0, η and η' electromagnetic form-factors[J]. Zeitschrift Für Physik C, 1991, 49(3): 401-409.

[28] ACCIARRI M, ADRIANI O, AGUILAR-BENITEZ M, et al. Measurement of η' (958) formation in two photon collisions at LEP-1[J]. Physics Letters B, 1998, 418(3-4): 399-410.

[29] KUBIS B, NIECKNIG F. Analysis of the $J/\psi \to \pi^0 \gamma^*$ transition form factor[J]. Physical Review D, 2015, 32(2): 5-28.

[30] ROSNER J L. Meson-photon transition form factors in the charmonium energy range[J]. Physical Review D, 2009, 79(9): 27-35.

[31] ZHAO Q. Understanding the radiative decays of vector charmonia to light pseudoscalar mesons [J]. Physics Letters B, 2011, 697(1): 52-57.

[32] AOKI Y, ARTHUR R, BLUM T, et al. Continuum limit physics from 2 + 1 flavor domain wall QCD[J]. Physical Review D, 2011, 83(7): 074508(1-129).

[33] ARTHUR R, BLUM T, BOYLE P A, et al. Domain wall QCD with near-physical pions[J]. Physical Review D, 2012, 87(9): 599-614.

[34] DURRS, FODOR Z, HOELBLING C, et al. The ratio FK/F_π in QCD [J]. Physical Review D, 2010, 81: 054507(1-15).

[35] JIANG S Z, ZHANG Y, LI C, et al. Computation of the p^6 order chiral Lagrangian coefficients [J]. Physical Review D, 2010, 81(1): 252-255.

[36] JIANGS Y, WEI Z L, CHEN Q S, et al. Computation of the $\mathscr{O}(p^6)$ order low-energy constants: An update[J]. Physical Review D, 2015, 92(2): 025014(1-30).

[37] BIJNENS J, ECKER G. Mesonic low-energy constants [J]. Annual Review of Nuclear & Particle Science, 2014, 64: 149-174.