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The MFCCA algorithm and its application in financial market: A new view of multifractal extension of DCCA

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Abstract: Multifractal extension of detrended cross-correlation analysis (DCCA) usually involves the trouble that the computation of arbitrary powers of the negative cross-covariances leads to complex values. However, a commonly adopted modulus processing method MFDXA often indicates significant multifractal cross-correlation signal when actually no fractality exists. Mulitfractal cross-correlation analysis (MFCCA) proposed by Oświecimka preserves the sign of the cross-covariances and settles the trouble above. MFCCA is a natural general extension of MFDFA and DCCA. Here it was demonstrated that MFCCA performs more effectively and powerfully than MFDXA from the view of the general two-component ARFIMA processes model. MFCCA can correctly identify the signal of multifractality behavior and show sensitivity to the varying of the weight parameter W.

Key words: multifractality; cross correlation analysis; detrended analysis; the general two-component ARFIMA processes; price-volume relationship; CSI 300 index

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MFCCA算法及其在金融市场中的应用:DCCA多重分形拓展的新视角

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摘要:基于降趋交叉分析法(DCCA)的多重分形情形拓展存在麻烦点,即负的交叉协方差的任意矩可能会导致复值的出现.通常采取模的处理方法MFDXA会在实际没有分形特征情形下检测出明显的多重分形信号.Oświecimka提出的多重分形降趋交互相关性分析法(MFCCA)保留了每个子区间降趋协方差符号这一

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重要信息，解决了上述麻烦点，同时能够准确识别多重分形交互关系信号，是降趋交互相关性分析法的自然拓展. 这里从一般形式两成分 ARFIMA 模型的角度出发，证明了 MFCCA 算法相比 MFDXA 算法更加有效. MFCCA 能够正确地识别分形特征，同时对权重参数 W 表现出一定的敏感性. 此外，将 MFCCA 算法应用于中国股票市场上，证实了 CSI 300 指数量价间只有大的波动才具有分形特征.

关键词：多重分形；相关性分析法；降趋分析法；一般两成分 ARFIMA 过程；量价关系；CSI 300 指数

0 Introduction

Correlation is an eternal topic in the academia and industry. For stationary series $\{x_i\}$ of $i=1,\cdots,N$, correlations between the elements of this series separated by τ steps are defined by the auto-correlation function[1]:

$$C(\tau)=\langle x_i x_{i+\tau}\rangle=\frac{1}{N-\tau}\sum_{i=1}^{N-\tau}\left(x_i-\frac{1}{N}\sum_{i=1}^{N}x_i\right)\left(x_{i+\tau}-\frac{1}{N}\sum_{i=1}^{N}x_i\right) \tag{1}$$

If $\{x_i\}$ is uncorrelated, then $E[C(\tau)]$ is zero for $\tau>0$. If short-range correlated, $E[C(\tau)]\propto\exp\{-\tau/\tau_\times\}$ with a time decay $\tau_\times$. If long-range correlated, $E[C(\tau)]\propto\tau^{-\gamma}$, $0<\gamma<1$. However, empirical data are often demonstrated non-stationary series, which are superimposed by noise or due to underlying trends of unknown origin. Eq. (1) can not be directly used for non-stationary series. Hurst et al.[2] was the first to propose a rescaled range analysis applied in non-stationary situations. Detrended fluctuation analysis (DFA), first introduced by Peng et al.[3] in 1990s, is a powerful tool to discover the scaling behavior of noisy data in the presence of trends without knowing their origin and shape. DFA describes the mono-fractal properties. However, many records do not exhibit this simple mono-fractal scaling behavior[1,4-5]. Multifractal detrending fluctuation analysis (MF-DFA)[6] is a natural generalization of DFA and works better than wavelet transform modulus maxima (WTMM)[7].

Cross-correlation is widely used when involved with several ingredients. Detrended cross-correlation analysis (DCCA) is proposed to investigate the long-range cross-correlations between two non-stationary time series, which is introduced by Podobnik et al.[8] and based on DFA. Subsequently, the multifractal extension of (MF-DCCA) of DCCA method was proposed by Zhou[9].

However, one problem arises, that is, complex value is obtained with the computation of the arbitrary powers of the negative cross-covariance. So far, a large quantity of previous research has employed the method of taking the absolute of the detrended cross-covariance, called MFDXA[10-14]. Oświecimka et al.[15] argued that the method of taking modulus so far available in the literature seriously distorts or amplifies the multifractal cross-correlation measures. Oświecimka took into account the sign of detrended cross-covariance of each box, eliminating the complexity of computing of arbitrary powers of the negative cross-covariances. The method was called MFCCA, which is a natural generalization of DCCA and performs better than MFDXA.

To support MFCCA, Oświecimka verified the reliability of MFCCA from the analysis of special form ARFIMA processes[8,16] and MSM (Markov-switching model)[17-19], showing that MFCCA is a robust and effective tool. As we know, two-component ARFIMA processes usually have a general form with the parameter W, which weights the strength of the correlation of the two processes. With the introduction of W, we verify the advantage of MFCCA over MFDXA.

Our paper is arranged as follows. In Section 2, the algorithms of MFCCA and MFDXA are described. In Section 3, we verify the effectiveness and advantage of the MFCCA algorithm over MFDXA from the general two-component ARFIMA processes. In Section 4, the tool of MFCCA algorithm is employed in investigating the correlation between the price and volume of Chinese CSI 300 index market, and the MFDXA algorithm is conducted to compare the results. In Section 5, we draw some conclusions.

1 Description of the MFCCA algorithm and the MFDXA algorithm

In this section, we will restate the MFCCA algorithm and the modulus version of MFDXA algorithm. Since the main procedures are the same for both algorithms, and the only differences are displayed in Step 3, we present them all together.

Consider two time series $\{x_i\}$, $\{y_i\}$, where $i=1,\cdots,N$.

Step 1 Determine the "profile"

$$\left.\begin{aligned} X(j) &= \sum_{i=1}^{j}[x_i - \langle x\rangle], \\ Y(j) &= \sum_{i=1}^{j}[y_i - \langle y\rangle],\ j = 1,\cdots,N \end{aligned}\right\} \quad (2)$$

Here $\langle\ \cdot\ \rangle$ denotes the whole average of the corresponding time series.

Step 2 Divide the profile $\{X(j)\}$, $\{Y(j)\}$ into $N_s = \langle N/s\rangle$ nonoverlapping segments of equal length s. We repeat this process from the opposite end since N/s is not always an integrate. Calculate the local trend of the $2N_s$ segments by a polynomial of order m ($P_{X,v}^{(m)}$ for X, $P_{Y,v}^{(m)}$ for Y). $m=1$ will be conducted in this paper. Then determine the detrended cross-covariance for each segment ν,

$$\left.\begin{aligned} &F_{xy}^2(v,s) = \\ &\quad \frac{1}{s}\sum_{k=1}^{s}\{(X[(v-1)s+k]) - P_{X,v}^{(m)}(k) \times \\ &\quad (Y[(v-1)s+k]) - P_{Y,v}^{(m)}(k)\},\ v = \overline{1,N_s}; \\ &F_{xy}^2(v,s) = \\ &\quad \frac{1}{s}\sum_{k=1}^{s}\{(X[N-(v-N_s)s+k]) - P_{X,v}^{(m)}(k) \times \\ &\quad (Y[N-(v-N_s)s+k]) - P_{Y,v}^{(m)}(k)\}, \\ &\qquad v = \overline{1+N_s,\ 2N_s} \end{aligned}\right\} \quad (3)$$

Step 3 Compute the q-th order covariance.

In the general case, $F_{xy}^2(v,s)$ can take both positive and negative values. In the MFCCA algorithm, the sign of the covariance should be taken into account when calculating the q-th order covariance function as follows:

$$F_{xy}^q(s) = \frac{1}{2N_s}\sum_{v=1}^{2N_s}\mathrm{sign}(F_{xy}^2(v,s))\,|F_{xy}^2(v,s)|^{\frac{q}{2}} \quad (4)$$

The parameter q can take any real number except zero. However, when $q=0$, the L'Hospital law can be employed, and we can then obtain the following logarithmic version,

$$F_{xy}^0(s) = \frac{1}{2N_s}\sum_{v=1}^{2N_s}\mathrm{sign}(F_{xy}^2(v,s))\ln|F_{xy}^2(v,s)| \quad (5)$$

In the MFDXA algorithm, the absolute of the covariance of each segment ν is directly computed without any other manipulations as follows:

$$F_{xy}^q(s) = \frac{1}{2N_s}\sum_{v=1}^{2N_s}|F_{xy}^2(v,s)|^{\frac{q}{2}} \quad (6)$$

When $q=0$, the analogous version to the MFCCA algorithm is exhibited as follows:

$$F_{xy}^0(s) = \frac{1}{2N_s}\sum_{v=1}^{2N_s}\ln|F_{xy}^2(v,s)| \quad (7)$$

From Eqs. (4) and (6) we can see that, for the negative q, small values of $F_{xy}^2(v,s)$ are amplified, while for large $q>0$, its large values dominate. We repeat the above procedures for different values of s. In practice, it is reasonable to take $s_{\max} < N/5$.

Step 4 If the obtained $F_{xy}^q(s)$ fluctuates around zero, there is no fractal cross-correlation between the time series for the considered value of q. If the q-th order covariance obeys the power law, simultaneously, depending on different values of q, then the two time series exhibit the multifractal cross-correlation.

$$F_{xy}^q(s)^{\frac{1}{q}} = F_{xy}(q,s) \sim s^{h_q} \quad (8)$$

For $q=0$,

$$\exp(F_{xy}^0(s)) = F_{xy}(0,s) \sim s^{h_0} \quad (9)$$

For the monofractal cross-correlation, the exponent h_q is independent of q and equal to h as obtained from the DCCA algorithm. When $\{x_i\}$ are equal to $\{y_i\}$, the MFCCA algorithm and

MFDXA algorithm are equivalent.

2 MFDXA or MFCCA

A commonly adopted "moludus" variant of the MFDXA procedure has been in question in Ref. [15] considering the special case of two-component ARFIMA processes and the MSM model. In this paper, to exemplify the performance of MFDXA and MFCCA more in detail, we introduce the general form of two-component ARFIMA stochastic processes to generate coupled fractal signals with long range correlation. In this case, each variable depends not only on its own past, but also on the past values of the other variable.

$$y_i = W\sum_{n=1}^{\infty} a_n(d_1)\, y_{i-n} + (1-W)\sum_{n=1}^{\infty} a_n(d_2)\, y'_{i-n} + \varepsilon_i \tag{10}$$

$$y'_i = (1-W)\sum_{n=1}^{\infty} a_n(d_1)\, y_{i-n} + W\sum_{n=1}^{\infty} a_n(d_2)\, y'_{i-n} + \varepsilon'_i \tag{11}$$

Here ε_i and ε'_i denote two independent and identically distributed Gaussian variables with zero mean and unit variance. $a_n(d_{1(2)})$ are statistical weights defined by

$a_n(d_{1(2)}) = \Gamma(n - d_{1(2)})/(\Gamma(-d_{1(2)})\Gamma(1+n))$,

where Γ denotes the Gamma function. W is the weight parameter controlling the strength of the correlation of both series. It is easy to see that when $W=1$ the two series are decoupled, while when $W=0.5$, the two series are the strongest correlated.

To better elaborate the performance of both algorithms, we control the possible variable ingredients, setting $d_1 = 0.1$, $d_2 = 0.4$ for all situations we will study. Before generating the series, we produce two independent standard normal random series, fixed at each scenario. The only variable ingredient in our simulation is W, ranging from 0.5 to 1.0 step by 0.05, indicating the strengthes of the correlation of these two series weakening.

We perform MFDXA and MFCCA algorithms in each situation. Both procedures use the least square fit to estimate the local trend. We calculate the order q ranging from 0.5 to 10 intercepted by 40 steps of 0.5 length.

When $W=1$, the two series are decoupled with no cross correlation. There is no reason to expect them to be multifractally cross correlated[8]. The q-th order covariance calculated by the method of MFCCA oscillates around zero, implying that there are no power-law cross correlations, which is in agreement with the expectation. In Fig. 1, we present the q-th order covariance (when $q=2$) from MFCCA and MFDXA, respectively. MFDXA shows an obviously power-law character. When W moves towards 0.5, both methods catch the multifractal behavior. As we know, $\Delta h = h_{\max}(q) - h_{\min}(q)$ can measure the degree of the multifractality[14]. Fig. 2

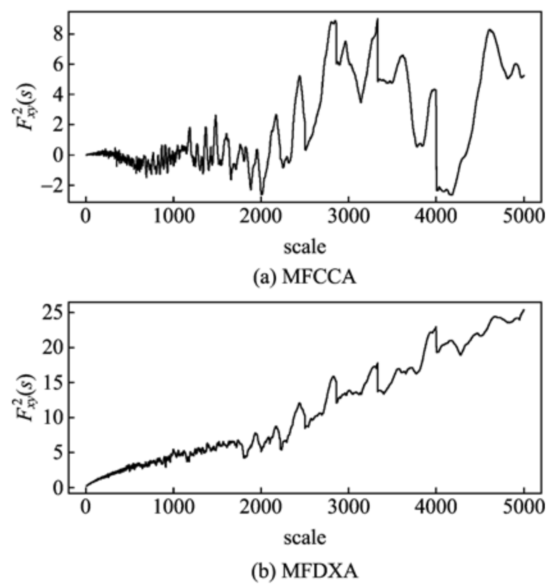

Fig. 1 $F^2_{xy}(s) \sim s$ using MFCCA and MFDXA

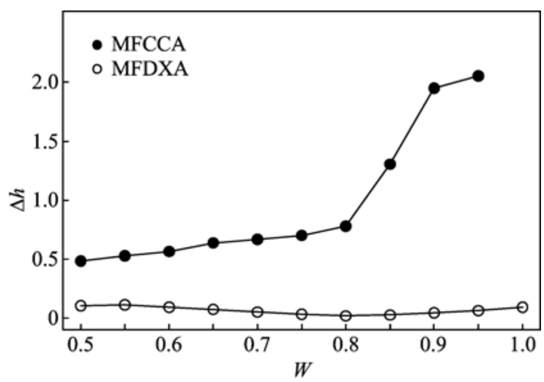

Fig. 2 $\Delta h \sim W$ using MFCCA and MFDXA displays Δh as a function of the weight W using MFDXA and MFCCA, respectively. However, for the situation $W=1$, the Δh obtained by the MFDXA algorithm is so small, 0.09, that it almost indicates a unifractal characteristic. This is a misleading signal. With W ranging from 0.5 to 1.0, the self-similarity between the two series will become more and more complex, leading to an increasingly stronger multifractality. From the solid points, the method of MFCCA coincides with our expectation of an increasing Δh. The Δh calculated from MFDXA in Fig. 2 exhibits little variability, and more importantly, it experiences a non-monotonous process. This implies that MFDXA is not sensitive to the vital weight parameter W. From the perspective of the general form of ARFIMA processes, we arrive at the conclusion that MFCCA algorithm is more effective and reliable than MFDXA. MFCCA is a natural general extension of MFDFA and DCCA. MFCCA algorithm can catch the true virtue of multifractality.

3 Analysis of the correlation between price and volume in Chinese market

The CSI 300 index is a capitalization-weighted stock market index designed to replicate the performance of 300 stocks traded in the Shanghai and Shenzhen Stock Exchanges. We select the CSI 300 index as our subject, because it is the best representative to reflect the price fluctuation and performance of China's A share market. High frequency data analyzed using the method of multifractal analysis is extremely rare in the previous literatures. At the same time, high frequency trading is becoming more and more popular in China. We choose the 5 min closing price and trading volume data downloaded from Wind database.

The sample interval is from 14:30, April 12, 2012 to 15:00, July 25, 2014. Over the period considered, this yields 26 597 data points. In this paper, returns are computed as logarithmic price return (that is $\ln(P_{t+1})-\ln(P_t)$, where P_t is the closing price at time t) and volume data are also calculated as logarithmic trading volume variation series (that is $\ln(V_{t+1})-\ln(V_t)$, where V_t is the trading volume at time t). Logarithmic price return and logarithmic trading volume time series are presented in Figs. 3 and 4, and their descriptive statistics are provided in Tab. 1.

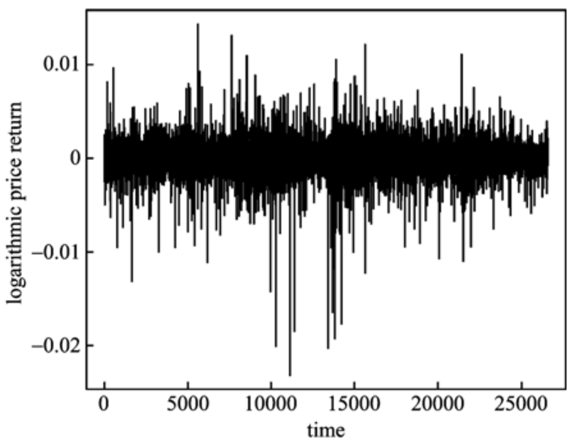

Fig. 3　Logarithmic price return

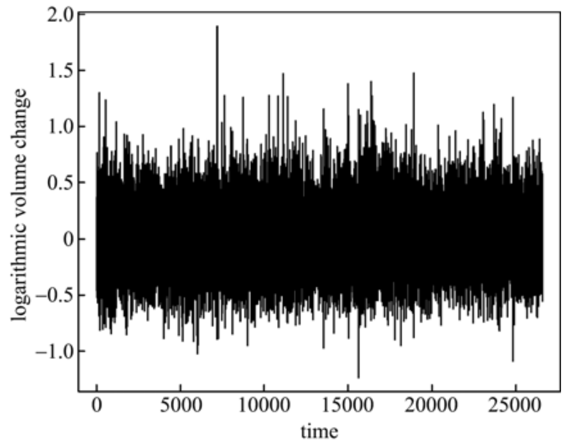

Fig. 4　Logarithmic trading volume

Tab. 1　The statistical description of return and volume series

	mean	max	min	SD	ske	kur	JB
return	−3.61E−6	0.014 279 17	−0.023 22	0.001 35	−0.875 03	19.756 47	435 775.1***
volume	4.08E−5	1.893 617	−1.234 38	0.244 493	0.310 332	1.649 358	3 439.651***

【Note】 *** denotes 1% significance level

Tab. 1 indicates that both skewness and kurtosis for the logarithmic price return and logarithmic trading volume show significant departures from normality. The Jarque-Bera statistics for the normality test suggest that for both series, the normality assumption can be rejected at the 1% significant level.

3.1 Multifractal analysis of logarithmic price return

We perform MFCCA for CSI 300 logarithmic price return and itself, in fact, from Eq. (3). This method is equal to MFDFA[6]. Fig. 5 (a) exhibits the detrended variance for the designated $s=100$, $F^2_{xx}(v,100)$ as the function of the box number v. Obviously, they are all positive values, thus avoiding the trouble that the detrended variance function $F^2_{xx}(v,s)$ for some q results in complex values.

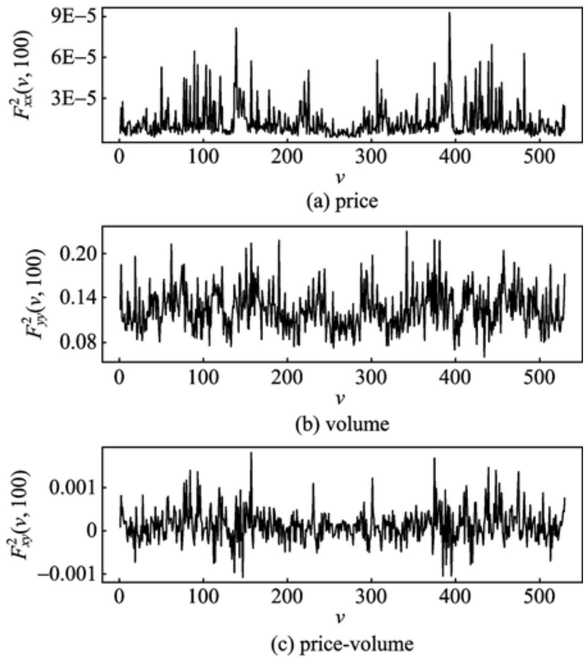

Fig. 5 Detrended (co)variance function for the designated $s=100$

In order to characterize the correlations in this case, the q-th order variance function $F_{xx}(q,s)$ are calculated for different q ranging from −10 to 10 intercepted by 40 steps of 0.5 length. The generalized hurst exponent $h(q)$ in Fig. 6(a). For the positive scenario, $h(q)$ is decreasing from 0.53 to 0.40, showing an obvious multifractal character. For the negative situation, $h(q)$ is ranging from 0.58 to 0.53, generally larger than the positive one. The q dependence of multifractal scaling exponent $\tau(q)$ has concave and nonlinear characteristics (see Fig. 6 (b)). To accurately characterize the strength of the multifractality, we also compute the locally Hölder exponent $\alpha(q)$.

$\Delta\alpha=\alpha_{\max}-\alpha_{\min}=0.647-0.304=0.343$, which implicates we can claim that CSI 300 logarithmic price return shows slightly strong multifractal characteristics. When $q=2$, $h(q)=0.53$, indicating that on the whole, the price return has weak persistence, and the cross-correlation for logarithmic price return is long range positively correlated, that is, the high price will probably be followed with higher price, and the low price will trigger lower price. To some extent, it also explains the herd-effect.

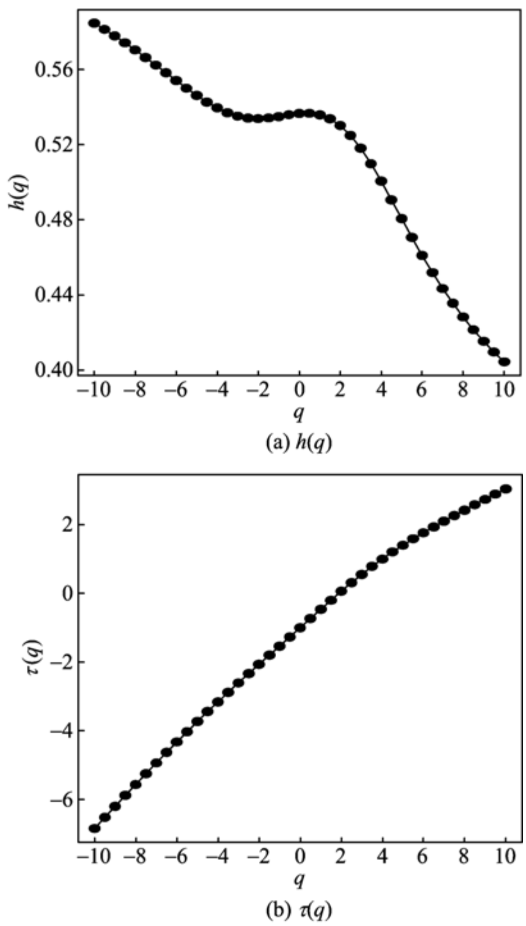

Fig. 6 Hurst exponent and scaling exponent for price return

3.2 Multifractal analysis of logarithmic trading volume

We do the same for logarithmic trading volume as the logarithmic price return. Fig. 5(b) shows that the detrended variances $F_{yy}^2(v,100)$ are all positive values like those of the price return series. However, compared with price, the scenario for volume is more complicated and interesting. Firstly, there exists an obvious cross-over point[20] for the q-th order detrended variance, about $\ln(s^*)=4.1$, that is 300 min, almost 1.25 trading days (see in Fig. 7, $q=10,\cdots,-10$ from top to bottom). For $s<s^*$ and $s>s^*$ the generalized hurst exponent $h(q)$ is figured out in Fig. 8(a). For the part of $s<s^*$, the logarithmic trading volume shows much more strongly multifractal behavior than the price return. The evident concavity of the scaling exponent $\tau(q)$ also recognizes the strong multifractality for the volume series (see in Fig. 8(b)). In this situation, $\Delta\alpha=1.58$, much larger than that of price. $h(2)=0.38$, anti-persistence is illustrated.

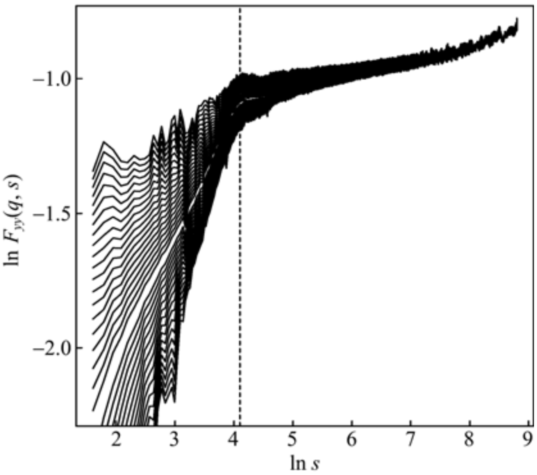

Fig. 7　Logarithmic q-th order detrened variance

While for the part of $s>s^*$, the range of the generalized hurst exponent is so narrow, that is, 0.065 to 0.048, exhibiting a linear behavior. The apparent linearity of the scaling exponent implicates the almost monofractality of the logarithmic trading volume for $s>s^*$ (see in Fig. 8(b)).

The existence of the cross-over point in this situation indicates that one simple multifractality model can not completely depict the nonlinear character of volume series.

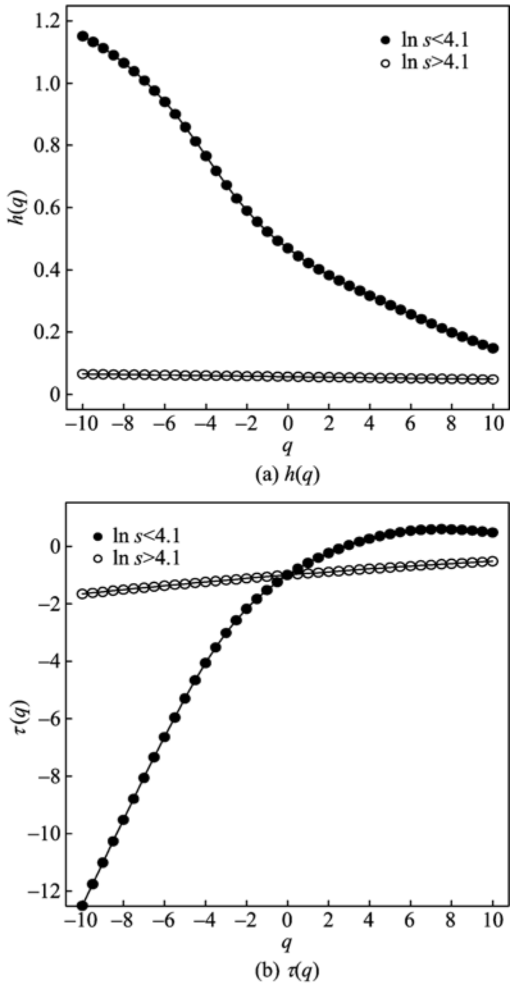

Fig. 8　Hurst exponent and scaling exponent for volume change

3.3 Multifractal analysis of cross-correlation between price and volume of CSI 300 index

Like the MFDFA method conducted above, the order $m=1$ is chosen to estimate the polynomial trend of each segment. Fig. 5(c) exhibits the detrended covariance for the designated $s=100$, $F_{xy}^2(v,100)$ as the function of the box number v. It is easy to notice that the detrended cross-covariance function $F_{xy}^2(v,100)$ takes both positive and negative values. Taking the modules of the cross-covariance will always ignore the important information. Reserving the sign of the cross-covariance is a wisdom disposal route. The values of parameter q range from -10 to 10 step by 0.5. For the negative qs, we arrive at the same conclusion as Refs. [9,15], that is, the q-th order covariance $F_{xy}^{q}(s)$ fluctuates around zero for the smaller s, even though for the larger s, $F_{xy}^{q}(s)$ are far larger than zero, but do not develop scaling behavior. So in this situation, Eq. (8) is not satisfied, which means that there is no fractal cross-correlation between the price and volume series under the negative qs.

Fig. 9(a) displays the relationship between the cross-correlation exponent $h(q)$ and q for the positive situation. $h(q)$ is a nonlinear function depending on q, which implicates that multifractal cross-correlation really exists between the price and volume in Chinese stock market. $h(2)=0.44$, illustrates that the relationship between the price and volume is anti-persistent rather than perfectly effective. The scaling exponent $\tau(q)$ is calculated in Fig. 9(b), $\Delta\alpha = 0.13$ implicates that the multifractality is very weak.

We also conduct the MFDXA algorithm on the price-volume relationship. As a different result from the MFCCA algorithm, we find that for both the positive and negative qs, the detrended covariances follow the nice power law and develop scaling behavior. Like the MFCCA algorithm we have done, Fig. 10 exhibits the hurst exponent and scaling exponent when q ranges from -10 to 10 by 0.5. Fig. 10(b) shows a stronger concavity of the scaling exponent $\tau(q)$ than that obtained from MFCCA algorithm. $\Delta\alpha = 0.93$, which is much larger than the previous result. The mulfractality under MFDXA is much stronger than MFCCA, moreover, multifractality also exists among the small fluctuations.

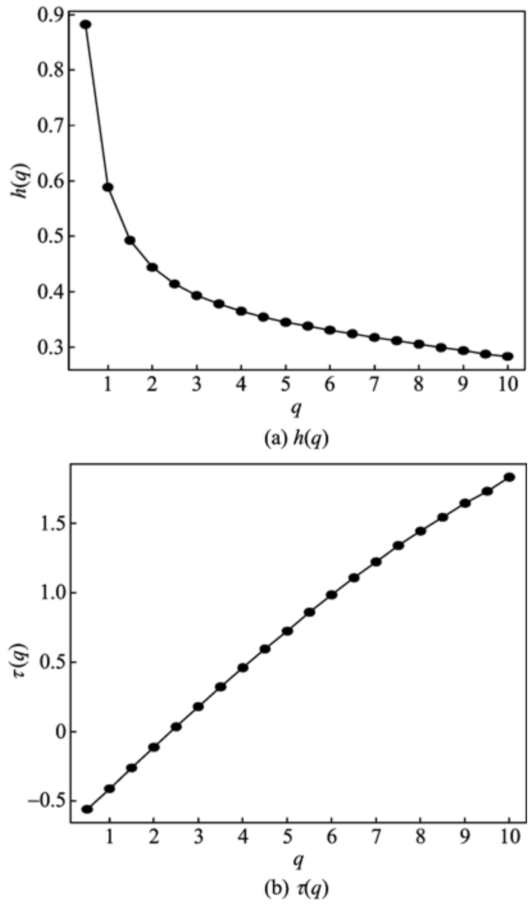

Fig. 9 Hurst exponent and scaling exponent by MFCCA

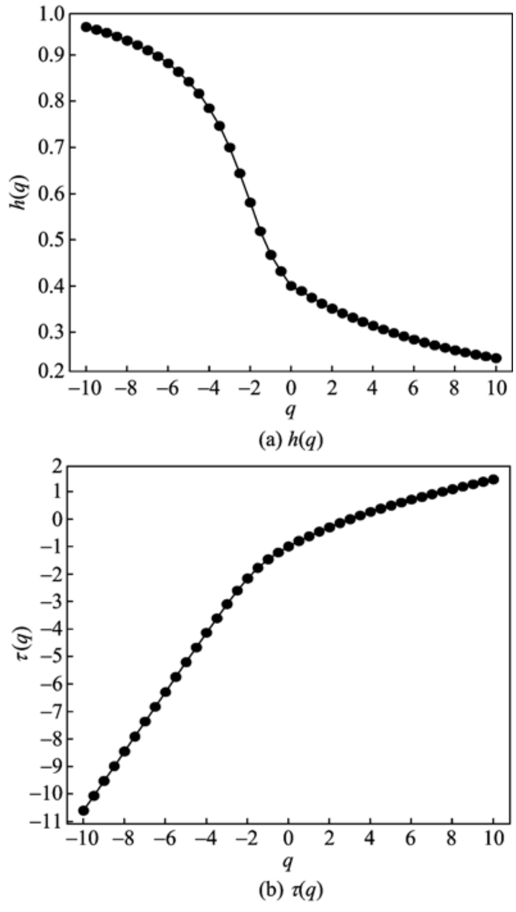

Fig. 10 Hurst exponent and scaling exponent by MFDXA

4 Conclusion

From the perspective of the general two-component ARFIMA processes, we demonstrate that the algorithm, MFCCA, is more effective and reliable than the traditional modulus processing method, MFDXA. The preservation of the sign of the cross-covariance fluctuation function brings a new challenge to the multifractality research on cross-correlation. MFCCA is a natural general extension of MFDFA and DCCA. Furthermore, the application of MFCCA to the price-volume of CSI 300 index in Chinese high frequency trading market, indicates that multifractality behavior is weak and appears selectively only for large fluctuations. This outcome suggests that multifractality of the price-volume of China's market might be temporal relations only between large events. Employing the MFCCA algorithm, we can obtain a more precise description of financial market when talking about the cross relationship. Further research will be conducted, such as how to utilize this relationship to oversee and stablize the financial market or to obtain better portfolio return.

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