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A generalized interior shock layer solution to nonlinear singularly perturbed equations

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Abstract: A class of singular perturbation problem of the reaction diffusion initial value equation was studied. Under suitable conditions, the generalized outer solution to reduced problems was considered. Then the interior shock and boundary layer correction solutions to the original problem were constructed by using the theory of generalized functions. Finally, using the fixed point theorem, the uniform validity of the generalized asymptotic solution with interior shock and initial layers was proved.

Key words: singular perturbation; nonlinear equation; shock layer

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非线性奇摄动问题的广义内部冲击层解

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摘要: 研究了一类广义奇摄动反应扩散方程初始边值问题. 在适当的假设下, 考虑了退化问题的广义解, 然后利用广义函数理论构造了原问题的冲击层和边界层渐近解. 再利用不动点定理证明了具有广义内部冲击层的渐近解的一致有效性.

关键词: 奇摄动; 非线性方程; 冲击层

0 Introduction

Nonlinear shock waves are an attractive subject in the mathematics circles[1-2]. Many approximate methods have been developed, such as the methods of multiple scales, the methods of

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matched asymptotic expansion, the method of averaging and the boundary layer method etc. Recently, many scholars have done a great deal of work[1-7]. Using the asymptotic method the authors also studied a class of nonlinear problems[8-22]. In this paper, using a special method, we study the shock layer solution to a class of generalized initial boundary value problems for the reaction diffusion equation.

Now we consider the following reaction equation with an initial boundary value problem

$$\left.\begin{aligned}&\frac{\partial u}{\partial t}-\varepsilon^{2m}L^m[u]=f(t,x,u),\\&\quad t\in(0,T_0),\ x\equiv(x_1,x_2,\cdots,x_n)\in\Omega\subset R^n\end{aligned}\right\}\tag{1}$$

$$\left.\begin{aligned}&\frac{\partial^j u}{\partial n^j}(t,x)=g_j(x),\\&\quad t\in(0,T_0),\ x\in\partial\Omega,\ j=0,1,\cdots,m-1\end{aligned}\right\}\tag{2}$$

$$u(0,x)=h(x),\ x\in\Omega\tag{3}$$

where ε is a small positive parameter and T_0 is a large enough positive constant, Ω signifies a bounded domain with boundary $\partial\Omega$ of class C^∞ and

$$L\equiv\sum_{i,j=1}^{n}D^i(a_{ij}(x)D^j),$$

$$D_0=\frac{\partial}{\partial t},\ D_j=\frac{\partial}{\partial x_j}(j=0,1,\cdots,n),$$

$$D^\alpha=D_1^{\alpha_1}D_2^{\alpha_2}\cdots D_n^{\alpha_n},\ |\alpha|=\sum_{j=1}^{n}\alpha_j,$$

with $m>1$, $x^\alpha=x_1^{\alpha_1}x_2^{\alpha_2}\cdots x_n^{\alpha_n}$, and the coefficients $a_{ij}(i,j=1,2,\cdots,n)$ are real value functions of class $C^\infty(\Omega)$, L is uniformly elliptic in $\bar{\Omega}$:

$$\sum_{i,j=1}^{n}\xi^i a_{ij}(x)\xi^j\geqslant\lambda_l\,|\xi|^{2n}:=\lambda\sum_{i=1}^{n}\xi_i^2,$$

$$\forall\,\xi\in R^n,\ x\in\bar{\Omega},\ \lambda>0,$$

$\frac{\partial}{\partial n}$ denotes differentiation in the outward normal direction on $\partial\Omega$.

Instead of the initial boundary value problem (1)~(3), we consider the generalized reaction diffusion equation initial boundary value problem

$$\left.\begin{aligned}&(\psi,D_0u)-\varepsilon^{2m}B^m[\psi,u]=(\psi,f),\\&\quad(t,x)\in(0,\infty)\times\Omega,\ \forall\,\psi\in C_0^\infty(\Omega)\end{aligned}\right\}\tag{4}$$

$$\left.\begin{aligned}&\left(\psi,\frac{\partial^j u}{\partial n^j}\right)=(\psi,g_j),\\&\quad x\in\partial\Omega,\ \forall\,\psi\in C_0^\infty(\Omega),\ j=0,1,\cdots,m-1\end{aligned}\right\}\tag{5}$$

$$(\psi,u)=(\psi,h),\ t=0,\ x\in\Omega,\ \forall\,\psi\in C_0^\infty(\Omega)\tag{6}$$

where $B[\psi,u]\equiv\sum_{i,j=1}^{n}(D^i\psi,a_{ij}D^ju)$, $C_0^\infty(\Omega)$ is the subset of $C^\infty(\Omega)$ consisting of functions with compact support in Ω, and $B[x,y]$ denotes the bilinear form associated with $L[u]$, whose form is defined for a_{ij} bounded in Ω, and for z and y belonging to Sobolev space $H^j(\Omega)$ with the finite norm:

$$\|\psi\|_j=\{\sum_{|\alpha|\leqslant j}\int_\Omega|D^\alpha\psi(x)|^2\mathrm{d}x\}^{\frac{1}{2}},$$

$$\forall\,\psi\in C^j(\Omega),\ j=0,1,$$

and (u,v) is defined as the scalar product in $H_0^j(\Omega)$.

It is our task to study the solution $u\in H_0^m(\Omega)$ for the generalized reaction diffusion equation initial value problem (4)~(6), where space $H_0^m(\Omega)$ signifies the completion of the space $C_0^\infty(\Omega)$.

We assume that

[H_1] The coefficients a_{ij} of the operator L are bounded in Ω.

[H_2] $|a_{ij}(x)-a_{ij}(\bar{x})|\leqslant c_l(|x-\bar{x}|)$ for $i,j=1,2,\cdots,n$, $\forall\,x,y\in\Omega$, and $c_l(|x-\bar{x}|)\to0$, for $|x-\bar{x}|\to0$.

[H_3] f is a sufficiently smooth real value function except $x\in S$, where S is the $(n-1)$-dimensional manifold in Ω.

[H_4] $g_j(j=0,1,\cdots,m-1)$ and h are sufficiently smooth real value functions with regard to their variables in correspondence ranges.

[H_5] There exists constants C_1 independent of v and u such that

$$|B^m[u,v]|\leqslant C_1\|u\|_m\cdot\|v\|_m,\ \forall\,u,v\in H_0^m.$$

And there exists constant C_2 independent of z such that

$$|B^m[v,v]|\leqslant C_2\|v\|_m^2,\ \forall\,v\in H_0^m.$$

[H_6] There are positive constants δ_1,δ_2, such that

$$\delta_1\leqslant\frac{\partial f}{\partial u}(t,x,u)\leqslant\delta_2<C_2,$$

$$\forall\,(t,x)\in[0,T_0]\times\bar{\Omega},\ \forall\,u\in R.$$

1 The outer solution

Consider the reduced problem of Eqs. (4)~(6):

$$\left.\begin{array}{l}(\psi, D_0 u) = (\psi, f),\\ \quad (t,x) \in (0,T_0)\times\Omega, \forall\psi\in C_0^\infty(\Omega)\end{array}\right\} \tag{7}$$

$$(\psi, u) = (\psi, h),\ t = 0, x\in\Omega, \forall\psi\in C_0^\infty(\Omega) \tag{8}$$

From the hypotheses, there exists in the generalized initial boundary value problem (7) and (8) a solution $U_0(x)\in H_0^1(\overline{\Omega})$, which is discontinuous at the $(n-1)$-dimensional manifold S.

Now we construct the outer asymptotic solution U to generalized initial boundary value problem (4)～(6). Let

$$U = \sum_{i=0}^{\infty} U_i(t,x)\varepsilon^i \tag{9}$$

Substituting (9) into Eqs. (7) and (8), developing the terms in ε and equating the coefficients of the same powers of ε, for the coefficients of $\varepsilon^i(i=1,2,\cdots)$, we obtain

$$\left.\begin{array}{l}(\psi, D_0 U_i) = (\psi, F_i) + B^m[\psi, U_{i-2m}],\\ \quad (t,x)\in(0,T_0)\times\Omega, \forall\psi\in C_0^\infty(\Omega)\end{array}\right\} \tag{10}$$

$$(\psi, U_i) = 0,\ t=0, x\in\Omega, \forall\psi\in C_0^\infty(\Omega) \tag{11}$$

where $F_i(i=1,2,\cdots)$ are known functions of U_j $(j\leqslant i-1)$. The negative subscripts in Eqs. (10) and (11) and thereafter in this paper are all assumed to be zero. From Eqs. (10) and (11), we can obtain successively determined solutions U_i $(i=1,2,\cdots)$. Thus we have outer solution (9). But it may not be continuous at $x\in S$ and may not satisfy the boundary conditions in Eq. (5), so we need to construct the interior shock layer and boundary layer corrections V and W.

2 Interior shock layer correction

We first construct a local coordinate system $(\bar\rho, \bar\phi)$ near the $(n-1)$-dimensional manifold S. Define the coordinate of every point $\overline{Q}$ in the neighborhood of S in the following way: The coordinate $\bar\rho(\leqslant\bar\rho_0)$ is the distance from point $\overline{P}$ to the $(n-1)$-dimensional manifold S along its normal vector, where $\bar\rho_0$ is small enough such that the normals on every point of S do not intersect each other in this neighborhood of the $(n-1)$-dimensional manifold S. The $\bar\phi=(\bar\phi_1, \bar\phi_2, \cdots, \bar\phi_{n-1})$ is a nonsingular coordinate system on the $(n-1)$-dimensional manifold S, and the coordinate $\bar\phi$ of the point $\overline{Q}$ is equal to the coordinate $\bar\phi$ of the point $\overline{P}$ at which the inner normal through the point $\overline{Q}$ intersects S.

In the neighborhood of S: $0\leqslant\bar\rho\leqslant\bar\rho_0$, we have

$$\overline{B}^m[\phi, u] = \sum_{i,j=1}^{n}(\overline{D}^i\phi, \bar a_{ij}^{2m}\overline{D}^j u),$$

$$\overline{L}^m = \bar a_{mm}^{2m}\frac{\partial^{2m}}{\partial\bar\rho^{2m}} + \overline{L}^{2m-1},\ \bar a_{mm}^{2m} > 0,$$

$$\overline{D}_1 = \frac{\partial}{\partial\bar\rho}, \overline{D}_{j+1} = \frac{\partial}{\partial\bar\phi_j}, j = 1,2,\cdots,n-1,$$

where the construction of $\overline{L}^{2m-1}$ is omitted.

From Eqs. (4)～(6), we have

$$\left.\begin{array}{l}(\psi, u) - \varepsilon^{2m}\overline{B}^m[\psi, u] = (\psi, f(t,x,u)),\\ \quad (t,x)\in(0,T_0)\times\Omega, \forall\psi\in C_0^\infty(\Omega)\end{array}\right\} \tag{12}$$

$$\left.\begin{array}{l}\left(\psi, \dfrac{\partial^j u}{\partial n^j}\right) = (\psi, g_j),\ (t,x)\in(0,T_0)\times\partial\Omega,\\ \forall\psi\in C_0^\infty(\Omega),\ j=0,1,\cdots,m-1\end{array}\right\} \tag{13}$$

$$(\psi, u) = (\psi, h),\ t=0, x\in\Omega, \forall\psi\in C_0^\infty(\Omega) \tag{14}$$

Introduce a stretched variable[1]:

$$\sigma = \bar\rho/\varepsilon \tag{15}$$

and

$$u = U + V \tag{16}$$

where

$$V = \sum_{i=0}^{\infty} v_i(t,\sigma)\varepsilon^i \tag{17}$$

From Eq. (15), substituting Eqs. (16), (17) into Eqs. (12)～(14), developing the terms in ε and equating the coefficients of the same powers of ε, for the coefficients of $\varepsilon^i(i=0,1,\cdots)$, we obtain

$$\left.\begin{array}{l}(\psi, D_0 v_i) - B_m[\psi, v_i] = (\psi, \overline{F}_i),\\ \quad (t,\bar\rho)\in(0,T_0)\times(0,\rho_0),\\ \quad \forall\psi\in C_0^\infty(\Omega),\ i=0,1,2,\cdots\end{array}\right\} \tag{18}$$

$$\left.\begin{array}{l}\left(\psi, \dfrac{\partial^j v_i}{\partial n^j}\right) = (\psi, U_j),\ \bar\rho=\pm 0,\\ \quad \forall\psi\in C_0^\infty(\Omega), j=0,1,\cdots,m-1\end{array}\right\} \tag{19}$$

$$(\psi, v_i) = 0,\ t=0, 0\leqslant\bar\rho\leqslant\bar\rho_0, \forall\psi\in C_0^\infty(\Omega) \tag{20}$$

where $\overline{F}_i(i=0,1,2,\cdots)$ are known functions of v_j $(j\leqslant i-1)$. From Eqs. (18)～(20), we can obtain successively determined solutions $v_i\,(i=0,1,2,\cdots)$. And from the hypotheses, $v_i\,(i=0,1,2,\cdots)$ possess the following shock layer behavior:

$$\left.\begin{aligned} &v_i = O(\exp(-\bar{k}_i \mid \sigma \mid)) = O(\exp(-\bar{k}_i \frac{\mid \bar{\rho} \mid}{\varepsilon})), \\ &0 < \varepsilon \ll 1, i = 0,1,\cdots \end{aligned}\right\} \tag{21}$$

where $\bar{k}_i(i=0,1,\cdots)$ are positive constants.

Let $\bar{v}_i=\bar{\psi}(\bar{\rho})v_i$, where $\bar{\psi}(\bar{\rho})$ is a sufficiently smooth real value function in the neighborhood of $(n-1)$-dimensional manifold S: $0\leqslant\bar{\rho}\leqslant\bar{\rho}_0$, and satisfies

$$\bar{\psi}(\bar{\rho}) = \begin{cases} 1,\ 0\leqslant\bar{\rho}\leqslant\frac{1}{3}\bar{\rho}_0; \\ 0,\ \bar{\rho}\geqslant\frac{2}{3}\bar{\rho}_0. \end{cases}$$

For the convenience, we still substitute $\bar{v}_i$ by v_i below. Thus from Eq. (17), we obtain the interior shock layer V.

3 Boundary layer correction

We construct also a local coordinate system $(\tilde{\rho}, \tilde{\phi})$ near $\partial\Omega$. Define the coordinate of every point $\tilde{Q}$ in the neighborhood of $\partial\Omega$ in the following way: The coordinate $\tilde{\rho}(\leqslant\tilde{\rho}_0)$ is the distance from the point $\tilde{P}$ to the $\partial\Omega$ along its normal vector, where $\tilde{\rho}_0$ is small enough such that the inner normals on every point of $\partial\Omega$ do not intersect each other in this neighborhood of $\partial\Omega$. The $\tilde{\phi}=(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_{n-1})$ is a nonsingular coordinate system on the $(n-1)$-dimensional manifold $\partial\Omega$. The coordinate $\tilde{\phi}$ of the point $\tilde{Q}$ is equal to the coordinate $\tilde{\phi}$ of the point $\tilde{P}$ at which the inner normal through the point $\tilde{Q}$ intersects the $\partial\Omega$.

In the neighborhood of $(n-1)$-dimensional manifold $\partial\Omega$: $0\leqslant\tilde{\rho}\leqslant\tilde{\rho}_0$, we have

$$\tilde{B}^m[\psi, u] = \sum_{i,j=1}^{n}(\tilde{D}^i\psi, \tilde{a}_{ij}^{2m}\tilde{D}^j u),$$

$$\tilde{L}^m = \tilde{a}_{nn}^{2m}\frac{\partial^{2m}}{\partial\tilde{\rho}^{2m}} + \tilde{L}^{2m-1},\ \tilde{a}_{nn}^{2m} > 0,$$

$$\tilde{D}_1 = \frac{\partial}{\partial\tilde{\rho}}, \tilde{D}_{j+1} = \frac{\partial}{\partial\tilde{\phi}_j}, j = 1,2,\cdots,n-1,$$

where the construction of $\tilde{L}_{2m-1}$ is also omitted.

From Eqs. (4)～(6), we have

$$\left.\begin{aligned} &(\psi, D_0 u) - \tilde{B}^m[\psi, u] = (\psi, f(t,x,u)), \\ &(t,x) \in (0,T_0)\times\Omega, \forall\,\psi\in C_0^\infty(\Omega) \end{aligned}\right\} \tag{22}$$

$$\left.\begin{aligned} &\left(\psi, \frac{\partial^j u}{\partial n^j}\right) = (\psi, g_0),\ (t,x)\in(0,T_0)\times\partial\Omega, \\ &\forall\,\psi\in C_0^\infty(\Omega),\ j = 0,1,\cdots,m-1 \end{aligned}\right\} \tag{23}$$

$$(\psi, u) = (\psi, h),\ t = 0, x\in\Omega, \forall\,\psi\in C_0^\infty(\Omega) \tag{24}$$

Introduce a stretched variable[1]:

$$\tau = \tilde{\rho}/\varepsilon \tag{25}$$

and

$$u = U + W \tag{26}$$

where

$$W = \sum_{i=0}^{\infty} w_i(\tau)\varepsilon^i \tag{27}$$

From Eq. (25), substituting Eqs. (26), (27) into Eqs. (22)～(24), developing the terms in ε and equating the coefficients of the same powers of ε, for the coefficients of $\varepsilon^i(i=0,1,\cdots)$, we obtain

$$\left.\begin{aligned} &(\psi, D_0 w_i) - \tilde{B}^m[\psi, w_i] = (\psi, \tilde{F}_i), \\ &\forall\,\psi\in C_0^\infty(\Omega) \end{aligned}\right\} \tag{28}$$

$$\left.\begin{aligned} &\left(\psi, \frac{\partial^j w_i}{\partial\tau^j}\right) = 0,\ \tau = 0, \\ &\forall\,\psi\in C_0^\infty(\Omega), j = 0,1,\cdots,k-1 \end{aligned}\right\} \tag{29}$$

$$(\phi, w_i) = 0,\ t = 0, \forall\,\phi\in C_0^\infty(\Omega) \tag{30}$$

where $\tilde{F}_i(i=1,2,\cdots)$ are known functions of w_j $(j\leqslant i-1)$. From Eqs. (28)～(30), we can obtain successively determined solutions $w_i(i=0,1,\cdots)$. And from the hypotheses, $w_i(i=0,1,\cdots)$ possess the following boundary layer behavior:

$$\left.\begin{aligned} &w_i = O(\exp(-\tilde{k}_i\tau)) = O\left(\exp\left(-\tilde{k}_i\frac{\tilde{\rho}}{\varepsilon}\right)\right), \\ &0 < \varepsilon \ll 1, i = 0,1,\cdots \end{aligned}\right\} \tag{31}$$

where $\tilde{k}_i(i=0,1,\cdots)$ are positive constants.

Let $\tilde{w}_i=\tilde{\psi}(\tilde{\rho})w_i$, where $\tilde{\psi}(\tilde{\rho})$ is a sufficiently smooth real value function in the neighborhood of $\partial\Omega$: $0\leqslant\tilde{\rho}\leqslant\tilde{\rho}_0$, and satisfies

$$\tilde{\psi}(\tilde{\rho}) = \begin{cases} 1,\ 0\leqslant\tilde{\rho}\leqslant\frac{1}{3}\tilde{\rho}_0; \\ 0,\ \tilde{\rho}\geqslant\frac{2}{3}\tilde{\rho}_0. \end{cases}$$

For convenience, we still substitute $\widetilde{w}_i$ with w_i below. Thus from Eq. (27), we obtain the boundary layer correction W.

4 The remainder estimation

Now we prove that

$$\left.\begin{aligned} &u = \sum_{i=0}^{\infty}[U_i + v_i + w_i]\varepsilon^i, \\ &(t,x) \in [0,T_0]\times\overline{\Omega}, 0<\varepsilon\ll 1 \end{aligned}\right\} \tag{32}$$

is a uniformly valid asymptotic expansion for the generalized solution to reaction diffusion equation initial boundary value problem (4)~(6).

Let

$$\left.\begin{aligned} &u(t,x) = \sum_{i=0}^{M}[U_i(t,x) + w_i(t,\sigma) + z_i(t,\tau)]\varepsilon^i + r, \\ &t>0, x\in\overline{\Omega}, 0<\varepsilon\ll 1 \end{aligned}\right\} \tag{33}$$

where $r\in H_0^r((0,T_0)\times\Omega)$ and from Eqs. (4)~(6), (21), (31) and (33), for the $\varepsilon>0$ small enough and from the hypotheses, we have

$$\begin{aligned} &(\psi,r) - \varepsilon^{2m}B^m[\psi,r] - (\psi, f(t,x,r)) = \\ &\varepsilon^{2m}B^m[\psi, u - \sum_{i=0}^{M}[U_i(x)+v_i(\sigma)+w_i(\tau)]\varepsilon^i] - \\ &(\psi, f(t,x,u - \sum_{i=0}^{M}[U_i(x)+v_i(\sigma)+w_i(\tau)]\varepsilon^i)) = \\ &(\psi, D_0U_0) = (\psi, f) + \\ &\sum_{i=1}^{M}[(\psi,D_0U_i) - (\psi,F_i) - B^m[\psi,U_{i-2m}]]\varepsilon^i + \\ &\sum_{j=1}^{M}[(\psi,D_0v_i) - B^m[\psi,v_i] - (\psi,\overline{F}_i)]\varepsilon^i + \\ &\sum_{i=1}^{M}[(\psi,D_0w_i) - \widetilde{B}^m[\psi,w_i] - (\psi,\widetilde{F}_i)]\varepsilon^i + \\ &O(\varepsilon^{M+1}) = O(\varepsilon^{M+1}), \\ &(t,x)\in(0,T_0)\times\Omega, \forall\psi\in C_0^\infty(\Omega), \\ &\left(\psi, \frac{\partial^j r}{\partial n^j}\right) = O(\varepsilon^{M+1}),\ (t,x)\in(0,T_0)\times\partial\Omega, \\ &\forall\phi\in C_0^\infty(\Omega), j=0,1,\cdots,m-1, \\ &(\psi,r) = O(\varepsilon^{M+1}),\ t=0, x\in\Omega, \forall\phi\in C_0^\infty(\Omega). \end{aligned}$$

Thus we have

$$\|r\|_0 = O(\varepsilon^{M+1}),\ (t,x)\in[0,T_0]\times\overline{\Omega}, 0<\varepsilon\ll 1.$$

From the fixed point theorem for functional analytic[1-2], we have the following theorem:

Theorem 1 Under the hypotheses $[H_1]$~$[H_6]$, for the $\varepsilon>0$ small enough and $\forall(t,x)\in[0,T_0]\times\overline{\Omega}$, the nonlinear generalized reaction diffusion equation initial boundary value problem (4)~(5) has a uniformly valid asymptotic expansion (33) for the generalized solution $u(t,x)\in H_0^1([0,T_0]\times\overline{\Omega})$, and holds

$$\begin{aligned} &\| u - \sum_{j=0}^{M}[U_j + v_j + w_j]\varepsilon^j \|_0 = O(\varepsilon^{M+1}), \\ &(t,x)\in[0,T_0]\times\overline{\Omega}, 0<\varepsilon\ll 1, \end{aligned}$$

where $\sum_{j=0}^{M}U_j\varepsilon^j$ is the asymptotic representation of the outer solution for the generalized initial boundary problem (4)~(6) and $v_i\left(t,\frac{\bar{\rho}}{\varepsilon}\right)$, $i=0,1,\cdots$, are the correction terms which possess interior shock layer behavior Eq. (21) and $w_i\left(t,\frac{\bar{\rho}}{\varepsilon}\right)$, $i=0,1,\cdots$, are the correction terms which possess boundary layer behavior Eq. (31).

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