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Ruin probability of the Sarmanov structure among finance and insurance risks with regularly varying tails

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Abstract: A discrete-time insurance risk model was considered, in which the insurance risks and financial risks follow jointly multivariate Sarmanov distributions, and the asymptotic formula for ruin probability was obtained.

Key words: ruin probability; Sarmanov distribution; regular varying; quasi-asymptotically independent

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带有 Sarmanov 相依结构正则变化尾的金融风险模型的破产概率

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摘要:研究了一类离散时间保险风险模型,其中,保险风险和金融风险服从多元联合 Sarmanov 分布,并获得了破产概率的渐近形式.

关键词:破产概率;Sarmanov 分布;正则变化;拟渐近独立

0 Introduction and model

Following Refs. [1-5], we consider a discrete-time stochastic risk model, within period n, a real-valued random variable (r. v.) X_n is interpreted as the net payout of the insurance, and these random variables are assumed to follow Farlie-Gumbel-Morgenstern (FGM) distribution type with identical marginal F. Suppose that the insurer positions himself in a discrete-time financial market consisting of a risk-free bond with a constant periodic interest rate r and a risky stock with a periodic stochastic return rate Δ_n supported on $(-1,\infty)$. Moreover, suppose that, at the beginning of each period n the insurer invests a fraction $\pi\in[0,1)$ of his current wealth in the stock

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and keeps the remaining wealth in the bond. Denote the insurer's wealth at time n by U_n, and let the initial wealth of the insurer be $x \geqslant 0$. Thus we have the recursive equation

$$\left.\begin{aligned} U_n &= [(1-\pi)(1+r)+\pi(1+\Delta_n)]U_{n-1} - X_n, \\ U_0 &= x,\ n \in \mathbb{N} \end{aligned}\right\} \tag{1}$$

As usual, the finite time ruin probability is defined as follows:

$$\psi(x,\ n) = P(\min_{0 \leqslant i \leqslant n} U_i < 0 \mid U_0 = x),\ n = 1,2,\cdots \tag{2}$$

Denote

$$Y_n := 1 + \Delta_n,$$

$$f(\pi, Y_n) := \frac{1}{(1-\pi)(1+r)+\pi Y_n},$$

where Y_n represents the inflation rate stochastic accumulation factor of the risky stock and $f(\pi, Y_n)$ the overall deflation rate/stochastic discount factor of the investment portfolio during period n. Obviously, $f(\pi, Y_n)$ are bounded from above by positive constants. According to Refs. [4,6], we call X_1, X_2, $\cdots$ insurance risks and Y_1, Y_2, $\cdots$ financial risks.

We shall assume that the loss distribution is regularly varying tailed. A distribution F is said to be regularly varying tailed with regularity index $\alpha > 0$, if $\overline{F}(x) = 1 - F(x) > 0$ for all x and

$$\lim_{x \to \infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = y^{-\alpha}$$

holds for all $y > 0$, denoted by $F \in \mathscr{R}_{-\alpha}$. For details of $\mathscr{R}_{-\alpha}$ see Refs. [7-8].

Throughout this paper we assume that $\{X_n, n \geqslant 1\}$ is a sequence of identically distributed random variables (r. v. s) with common distribution F on $(-\infty, +\infty)$, $\{Y_n, n \geqslant 1\}$ is another sequence of nonnegative r. v. s with marginal distribution G_n, $n \geqslant 1$, respectively. For the asymptotic behavior of ruin probability has been considered widely, see Refs. [3-5,9]. In fact, the independence assumption is far unrealistic for applied problems, so people have started to consider some dependence structure. Ref. [5] obtained an exact asymptotic formula for ruin probability under the condition that any n-dimensional distribution of the financial risks Y_1, Y_2, $\cdots$ is a multivariate FGM distribution, where, they still assume that the two sequences $\{X_n, n \geqslant 1\}$ and $\{Y_n, n \geqslant 1\}$ are mutually independent.

Recently, a lot of research has been focused on random weighted sums for the case that $\{X_n, n \geqslant 1\}$ are dependent but independent of the sequence of $\{Y_n, n \geqslant 1\}$, such as Refs. [10-12], etc. From a more realistic point of view, it is more interesting to study the case that insurance risk X and financial risk Y are dependent. However, the product of dependent r. v. s has not been well studied, there being only a few papers on this topic (Refs. [13-15]).

In this paper we assume that these random vectors from the two sequences jointly follow multivariate Sarmanov distributions, that is to say, $X_1, \cdots, X_m, Y_1, \cdots, Y_n$ are dependent through a multivariate Sarmanov distribution for any $m, n \in \mathbb{N}$. Under these assumptions we obtain asymptotical relations for finite-time ruin probability of this stochastic risk model.

The rest of this paper is organized as follows: Section 1 introduces preliminaries and presents our main results, Section 2 gives some necessary lemmas and provides the proof of the obtained theorem.

1 Preliminaries and main results

It is well known that correlation coefficients of FGM distributions lies between $-1/3$ and $1/3$ (Ref. [16]). Ref. [17] showed that the range of correlation coefficients can be widened by considering the iterated generalizaiiori of FGM distribution proposed by Ref. [18]. To overcome this limitation several authors suggested various generalizations. Refs. [19-20] proposed different extensions of the FGM. Then there is the Sarmanov family of distributions, of which FGM is a special case, see Refs. [21-22].

Let $\phi_i(x)$, $i = 1, \cdots, n$, be a set of bounded nonconstant functions such that

$$\int_{-\infty}^{\infty} \phi_i(x_i)\ F(\mathrm{d}x_i) = 0$$

for all $1 \leqslant i \leqslant n$. We say a random vector $Y_1, \cdots, Y_n$ jointly follows a Sarmanov distribution, if it has the distribution of the form

$$P(Y_1 \in \mathrm{d}y_1, \cdots, Y_n \in \mathrm{d}y_n) = \Big(1 + \sum_{1 \leqslant k < l \leqslant n} w_{kl} \phi_k(y_k) \phi_l(y_l)\Big) \prod_{j=1}^{n} G_j(\mathrm{d}y_j) \quad (3)$$

where $G_1, \cdots, G_n$ are corresponding distribution functions of $Y_1, \cdots, Y_n$, respectively, $\phi_1, \cdots, \phi_n$ are kernels, and w_{kl}, $1 \leqslant k < l \leqslant n$, are real numbers which satisfy the condition

$$1 + \sum_{1 \leqslant k < l \leqslant n} w_{kl} \phi_k(y_k) \phi_l(y_l) \geqslant 0,\ (y_1, \cdots, y_n) \in \mathbb{R}^n \quad (4)$$

The definition of Sarmanov distribution above is slightly different from and more general than the original given in Refs. [23-24], see Ref. [9]. Note that when all of the w_{kl} are equal to zero, then (3) reduces to the independent case. By Theorem 5 of Ref. [23], we know that any subset $(Y_{k1}, \cdots, Y_{km})$, $1 \leqslant k_1 < k_2 < \cdots < k_m \leqslant n$, is also a Sarmanov distribution of the form

$$P\Big(\bigcap_{j=1}^{m} (Y_{k_j} \in \mathrm{d}y_j)\Big) = \Big(1 + \sum_{1 \leqslant i < j \leqslant m} w_{k_i k_j} \phi_{k_i}(y_i) \phi_{k_j}(y_j)\Big) \prod_{j=1}^{m} G_{k_j}(\mathrm{d}y_j).$$

Note that

$$\mathrm{Cov}(Y_i, Y_j) = w_{ij} E[Y_i \phi(Y_i)] E[Y_j \phi(Y_j)],$$

therefore we can choose w_{ij} and ϕ_i, ϕ_j such that Y_i and Y_j are positively dependent or negatively dependent. If $\phi_j = 1 - 2G_j$, then (3) leads to the well-known FGM distribution.

Throughout this paper we assume that $(X_1, \cdots, X_m, Y_1, \cdots, Y_n)$ jointly follows a Sarmanov distribution of the form

$$P\Big(\bigcap_{i=1}^{m} (X_i \in \mathrm{d}x_i), \bigcap_{j=1}^{n} (Y_j \in \mathrm{d}y_j)\Big) = \prod_{i=1}^{m} F_i(\mathrm{d}x_i) \prod_{j=1}^{n} G_j(\mathrm{d}y_j) \cdot \Big(1 + \sum_{1 \leqslant k < l \leqslant n} w_{kl} \phi_k(y_k) \phi_l(y_l) + \sum_{1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n} c_{ij} (1 - 2F_i(x_i)) \phi_j(y_j) + \sum_{1 \leqslant i < j \leqslant m} a_{ij} (1 - 2F_i(x_i))(1 - 2F_j(x_j))\Big) \quad (5)$$

From (5), for any $n \in \mathbb{N}$, $m \in \mathbb{N}$ we know that $X_1, \cdots, X_m$ jointly follows a multivariate FGM distribution and $Y_1, \cdots, Y_n$ follows a general Sarmanov distribution. And the two sequences are not independent.

For $m, n \in \mathbb{N}$, let $(X_1^*, \cdots, X_m^*, Y_1^*, \cdots, Y_n^*)$ be an independent copy of $(X_1, \cdots, X_m, Y_1, \cdots, Y_n)$. That is to say, the former has the same marginal distributions as the latter which has independent components. Let $(X_{11}^*, \cdots, X_{m1}^*, Y_{11}^*, \cdots, Y_{n1}^*)$, $(X_{12}^*, \cdots, X_{m2}^*, Y_{12}^*, \cdots, Y_{n2}^*)$, $(X_1^*, \cdots, X_m^*, Y_1^*, \cdots, Y_n^*)$ be three independent and identically distributed random variables. Denote $X_{j\vee}^* := X_{j1}^* \vee X_{j2}^*$, we then know that $X_{j\vee}^*$ has density function $2F_j(x) F_j(\mathrm{d}x)$.

For the sake of simplicity, denote

$$\mu_j(\pi, \alpha) = E[((1-\pi)(1+r) + \pi Y_j)^{-\alpha}],$$

$$\nu_j(\pi, \alpha) = E[\phi_j(Y_j)((1-\pi)(1+r) + \pi Y_j)^{-\alpha}].$$

Hereafter all limit relationships are for $x \to \infty$ unless stated otherwise, and for two positive functions $f(x)$ and $g(x)$, we write $f(x) \sim g(x)$ if $\lim_{x \to \infty} f(x)/g(x) = 1$.

Now we give our main results as follows.

Theorem 1.1 Let $X_1, \cdots, X_n, Y_1, \cdots, Y_n$ jointly follow a multivariate Sarmanov distribution given in (5) with $F \in \mathscr{R}_{-\alpha}$. Assume that for some $p > \alpha$, $EY_j^p < \infty$ for all $j = 1, 2, \cdots, n$. Then

$$\psi(x, n) \sim \overline{F}(x) \cdot \sum_{i=1}^{n} \Big(\prod_{j=1}^{i} \mu_j(\pi, \alpha) \Big[1 + \sum_{1 \leqslant k < l \leqslant i} w_{kl} \frac{\nu_k(\pi, \alpha) \nu_l(\pi, \alpha)}{\mu_k(\pi, \alpha) \mu_l(\pi, \alpha)}\Big] - \sum_{1 \leqslant j \leqslant i} c_{ij} \nu_j(\pi, \alpha) \prod_{\substack{l=1 \\ l \neq j}}^{i} \mu_l(\pi, \alpha) \Big)$$

In particular, if all kernels ϕ_j are identical and all marginal distributions are identical, then

$$\psi(x, n) \sim \overline{F}(x) \sum_{i=1}^{n} \Big(\mu_1^i(\pi, \alpha) \Big[1 + \Big[\frac{\nu_1(\pi, \alpha)}{\mu_1(\pi, \alpha)}\Big]^2 \sum_{1 \leqslant k < l \leqslant i} w_{kl}\Big] - \nu_1(\pi, \alpha) \mu_1^{i-1}(\pi, \alpha) \sum_{1 \leqslant j \leqslant i} c_{ij} \Big).$$

Now we consider a special case that $\{(X_i, Y_i), i \geqslant 1\}$ are independent and identical distributed random vectors. In this case, $a_{ij} = 0$, $w_{ij} = 0$ for all $i, j \in \mathbb{N}$, $c_{ii} = \theta$, $c_{ij} = 0$ for all $i \neq j$. we have the following corollary:

Corollary 1.1 Let $\{(X_i, Y_i), i\geqslant 1\}$ be a sequence of independent and identical distributed random vectors, and $c_{ii}=\theta, -1\leqslant\theta\leqslant 1$. Let $F\in\mathscr{R}_{-\alpha}$ for some $\alpha>0$ and $EY^p<\infty$ for some $p>\alpha$, then we have for all $n=1,2,\cdots,\infty$,

$$\psi(x,n)\sim\frac{1-\mu_1^n(\pi,\alpha)}{1-\mu_1(\pi,\alpha)}(\mu_1(\pi,\alpha)-\theta\nu_1(\pi,\alpha))\overline{F}(x) \tag{6}$$

2 Proof of the theorem

For the case that Z and W are independent, it is well-known that if $Z\in\mathscr{R}_{-\alpha}$ for some $\alpha>0$ and $EW^{\alpha+\delta}<\infty$ for some $\delta>0$, then

$$\lim_{x\to\infty}\frac{P(ZW>x)}{P(Z>x)}=EW^{\alpha}.$$

This result is usually called Breiman's theorem, see Ref.[25]. Now let us introduce the definition of the dependence structure of random variables: quasi-asymptotic independence first introduced by Ref.[26].

Definition 2.1 Two nonnegative random variables X_1 and X_2 with distributions F_1 and F_2, respectively, are said to be quasi-asymptotically independent if

$$\lim_{x\to\infty}\frac{P(X_1>x,X_2>x)}{\overline{F}_1(x)+\overline{F}_2(x)}=0 \tag{7}$$

By the definition of quasi-asymptotical independence, we know that if $X_1,\cdots,X_n$ follow a joint n-dimension FGM distribution, then $X_1,\cdots,X_n$ are pairwise quasi-asymptotically independent. The following two lemmas come from Ref.[26] after some minor modifications.

Lemma 2.1 Let $X_1,\cdots,X_n$ be n pairwise quasi-asymptotically independent real-valued random variables with marginal distributions $F_1\in\mathscr{R}_{-\alpha},\cdots,F_n\in\mathscr{R}_{-\alpha}$, respectively. $S_n=\sum_{i=1}^{n}X_i$, then it holds that

$$P(S_n>x)\sim\sum_{i=1}^{n}\overline{F}_i(x) \tag{8}$$

Lemma 2.2 Let Z_1 and Z_2 be two quasi-asymptotically independent random variables distributed by $H_1\in\mathscr{R}_{-\rho}$ and $H_2\in\mathscr{R}_{-\tau}$ respectively. And let W_1 and W_2 be two nonnegative random variables independent of Z_1 and Z_2 such that $EW_1^p<\infty$ and $EW_2^p<\infty$ for some $p>\rho\vee\tau$. Then the random variables Z_1W_1 and Z_2W_2 are quasi-asymptotically independent.

The following lemma extends Breiman's theorem to dependent and multivariate case.

Lemma 2.3 Suppose that for any $n\in\mathbb{N}$, $X_1,\cdots,X_n,Y_1,\cdots,Y_n$ jointly follows a Sarmanov distribution of the form (5), $F\in\mathscr{R}_{-\alpha}$ and for some $p>\alpha$, $EY_j^p<\infty$ for all $j=1,2,\cdots,n$. Then

$$P(X_n\prod_{j=1}^{n}((1-\pi)(1+r)+\pi Y_j)^{-1}>x)\sim \overline{F}_n(x)\ C(n) \tag{9}$$

where

$$C(n)=\prod_{j=1}^{n}\mu_j(\pi,\alpha)\left[1+\sum_{1\leqslant k<l\leqslant n}w_{kl}\frac{\nu_k(\pi,\alpha)\nu_l(\pi,\alpha)}{\mu_k(\pi,\alpha)\mu_l(\pi,\alpha)}\right]-\sum_{1\leqslant j\leqslant n}c_{nj}\nu_j(\pi,\alpha)\prod_{\substack{i=1\\ i\neq j}}^{n}\mu_i(\pi,\alpha).$$

Proof Denote the distribution function of $(X_n,Y_1,\cdots,Y_n)$ by $H(x_n,y_1,\cdots,y_n)$. By the form (5) of Sarmanov distribution, we have

$$H(dx_n,dy_1,\cdots,dy_n)=F_n(dx_n)\prod_{j=1}^{n}G_j(dy_j)\Big(1+\sum_{1\leqslant k<l\leqslant n}w_{kl}\phi_k(y_k)\phi_l(y_l)+\sum_{1\leqslant j\leqslant n}c_{nj}(1-2F_n(x_n))\phi_j(y_j)\Big)=$$

$$F_n(dx_n)\prod_{j=1}^{n}G_j(dy_j)\Big(1+\sum_{1\leqslant k<l\leqslant n}w_{kl}\phi_k(y_k)\phi_l(y_l)\Big)+F_n(dx_n)\prod_{j=1}^{n}G_j(dy_j)\Big(\sum_{1\leqslant j\leqslant n}c_{nj}(1-2F_n(x_n))\phi_j(y_j)\Big).$$

Note that $2F_n(x)F_n(dx)$ is the density function of $X_{n\vee}{}^{*}=X_{n1}^{*}\vee X_{n2}^{*}$, and let $(\widetilde{Y}_1^{*},\cdots,\widetilde{Y}_n^{*})$ be a random vector independent of $(X_1^{*},\cdots,X_m^{*},Y_1^{*},\cdots,Y_n^{*})$ with distribution $\widetilde{G}_j$, defined by

$$\widetilde{G}_j(dy)=\phi_j(y_j)G_j(dy).$$

By Ref.[27], $EY_j^p<\infty$ for some $p>\alpha$ and Breiman's theorem, we have

$$P\Big(X_n\prod_{j=1}^{n}f(\pi,Y_j)>x\Big)=P\Big(X_n^{*}\prod_{j=1}^{n}f(\pi,Y_j)>x\Big)+$$

$$\sum_{1\leqslant j\leqslant n} c_{nj} P\big(X_n^* f(\pi,\widetilde{Y}_j^*)\prod_{\substack{i=1\\ i\neq j}}^{n} f(\pi,Y_i^*) > x\big)-$$

$$\sum_{1\leqslant j\leqslant n} c_{nj} P\big(X_{n\vee}^* f(\pi,\widetilde{Y}_j^*)\prod_{\substack{i=1\\ i\neq j}}^{n} f(\pi,Y_i^*) > x\big)=:$$

$$P_1+P_2-P_3.$$

$$P_1 \sim \overline{F}_n(x) E\Big[\prod_{j=1}^{n}((1-\pi)(1+r)+\pi Y_j)^{-\alpha}\Big]=$$

$$\overline{F}_n(x)\prod_{j=1}^{n}\mu_j(\pi,\alpha)\left[1+\sum_{1\leqslant k<l\leqslant n} w_{kl}\frac{\nu_k(\pi,\alpha)\nu_l(\pi,\alpha)}{\mu_k(\pi,\alpha)\mu_l(\pi,\alpha)}\right].$$

$X_i^*\in\mathscr{R}_{-\alpha}$ implies $P(X_{i\vee}^*>x)\sim 2P(X_i^*>x)$ and $X_{i\vee}^*\in\mathscr{R}_{-\alpha}$, then

$$P_2\sim\sum_{1\leqslant j\leqslant n} c_{nj}\overline{F}_n(x)E[\phi_j(Y_j)f(\pi,Y_j)^{-\alpha}]\cdot$$

$$E\Big[\prod_{\substack{i=1\\ i\neq j}}^{n} f(\pi,Y_i^*)^{-\alpha}\Big]=$$

$$\overline{F}_n(x)\sum_{1\leqslant j\leqslant n} c_{nj}\nu_j(\pi,\alpha)\prod_{\substack{i=1\\ i\neq j}}^{n}\mu_i(\pi,\alpha),$$

$$P_3\sim 2P_2\sim 2\overline{F}_n(x)\sum_{1\leqslant j\leqslant n} c_{nj}\nu_j(\pi,\alpha)\prod_{\substack{i=1\\ i\neq j}}^{n}\mu_i(\pi,\alpha).$$

Thus we have

$$P\big(X_n\prod_{j=1}^{n} f(\pi,Y_j)>x\big)\sim$$

$$\overline{F}_n(x)\Big(\prod_{j=1}^{n}\mu_j(\pi,\alpha)\big(1+\sum_{1\leqslant k<l\leqslant n} w_{kl}\frac{\nu_k(\pi,\alpha)\nu_l(\pi,\alpha)}{\mu_k(\pi,\alpha)\mu_l(\pi,\alpha)}\big)-$$

$$\sum_{1\leqslant j\leqslant n} c_{nj}\nu_j(\pi,\alpha)\prod_{\substack{i=1\\ i\neq j}}^{n}\mu_i(\pi,\alpha)\Big).\qquad\square$$

Lemma 2.4 Let $X_1,\cdots,X_n,Y_1,\cdots,Y_n$ jointly follow a multivariate Sarmanov distribution given in (5) with $F_i\in\mathscr{R}_{-\alpha}$, $1\leqslant i\leqslant n$ and assume that for some $p>\alpha$, $EY_j^p<\infty$ for all $j=1,2,\cdots,n$. Then for any $1\leqslant k<l\leqslant n$, the random variables $X_lY_1\cdots Y_l$ and $X_kY_1\cdots Y_k$ are quasi-asymptotically independent.

Proof Let $H(x_l, x_k, y_1, \cdots, y_n)$ be the distribution function of $X_l,X_k,Y_1,\cdots,Y_n$, then

$$H(dx_l,dx_k,dy_1,\cdots,dy_n)=$$

$$F_l(dx_l)F_k(dx_k)\prod_{j=1}^{n}G_j(dy_j)\cdot$$

$$((1+\sum_{1\leqslant k<l\leqslant n} w_{kl}\phi_k(y_k)\phi_l(y_l))+$$

$$a_{kl}(1-2F_k(x_k))(1-2F_l(x_l))+$$

$$\sum_{1\leqslant j\leqslant n} c_{kj}(1-2F_k(x_k))\phi_j(y_j)+$$

$$\sum_{1\leqslant j\leqslant n} c_{lj}(1-2F_l(x_l))\phi_j(y_j)).$$

We cut the probability of $P(X_l\prod_{j=1}^{l} f(\pi,Y_j)>x$, $X_k\prod_{j=1}^{k} f(\pi,\ Y_j)>x)$ into four parts, that is

$$P(X_l\prod_{j=1}^{l} f(\pi,Y_j)>x, X_k\prod_{j=1}^{k} f(\pi,Y_j)>x)=$$

$$\int_{x_l\prod_{j=1}^{l} f(\pi,Y_j)>x,\,x_k\prod_{j=1}^{k} f(\pi,Y_j)>x} H(dx_l,dx_k,dy_1,\cdots,dy_k)=:$$

$$Q_1+Q_2+Q_3+Q_4\qquad(10)$$

First we deal with Q_1. From Lemma 2.3 we know that

$$P(X_l\prod_{j=1}^{l} f(\pi,Y_j)>x)\sim\overline{F}_l(x)\ C(l).$$

By Lemma 2.2, we know that $X_l^*\prod_{j=1}^{l} f(\pi,Y_j)$ and $X_k^*\prod_{j=1}^{k} f(\pi,Y_j)$ are quasi-asymptotically independent. Following the definition of quasi-asymptotically independent and Breiman's theorem, we have

$$Q_1=$$

$$\int_{x_l\prod_{j=1}^{l} f(\pi,Y_j)>x,\,x_k\prod_{j=1}^{k} f(\pi,Y_j)>x}\big(1+\sum_{1\leqslant k<l\leqslant n} w_{kl}\phi_k(y_k)\phi_l(y_l)\big)\cdot$$

$$F_l(dx_l)F_k(dx_k)\prod_{j=1}^{n}G_j(dy_j)=$$

$$P(X_l^*\prod_{j=1}^{l} f(\pi,Y_j)>x, X_k^*\prod_{j=1}^{k} f(\pi,Y_j)>x)=$$

$$o(1)(P(X_l^*\prod_{j=1}^{l} f(\pi,Y_j)>x)+$$

$$P(X_k^*\prod_{j=1}^{k} f(\pi,Y_j)>x))=$$

$$o(1)(P(X_l>x)+P(X_k>x))=$$

$$o(1)\ (P(X_l\prod_{j=1}^{l} f(\pi,Y_j)>x)+$$

$$P(X_k\prod_{j=1}^{k} f(\pi,Y_j)>x)).$$

Next we consider Q_2,

$$Q_2=\int_{\{x_l\prod_{j=1}^{l} f(\pi,Y_j)>x,\,x_k\prod_{j=1}^{k} f(\pi,Y_j)>x\}} a_{kl}(1-2F_k(x_k))\cdot$$

$$(1-2F_l(x_l))F_l(dx_l)F_k(dx_k)\prod_{j=1}^{n}G_j(dy_j)=$$

$$a_{kl}\Big[P\big(X_l^*\prod_{j=1}^{l} f(\pi,Y_j^*)>x, X_k^*\prod_{j=1}^{k} f(\pi,Y_j^*)>x\big)-$$

$$P\big(X_{l\vee}^*\prod_{j=1}^{l} f(\pi,Y_j^*)>x, X_k^*\prod_{j=1}^{k} f(\pi,Y_j^*)>x\big)-$$

$$P\Big(X_l^{*}\prod_{j=1}^{l}f(\pi,Y_j^{*})>x,X_{k\vee}^{*}\prod_{j=1}^{k}f(\pi,Y_j^{*})>x\Big)+$$
$$P\Big(X_{l\vee}^{*}\prod_{j=1}^{l}f(\pi,Y_j^{*})>x,X_{k\vee}^{*}\prod_{j=1}^{k}f(\pi,Y_j^{*})>x\Big)\Big]=:$$
$$a_{kl}(Q_{21}-Q_{22}-Q_{23}+Q_{24}).$$

Note that $P(X_{k\vee}^{*}>x)\sim 2P(X_k>x)$, by a similar argument to Q_1 we have

$$Q_{2j}=o(1)\Big(P\big(X_l\prod_{j=1}^{l}f(\pi,Y_j)>x\big)+P\big(X_k\prod_{j=1}^{k}f(\pi,Y_j)>x\big)\Big),\ j=1,2,3,4.$$

Armed with the same technic to Q_2, we have

$$Q_i=o(1)\Big(P\big(X_l\prod_{j=1}^{l}f(\pi,Y_j)>x\big)+P\big(X_k\prod_{j=1}^{k}f(\pi,Y_j)>x\big)\Big),\ i=3,4.$$

Thus the random variables $X_lY_1\cdots Y_l$ and $X_kY_1\cdots Y_k$ are quasi-asymptotically independent. □

Proof of Theorem 1.1 For a real number x, its positive part is denoted by

$$x^{+}=\max\{x,0\}=x\vee 0.$$

Clearly, $\{Y_i, i\geqslant 1\}$ are nonnegative,

$$\sum_{i=1}^{n}X_i\prod_{j=1}^{i}Y_j\leqslant\max_{1\leqslant k\leqslant n}\sum_{i=1}^{k}X_i\prod_{j=1}^{i}Y_j\leqslant\sum_{i=1}^{n}X_i^{+}\prod_{j=1}^{i}Y_j,\quad n\geqslant 1.$$

Lemma 2.3 implies $X_i\prod_{j=1}^{i}Y_j\in\mathscr{R}_{-\alpha}$ for all $i=1,\cdots,n$. Lemma 2.4 gives that $X_l\prod_{j=1}^{l}Y_j$ and $X_k\prod_{j=1}^{k}Y_j$ are quasi-asymptotically independent for all $1\leqslant l\neq k\leqslant n$. By Lemma 2.1, we have

$$P\Big(\sum_{i=1}^{n}X_i^{+}\prod_{j=1}^{i}Y_j>x\Big)\sim\sum_{i=1}^{n}P\Big(X_i^{+}\prod_{j=1}^{i}Y_j>x\Big),$$

and

$$P\Big(\sum_{i=1}^{n}X_i\prod_{j=1}^{i}Y_j>x\Big)\sim\sum_{i=1}^{n}P\Big(X_i\prod_{j=1}^{i}Y_j>x\Big).$$

Note that $P(X_i^{+}\prod_{j=1}^{i}Y_j>x)=P(X_i\prod_{j=1}^{i}Y_j>x)$, $x>0$, the rest proof of Theorem 1.1 is trivial. The proof is completed. □

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