第45卷第1期 中国科学技术大学学报 Vol. 45,No. 7
2015年7月 JOURNAL OF UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA Jul. 2015

Article ID:0253-2778(2015)07-0614-09

Non-fragile stabilization for networked control systems with input delay and randomly occurring gain uncertainties

JANG Shun[1,2], PAN Feng[1]

(1. *Key Laboratory of Advanced Process Control for Light Industry of Ministry of Education*,
2. *Department of Automation, Jiangnan University*, *Wuxi* 214122, *China*)

Abstract: The problem of non-fragile stabilization is investigated for a class of nonlinear networked control systems with time-varying input delay and randomly occurring gain uncertainties. A binary switching sequence obeying a conditional probability distribution is introduced to govern the randomly occurring gain fluctuation in controller implementation, which could better reflect the random nature of network-induced phenomena. Attention is focused on the design of a non-fragile static output feedback controller such that the closed-loop systems is mean-square asymptotically stable in the presence of network-induced delay and gain uncertainties. Intensive stochastic analyses and novel inequality bounding techniques are carried out to achieve the existence condition of the stabilization controller, and the desired controller gain can be derived by solving a nonconvex feasibility problem via a modified cone complementary linearization algorithm. Finally, a numerical example is provided to illustrate the effectiveness and superiority of the proposed stabilization method.

Key words: networked control systems; non-fragile control; time-varying delays; randomly occurring uncertainty; stabilization

CLC number: TP273 **Document code**: A doi: 10.3969/j.issn.0253-2778.2015.07.013

Citation: JANG Shun, PAN Feng. Non-fragile stabilization for networked control systems with input delay and randomly occurring gain uncertainties[J]. Journal of University of Science and Technology of China, 2015,45(7): 614-622.

有随机增益不确定性的网络化时滞系统非脆弱镇定

姜 顺,潘 丰

(轻工过程先进控制教育部重点实验室,江南大学自动化系,江苏无锡 214122)

摘要:研究了带有输入时滞和随机发生的增益不确定性的网络化控制系统的非脆弱镇定问题.目标是设计一个非脆弱的静态输出反馈控制器,使得闭环系统在有输入时滞和增益不确定情况下保持均方渐近稳定.为了更好地反映网络诱导现象的随机特性,引入了服从一定概率分布的二进制切换序列描述控制器在实现过程中随机发生的增益浮动.然后利用随机稳定性理论和新的矩阵不等式界定方法得到了控制器存在的充分条件.据此,控制器增益矩阵可通过给出的锥补线性化算法求解.最后提供了一个数值例子验证了方法的有效性和优越性.

Received: 2015-01-20; **Revised**: 2015-04-16

Foundation item: Supported by National Natural Science Foundation (NNSF) of China (61403168, 61273131).

Biography: JIANG Shun (corresponding author), male, born in 1981, PhD/associate professor.
Research field: networked control systems. Email: haveshun@sina.com

关键词：网络化控制系统；非脆弱控制；时变时滞；随机发生的不确定性；镇定

0 Introduction

In the past decade, with the rapid development of network technology and the wide spread of the Internet, the use of networks in control systems has become popular[1-5], which makes it possible for us to remotely control the large distributed systems. Networked control systems (NCSs) are a type of distributed control systems where the information of system components is exchanged via communication networks. Compared with the conventional point-to-point system connection, the new network-based control scheme can reduce system wiring, ease installation and maintenance and increase the reliability. While NCSs have many appealing advantages, the insertion of the communication network in control loops has also brought some interesting and challenging problems due to inherent network-limited bandwidth, including network-induced delays[6-7], packet dropouts[8-9], quantization errors[10] and so on, which could deteriorate system performance and may even destablize the system.

Recently, the stabilization problem of NCSs has attracted considerable research interest and fruitful results have been reported in the Refs. [11-15]. Generally speaking, based on the assumption that system state information is completely available, state feedback control is the most commonly used stabilization method. For example, in Ref. [14], by assuming that the network-induced delay takes values in a set and the occurrence probabilities of delay values are known, the stabilization problem has been investigated for NCSs. Under the consideration of bounded packet loss, the problem of state feedback stabilization has been studied for linear systems over networks in Ref. [8]. In Ref. [11], by constructing a new Lyapunov functional and by making use of novel bounding techniques for some cross terms to reduce the conservatism, a delay-dependent H_∞ stabilization criterion has been derived for continuous-time NCSs. On the other hand, in view of the unavailability of state information in practical engineering systems, output feedback stabilization has stirred increasing research attention due mainly to its practicability. Just to mention a few: in Ref. [12], by taking network-induced delay, random packet loss and quantization error into consideration simultaneously, a static output feedback controller has been designed to stabilize the nonlinear NCSs. According to the different status of packet dropouts in the backward and the forward channels, the NCSs with short communication delays and packet dropouts have been modeled as switched systems with four subsystems in Ref. [9] and the observer-based output feedback controller has been utilized to exponentially stabilize the original systems. In Ref. [15], by employing two Markov chains to model sensor-to-controller (S-C) and controller-to-actuator (C-A) random network-induced delays, a dynamic output feedback controller has been designed via iterative LMI approach.

It is worth noting that almost all the reference mentioned above has been based on the implicit assumption that the controller can be implemented exactly. Unfortunately, such an assumption may not be true in some practical engineering applications. For example, in practice, controllers may have a certain degree of errors owing to round-off in numerical computation, finite word length in digital systems, the imprecision inherent in analog systems and the need for additional tuning of parameters in the finial implementation. Hence, how to design a controller insensitive to the variations in its gain, i. e., the controller is non-fragile, has received particular attention in recent years[16-18]. However, so far, the non-fragile stabilization problem for nonlinear NCSs with time-varying delays and gain uncertainties has not been adequately investigated, not to mention the case where gain uncertainties occur in a probabilistic way.

In this paper, we will be concerned with the non-fragile stabilization problem for a class of nonlinear NCSs with time-varying input delays. A unified static output feedback control law is adopted to take network-induced delays and gain uncertainties into consideration simultaneously. In light of the fact that some network-induced phenomena may lead to intermittent parameter changes in practical systems, here the occurrence of controller gain uncertainties is characterized in a probabilistic way, which is quite different from common deterministic and persistent uncertainties considered in traditional non-fragile control/filter problems. The main purpose of the paper is to design a non-fragile static output feedback controller such that the resulting closed-loop system is stabilized in the presence of network-induced delays and randomly occurring gain uncertainties. By utilizing Lyapunov stability theory and some new inequality bounding techniques, the existence condition of the stabilization controller is formulated in the form of nonlinear matrix inequalities, and the desired controller gain can be derived by solving a nonconvex feasibility problem via the provided cone complementary linearization (CCL) algorithm. Finally, a numerical example is exploited to demonstrate the effectiveness of the proposed stabilization method.

Notation the notation used in the paper is fairly standard. The superscript "T" denotes matrix transposition. $\mathbb{R}^n$ stands for the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ is the set of all the real matrices of dimension $m\times n$. I and 0 represent the identity matrix and zero matrix, respectively. The notation $X>0$ (respectively, $X\geqslant 0$) means that X is real symmetric and positive definite (respectively, positive semi-definite). Prob{ • } means the occurrence probability of the event " • ", and $E\{x|y\}$ stands for the expectation of x conditional on y. In symmetric block matrices, we use an asterisk * to represent a term that is induced by symmetry and diag{…} denotes a block-diagonal matrix.

1 Problem statements and formulations

Consider a class of discrete-time systems with sector-bounded nonlinearity described as follows:

$$\left.\begin{aligned} x(\kappa+1)&=Ax(\kappa)+Ff(\kappa,x(\kappa))+Bu(\kappa)\\ y(\kappa)&=Cx(\kappa)\end{aligned}\right\} \tag{1}$$

where $x(\kappa)\in\mathbb{R}^n$ is the state vector, $u(\kappa)\in\mathbb{R}^p$ is the control input, $y(\kappa)\in\mathbb{R}^m$ is the measured output, and A, B, C, F are known real constant matrices with appropriate dimensions. The nonlinear vector-valued function $f(\cdot,\cdot)$ is assumed to satisfy the following global sector-bounded condition:

$$[f(\kappa,x(\kappa))-S_1\tilde{x}(\kappa)]^T[f(\kappa,x(\kappa))-S_2\tilde{x}(\kappa)]=0 \tag{2}$$

where $\tilde{x}(\kappa)\in\mathbb{R}^n$, $S_1,S_2\in\mathbb{R}^{m\times n}$ are known real constant matrices, and S_1-S_2 is a positive defined matrix.

Remark 1.1 It is customary that the nonlinear function $f(\cdot,\cdot)$ is said to belong to the sector $[S_1,S_2]$[19]. Note that the nonlinear description in Eq. (2) is quite general and can include the usual Lipschitz condition as a special case, and also covers several other classes of well-studied nonlinear systems. In recent years, the control analysis, filtering and fault detection problems for systems with sector nonlinearities have been intensively studied[20-22]

In this paper, we are interested in designing a static output feedback controller of the following form:

$$u(\kappa)=Ky(\kappa) \tag{3}$$

where K is the controller gain to be determined, such that the original system (1) is stabilized in a nonideal networked environment.

As is discussed in Introduction, since the measurements and control signals are transmitted over the communication networks with limited bandwidth, the network-induced delays are often inevitable in the shared communication channels.

In addition, inaccuracies or uncertainties do occur in the controller implementation due to some unexpected errors, e. g. actuator degradations, roundoff errors in numerical computation, additional tuning of parameters and so on. Thus, by taking the communication delays and uncertainties into consideration simultaneously, the non-fragile output feedback control law can be described preferably by

$$u(\kappa) = [K + \alpha(\kappa)\Delta K] y(\kappa - \tau(\kappa)) \tag{4}$$

where the matrix ΔK represents norm-bounded parameter uncertainties satisfying

$$\Delta K = M_1 F(\kappa) N_1.$$

where M_1 and N_1 are known real constant matrices with appropriate dimensions. $F(\kappa)$ is a time-varying uncertain matrix which satisfies $F^T(\kappa)F(\kappa) \leqslant \gamma^2 I$, where the positive scalar γ stands for the amplitude of the uncertainty. The stochastic variable $\alpha(\kappa) \in \mathbb{R}$ is a Bernoulli-distributed white sequence, which is introduced to characterize the random fluctuation of controller gain in the implementation. A natural assumption on $\alpha(\kappa)$ can be made as follows:

$$\left.\begin{aligned} &\text{Prob}\{\alpha(\kappa)=1\} = E\{\alpha(\kappa)\} = \bar{\alpha} \\ &\text{Prob}\{\alpha(\kappa)=0\} = 1 - E\{\alpha(\kappa)\} = 1 - \bar{\alpha} \end{aligned}\right\} \tag{5}$$

$\tau(\kappa)$enotes network-induced delay satisfying

$$\tau_m \leqslant \tau(\kappa) \leqslant \tau_M,$$

where τ_m and τ_M are known nonnegative integers representing the lower and upper bounds of the time-varying delays, respectively.

Remark 1. 2 A unified control law model described in Eq. (4) is proposed to take the randomly occurring parameter uncertainties and communication delay into consideration, simultaneously. Although the problem of non-fragile control has attracted particular research interests in recent years, the existing results have mainly concentrated on deterministic and persistent uncertainties[18,23,24] In reality, however, the occurrence of uncertainty in the controller gain may be random and intermittent owing to abrupt structural and parametric changes arising some network-induced phenomena, such as random failures and repairs of communication components, sudden environmental disturbances in network channels, changing subsystem interconnections, etc[25-26]. Therefore, it could better reflect the reality of the gain fluctuation in controller implementation to formulate the gain changes into a stochastic framework especially in an unreliable networked environment.

Substituting Eq. (4) as $u(\kappa)$ into Eq. (1), we obtain the closed-loop system as follows:

$$x(\kappa+1) = Ax(\kappa) + Ff(\kappa, x(\kappa)) + BK_1 x(\kappa - \tau(\kappa)) \tag{6}$$

where $K_1 = [K + \alpha(\kappa)\Delta K]C$.

It should be noticed that the closed-loop system (6) is actually a stochastic system with time-varying delays and parameter uncertainties, in which the stochastic variable $\alpha(\kappa)$ is employed to characterize the phenomenon of randomly occurring parameter uncertainties, and $\tau(\kappa)$ represents the network-induced delay. The randomicity of the parameter changes makes the analysis and synthesis more complicated than that of ordinary time-delay systems. To deal with the stochastic parameter systems, the following definition should be first introduced for the stability analysis.

Definition 1. 1 The stochastic parameter system (6) is said to be robustly asymptotically mean-square stable if

$$E\{\| x(\kappa) \|^2\} \to 0 \text{ as } \kappa \to \infty$$

for any initial conditions and all admissible uncertainties.

In the following, we will concentrate on stability analysis and controller design for system (6) with time-varying delays and stochastic parameter uncertainties.

2 Stability analysis

Before proceeding further, we first give the following lemmas that are essential in establishing our main results in this paper.

Lemma 2. 1[27] Let $Y_0(\eta)$, $Y_1(\eta)$, $\cdots$, $Y_\theta(\eta)$, be quadratic function of $\eta \in \mathbb{R}^n$, $Y_i(\eta) = \eta^T T_i \eta$, $i = 0, 1, 2, \cdots, \theta$ with $T_i = T_i^T$. Then the implication$Y_1(\eta) \leqslant 0, \cdots, Y_\theta(\eta) \leqslant 0 \Rightarrow Y_0(\eta) \leqslant 0$

holds if there exist nonnegative scalars $\lambda_1, \cdots, \lambda_\theta$ such that

$$T_0 - \sum_{i=1}^{\theta} \lambda_i T_i < 0$$

Lemma 2. 2[13] Suppose $\tau_m \leqslant \tau(\kappa) \leqslant \tau_M$, $\sum_i = \sum_i^{\mathrm{T}}$, $i=1,2$, and $\Phi = \Phi^{\mathrm{T}}$ are some constant matrices with appropriate dimensions, then $(\tau(\kappa)-\tau_m)\sum_1 + (\tau_m - \tau(\kappa))\sum_2 + \Phi \leqslant 0$ holds if and only if the following inequalities hold:

$$(\tau_M - \tau_m)\sum\nolimits_1 + \Phi < 0; (\tau_M - \tau_m)\sum\nolimits_2 + \Phi < 0.$$

The following theorem provides a sufficient condition under which the closed-loop system (6) is asymptotically mean-square stable.

Theorem 2. 1 Let the scalars τ_m, τ_M, $\bar{\alpha}$ and the controller gain matrix K be given. Then, the closed-loop system (6) is robustly asymptotically mean-square stable if there exist matrices $P>0$, $Q_i>0 (i=1,2,3)$, $R_j>0 (j=1,2)$, M, N, S, T and a scalar $\mu \geqslant 0$ such that the following matrix inequality.

$$\begin{bmatrix} \Theta_{11} & * & * & * & * & * \\ \Theta_{21} & \Theta_{22} & * & * & * & * \\ \Theta_{31} & 0 & \Theta_{33} & * & * & * \\ \Theta_{41} & 0 & 0 & \Theta_{44} & * & * \\ \tau_m N & 0 & 0 & 0 & -\tau_m R_1 & * \\ \Pi_\kappa & 0 & 0 & 0 & 0 & \Theta_{66} \end{bmatrix} < 0 \qquad (7)$$

holds for $\kappa=1,2$, where

$\Theta_{11} = \Sigma + \Omega + \Omega^{\mathrm{T}}$, $\tilde{\tau} = \tau_M - \tau_m + 1$;

$$\Sigma = \begin{bmatrix} \Xi & * & * & * & * \\ 0 & -Q_1 & * & * & * \\ 0 & 0 & -Q_3 & * & * \\ 0 & 0 & 0 & -Q_2 & * \\ 0.5\mu(S_1+S_2) & 0 & 0 & 0 & -\mu I \end{bmatrix};$$

$\Omega = [N^{\mathrm{T}} \; S^{\mathrm{T}} \; M^{\mathrm{T}} - N^{\mathrm{T}} - S^{\mathrm{T}} + T^{\mathrm{T}} - M^{\mathrm{T}} - T^{\mathrm{T}} \; 0]$;

$\Xi = Q_1 + Q_2 + \tilde{\tau} Q_3 - P - 0.5\mu(S_1^{\mathrm{T}} S_2 + S_2^{\mathrm{T}} S_1)$;

$$\Theta_{21} = \begin{bmatrix} A & 0 & BK_0 & 0 & F \\ 0 & 0 & \bar{\beta} BK_\Delta & 0 & 0 \end{bmatrix};$$

$$\Theta_{22} = \begin{bmatrix} -P^{-1} & * \\ 0 & -P^{-1} \end{bmatrix}, \; K_0 = (K + \bar{\alpha}\Delta K)C;$$

$K_\Delta = \Delta K C$, $\bar{\beta} = \sqrt{\bar{\alpha}(1-\bar{\alpha})}$, $\hat{\tau} = \tau_M - \tau_m$;

$$\Theta_{31} = \begin{bmatrix} \tau_M(A-I) & 0 & \tau_M BK_0 & 0 & \tau_M F \\ 0 & 0 & \tau_M \bar{\beta} BK_\Delta & 0 & 0 \end{bmatrix};$$

$$\Theta_{33} = \begin{bmatrix} \tau_M R_1^{-1} & * \\ 0 & \tau_M R_1^{-1} \end{bmatrix}, \; \Theta_{44} = \begin{bmatrix} \hat{\tau} R_2^{-1} & * \\ 0 & \hat{\tau} R_2^{-1} \end{bmatrix};$$

$$\Theta_{41} = \begin{bmatrix} \hat{\tau} M(A-I) & 0 & \hat{\tau} BK_0 & 0 & \hat{\tau} F \\ 0 & 0 & \hat{\tau}\bar{\beta} BK_\Delta & 0 & 0 \end{bmatrix};$$

$$\Pi_1 = \begin{bmatrix} \hat{\tau} N \\ \hat{\tau} S \end{bmatrix}, \; \Pi_2 = \begin{bmatrix} \hat{\tau} M \\ \hat{\tau} T \end{bmatrix}, \; \Theta_{66} = \begin{bmatrix} -\hat{\tau} R_1 & * \\ 0 & -\hat{\tau} R_2 \end{bmatrix}.$$

Proof is omitted here. If need, you can contact with author.

Note that the stability criterion (7) in Theorem 2. 1 is not a strict LMI condition due to the existence of uncertain terms and the coexistence of Lyapunov matrices and their inversions. In what follows, we are devoted to establishing an alternative sufficient condition for the controller design problem.

3 Controller design

Based on the stability criterion presented in section 2, the following theorem will present a sufficient condition for the existence of the non-fragile stabilization controller, which ensures that the closed-loop system (6) is robustly asymptotically mean-square stable in the presence of randomly occurring uncertainties and time-varying delays.

Theorem 3. 1 Let the scalars τ_m, τ_M and $\bar{\alpha}$ be given. Then, there exists a non-fragile output feedback controller in the form of (4) such that the closed-loop system (6) is robustly asymptotically mean-square stable if there exist matrices $P>0$, $Q_i>0 (i=1,2,3)$, $R_j>0 (j=1,2)$, $\chi>0$, $U>0$, $V>0$, M, N, S, T and scalars $\varepsilon>0$, $\mu>0$ satisfying:

$$\begin{bmatrix} \Psi_{11} & * & * & * & * & * & * \\ \Psi_{21} & \Psi_{22} & * & * & * & * & * \\ \Psi_{31} & 0 & \Psi_{33} & * & * & * & * \\ \Psi_{41} & 0 & 0 & \Psi_{44} & * & * & * \\ \tau_m N & 0 & 0 & 0 & -\tau_m R_1 & * & * \\ \Pi_\kappa & 0 & 0 & 0 & 0 & \Theta_{66} & \\ 0 & \Psi_{72} & \Psi_{73} & \Psi_{74} & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \qquad (8)$$

$$P\chi = I, \; R_1 U = I, \; R_1 V = I \qquad (9)$$

for $\kappa = 1, 2$, where Π_κ, Θ_{66} are given in

Eq. (7), and

$\Psi_{11}=\Theta_{11}+\mathrm{diag}\{0,0,\varepsilon\gamma^2 C^T N_1^T N_1 C,0,0\}$;

$\Psi_{21}=\begin{bmatrix} A & 0 & BKC & 0 & F \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, $\Psi_{22}=\begin{bmatrix} -\chi & * \\ 0 & -\chi \end{bmatrix}$;

$\Psi_{31}=-\tau_M\begin{bmatrix} (A-I) & 0 & BKC & 0 & F \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$;

$\Psi_{41}=\tau\begin{bmatrix} (A-I) & 0 & BKC & 0 & F \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, $\Psi_{33}=\tau_M\begin{bmatrix} U & * \\ 0 & U \end{bmatrix}$;

$\Psi_{44}=\hat{\tau}\begin{bmatrix} -V & * \\ 0 & -V \end{bmatrix}$, $\Psi_{72}=[\bar{\alpha}M_1{}^T B^T \quad \bar{\beta}M_1{}^T B^T]$;

$\Psi_{73}=\tau_M\Psi_{72}$, $\Psi_{74}=\hat{\tau}\Psi_{72}$

Proof In order to eliminate the uncertain terms, we rewrite Eq. (7) as the following form:

$$\begin{bmatrix} \Theta_{11} & * & * & * & * & * \\ \Psi_{21} & \Theta_{22} & * & * & * & * \\ \Psi_{31} & 0 & \Theta_{33} & * & * & * \\ \Psi_{41} & 0 & 0 & \Theta_{44} & * & * \\ \tau_m N & 0 & 0 & 0 & -\tau_m R_1 & * \\ \Pi_\kappa & 0 & 0 & 0 & 0 & \Theta_{66} \end{bmatrix}+$$

$$\frac{1}{\gamma}[0\ \Psi_{72}\ \Psi_{73}\ \Psi_{74}\ 0\ 0]^T F(\kappa)[\phi\ 0\ 0\ 0\ 0\ 0]+$$

$$\frac{1}{\gamma}[\phi\ 0\ 0\ 0\ 0\ 0]^T F^T(\kappa)[0\ \Psi_{72}\ \Psi_{73}\ \Psi_{74}\ 0\ 0]<0 \quad (10)$$

Because of $\frac{1}{\gamma^2}F^T(\kappa)F(\kappa)\leqslant I$, according to Schur complement, we know that Eq. (10) holds if and only if there exists scalar $\varepsilon>0$ such that

$$\begin{bmatrix} \Theta_{11} & * & * & * & * & * & * \\ \Psi_{21} & \Theta_{22} & * & * & * & * & * \\ \Psi_{31} & 0 & \Theta_{33} & * & * & * & * \\ \Psi_{41} & 0 & 0 & \Theta_{44} & * & * & * \\ \tau_m N & 0 & 0 & 0 & -\tau_m R_1 & * & \\ \Pi_\kappa & 0 & 0 & 0 & 0 & \Theta_{66} & * \\ 0 & \Psi_{72} & \Psi_{73} & \Psi_{74} & 0 & 0 & -\varepsilon I \end{bmatrix}<0 \quad (11)$$

Then, by defining $\chi=P^{-1}$, $U=R_1{}^{-1}$ and $V=R_2{}^{-1}$, we readily obtain Eqs. (8) and (9), and the proof is completed.

Notice that the design criteria in the above theorem are not strict LMI conditions due to the existence of matrix equality constraint (9). This means that the direct computation of the desired controller gain K becomes infeasible. However, as discussed in Ref. [28], we can use the CCL algorithm to formulate it into a sequential optimization problem subject to LMI constraints.

Using the CCL approach, we suggest the following nonlinear minimization problem involving LMI conditions instead of the original nonconvex feasibility problem formulated in Theorem 3. 1.

Minimize $\mathrm{tr}\{P\chi+R_1U+R_2V\}$

subject to Eq. (8) and

$$\begin{bmatrix} P & I \\ I & \chi \end{bmatrix}\geqslant 0,\ \begin{bmatrix} R_1 & I \\ I & U \end{bmatrix}\geqslant 0,\ \begin{bmatrix} R_2 & I \\ I & V \end{bmatrix}\geqslant 0 \quad (12)$$

According to the basic idea of CCL, if the solution to the above minimization problem is $3n$, that is,

Minimize $\mathrm{tr}\{P\chi+R_1U+R_2V=3n\}$,

then the conditions in Theorem 3. 1 are solvable. It is worth mentioning that although the above optimization problem (12) may not necessarily always find the global optimal solution, the adopted nonlinear minimization problem is much easier to solve than the original nonconvex feasibility problem.

Specifically, the iterative algorithm for designing the non-fragile output feedback controller can be outlined as follows.

Algorithm 3. 1 Find the non-fragile output feedback controller gain matrix K.

Step 1 Find a feasible set (P^0, $Q_1{}^0$, $Q_2{}^0$, $Q_3{}^0$, $R_1{}^0$, $R_2{}^0$, χ^0, U^0, V^0, K^0, M^0, N^0, S^0, T^0, μ^0, ε^0) satisfying Eqs. (8) and (12). Set $\kappa=0$.

Step 2 Solve the following minimization problem:

Minimize $\mathrm{tr}\{P^\kappa\chi+P\chi^\kappa+R_1{}^\kappa U+R_1U^\kappa+R_2{}^\kappa V+R_2V^\kappa\}$

subject to Eqs. (8) and (12).

Step 3 Substitute the obtained matrix variables into Eq. (11). If the condition (11) is satisfied with

$|\mathrm{tr}\{P^\kappa\chi+P\chi^\kappa+R_1{}^\kappa U+R_1U^\kappa+R_2{}^\kappa V+R_2V^\kappa\}-6n|<\delta$

for a sufficient small scalar $\delta>0$, then output the feasible solution, EXIT.

Step 4 If $\kappa>N$, where N is the maximum number of the iterations allowed, EXIT.

Step 5 Otherwise, set $\kappa=\kappa+1$, (P^κ, $Q_1{}^\kappa$, $Q_2{}^\kappa$, $Q_3{}^\kappa$, $R_1{}^\kappa$, $R_2{}^\kappa$, χ^κ, U^κ, V^κ, K^κ, M^κ, N^κ, S^κ,

T^{κ}, μ^{κ}, ε^{κ}) = (P, Q_1, Q_2, Q_3, R_1, R_2, χ, U, V, K, M, N, S, T, μ, ε) and go back to Step 2.

4 Numerical example

In this section, a numerical example is provided to illustrate the effectiveness of the proposed non-fragile stabilization scheme. Consider the discrete-time nonlinear system (1) and the non-fragile output feedback controller described by Eq. (4), where the system parameters are given as follows:

$$A=\begin{bmatrix}0.8187 & 0 & -0.955\\ 0.1722 & 0.8048 & -0.0094\\ 0 & 0 & 1.050\end{bmatrix},\ B=\begin{bmatrix}-0.85\\ 0.06\\ 0.25\end{bmatrix},$$

$$F=\begin{bmatrix}0.01 & 0 & 0\\ 0.01 & 0 & 0\\ 0 & 0 & 0.01\end{bmatrix},\ C=\begin{bmatrix}1 & 1 & 0\\ 1 & 0 & 1\end{bmatrix},\ x=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix},$$

$$f(\kappa,x)=\begin{bmatrix}-0.7x_1+0.05x_2+0.05x_3\\ -0.05x_1+0.85x_2\\ -0.05x_1-0.475x_3+\dfrac{x_3\sin x_1}{\sqrt{x_1{}^2+x_2{}^2+x_3{}^2}}\end{bmatrix},$$

$$S_1=\begin{bmatrix}-0.5 & 0.1 & 0\\ 0 & 0.9 & 0\\ -0.1 & 0 & -0.2\end{bmatrix},\ S_2=\begin{bmatrix}-0.9 & 0 & 0.1\\ -0.1 & 0.8 & 0\\ 0 & 0 & -0.75\end{bmatrix},$$

$$M_1=[0.2\quad -0.1],\ N_1=\begin{bmatrix}0.1 & -0.1\\ -0.2 & 0.3\end{bmatrix},$$

$$F(\kappa)=\begin{bmatrix}1.1\sin(\kappa) & 0\\ 0 & 1.1\cos(\kappa)\end{bmatrix}.$$

Since the eigenvalues of A are {0.8048, 0.8187, 1.05}, the system is unstable without a stabilization controller. It can be easily checked that the above nonlinear vector-value function $f(\kappa, x(\kappa))$ satisfies the sector-bounded condition (2). Assume that the occurrence probability of the controller gain uncertainties is $\bar{\alpha}=0.8$, and the time-varying communication delay $\tau(\kappa)$ uniformly distribute in the interval [1, 2]. By utilizing the CCL algorithm developed in section 3, we can obtain the non-fragile output feedback controller gain $K=[0.1959, -0.2921]$.

Similarly, if we select communication delay $\tau(\kappa)\in[1, 4]$, the corresponding controller gain can also be obtained from Theorem 3.1:

$$K=[0.1457, -0.2623].$$

To illustrate the stochastic stability of the closed-loop system, the initial condition of the state is chosen as $x(0)=[0.15, 0, -0.15]^{\mathrm{T}}$. For $\tau_M=2$ and $\tau_M=4$, the closed-loop state responses are depicted in Figs. 1 and 2, respectively. We can see that the state variables asymptotically converge to zero in spite of the existence of time delays and randomly occurring gain uncertainties. This shows effectiveness of the proposed non-fragile controller design procedure in Theorem 3.1. Furthermore, it is also obvious that a larger upper bound of time delay will lead to a slower convergence speed of the state. This means that the communication delay has a significant effect on system stability in networked environment.

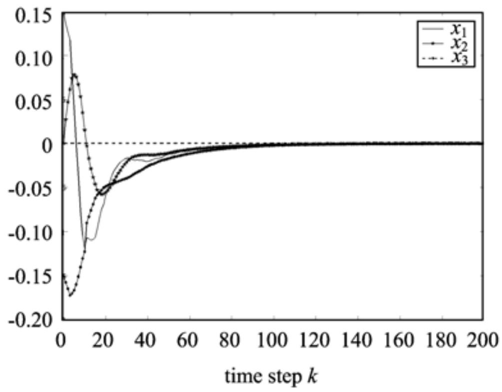

Fig. 1 State responses under the non-fragile stabilization controller with $\tau_M=2$

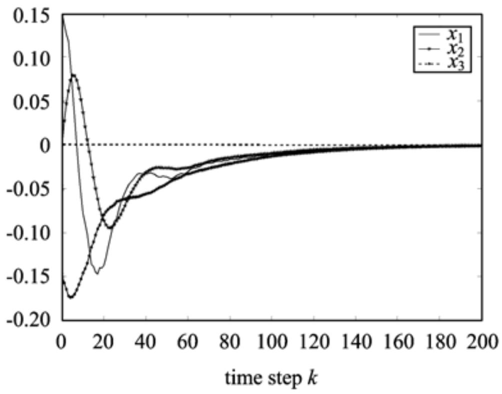

Fig. 2 State responses under the non-fragile stabilization controller with $\tau_M=4$

In order to show the advantages of the

proposed non-fragile stabilization method, we make a comparison between the non-fragile stabilization scheme and the conventional one. In the conventional stabilization scheme, the randomly occurring gain uncertainties are not taken into account during the controller design. According to Theorem 3.1 by setting $\bar{\alpha}=0$ and $\gamma=0$, a common output-feedback controller gain can be derived as follows:

$$K=[0.0993, -0.1992],$$

that is, a delayed output-feedback control law $u(\kappa)=[0.0993, 0.1992]y(k-\tau(\kappa))$ is adopted. In this case, the closed-loop system dynamics are shown in Fig. 3.

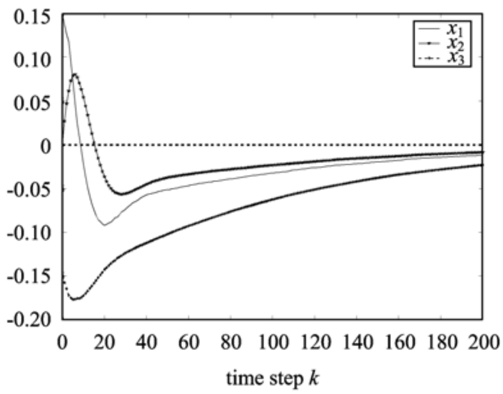

Fig. 3 State responses under the conventional stabilization controller

From the contrastive simulation results, it is clear that the convergence speed of the states under the non-fragile controller is much faster than that under the conventional one. This implies that it is essential to take randomly occurring gain uncertainties into consideration in controller design. The comparison also indicates the superiority of the non-fragile stabilization scheme developed in this paper.

5 Conclusion

In this paper, the non-fragile stabilization problem has been investigated for class of nonlinear NCSs with network-induced delays and gain uncertainties. Different from traditional deterministic and persistent uncertainty considered in most of the existing reference, the controller gain uncertainties under investigation has been allowed to occur in a probabilistic way. By defining new Lyapunov functional and by making use of novel inequality bounding techniques, the existence condition for the non-fragile stabilization controller bas been derived in the form of nonlinear matrix inequalities, and a modified CCL procedure has been exploited to solve the nonconvex feasibility problem. Finally, a simulation example has been provided to demonstrate the effectiveness of the theoretical results presented in this paper.

参考文献(References)

[1]Tabuada P. Event-triggered real-time scheduling of stabilizing control tasks[J]. IEEE Transactions on Automatic Control, 2007, 52(9): 1680-1685.

[2]Cuenca A, Salt J, Sala A, et al. A delay-dependent dual-rate PID controller over an Ethernet network[J]. IEEE Transactions on Industrial Informatics, 2011, 7(1): 18-29.

[3]Peng C, Han Q L, Yue D. To transmit or not to transmit: A discrete event-triggered communication scheme for networked Takagi-Sugeno fuzzy systems[J]. IEEE Transactions on Fuzzy systems, 2013, 21(1): 164-170.

[4]Katewa V, Gupta V. On a rate control protocol for networked estimation[J]. Automatica, 2013, 49(5): 1310-1317.

[5]Zhang L X, Gao H J, Kaynak O. Network-induced constraints in networked control systems-a survey[J]. IEEE Transactions on Industrial Informatics, 2013, 9(1): 403-416.

[6]Wu W, Fang H J. Integrated fault-tolerant control methodology for non-linear dynamical networked control systems subject to time-varying delay[J]. IET Control Theory and Applications, 2014, 8(8): 535-545.

[7]Yang R N, Liu G P, Shi P, et al. Predictive output feedback control for networked control systems[J]. IEEE Transactions on Industrial Electronics, 2014, 61(1): 512-520.

[8]Xiong J L, Lam J. Stabilization of linear systems over networks with bounded packet loss[J]. Automatica, 2007, 43(1):80-87.

[9]Zhang W A, Yu L. Output feedback stabilization of networked control systems with packet dropouts[J]. IEEE Transactions on Automatic Control, 2007, 52(9): 1705-1710.

[10]Gao H J, Chen T W. A new approach to quantized

feedback control systems[J]. Automatica, 2008, 44 (2): 534-542.

[11]JiangX F, Han Q L, Liu S R, et al. A new H∞ stabilization criterion for networked control systems[J]. IEEE Transactions on Automatic Control, 2008, 53 (4): 1025-1032.

[12] Jiang S, Fang H J. Quantized stabilization of discrete-time systems in a networked environment[J]. Applied Mathematical Modelling, 2014, 38(5-6): 1685-1697.

[13]Yue D, Tian E G, Wang Z D, et al. Stabilization of systems with probabilistic interval input delays and its application to networked control systems[J]. IEEE Transactions on Systems, Man, and Cybernetics-Part A: System and Humans, 2009, 39(4): 939-945.

[14]Gao H J, Meng X Y, Chen T W. Stabilization of networked control systems with a new delay characterization[J]. IEEE Transactions on Automatic Control, 2008, 53(9): 2142-2148.

[15]Shi Y, Yu B. Output feedback stabilization of networked control systems with random delays modeled by Markov chains[J]. IEEE Transactions on Automatic Control, 2009, 54(7): 1668-1674.

[16]Shu Z, Lam J, Xiong J L. Non-fragile exponential stability assignment of discrete-time linear systems with missing data in actuators[J]. IEEE Transactions on Automatic Control, 2009, 54(3): 625-630.

[17]Wang D, Shi P, Wang W, et al. Non-fragile H∞ control for switched stochastic delay systems with application to water quality process[J]. International Journal of Robust and Nonlinear Control, 2014, 24 (11): 1677-1693.

[18]Liu Y H, Niu Y G, Zou Y Y. Non-fragile observer-based sliding mode control for a class of uncertain switched systems[J]. Journal of Franklin Institute, 2014, 351(2): 952-963.

[19]Khalil H K. Nonlinear Systems[M]. 3ed, Upper Saddle River: Prentice-hall, 1996.

[20]Wang Z D, Daniel W C H, Liu Y R, et al. Robust H∞ control for a class of nonlinear discrete time-delay stochastic systems with missing measurements[J]. Automatica, 2009, 45(3): 684-691.

[21]Guo X G, Yang G H. Reliable H∞ filter design for a class of discrete-time nonlinear systems with time-varying delay[J]. Optimal Control Applications and Methods, 2010, 31(4): 303-322.

[22]Jiang S, Fang H J. Robust fault detection for networked control systems with nonlinear disturbances and imperfect measurements[J]. International Journal of Systems Science, 2013, 44(11): 2027-2038.

[23]Yang G H, Che W W. Non-fragile H∞ filter design for linear continuous-time systems[J]. Automatica, 2008, 44(11): 2849-2856.

[24]Gao L J, Wang D D, Wu Y Q. Non-fragile observer-based sliding mode control for Markovian jump systems with mixed mode-dependent time delays and input nonlinearity [J]. Applied Mathematics and Computation, 2014, 229: 374-395.

[25]Dong H L, Wang Z D, Gao H J. Fault detection for Markovian jump systems with sensor saturations and randomly varying nonlinearities[J]. IEEE Transactions on Circuits and Systems-I: Regular Papers, 2012, 59 (10): 2354-2362.

[26]Zhang S J, Wang Z D, Ding D R, et al. Fuzzy filtering with randomly occurring parameter uncertainties, interval delays, and channel fadings [J]. IEEE Transactions on Cybernetics, 2014, 44(3): 406-417.

[27]IwasakiT, Hara S. Generalized KYP lemma: Unified frequency domain inequalities with design application [J]. IEEE Transactions on Automatic Control, 2005, 50(1): 41-59.

[28]Gao H J, Chen T W, Lam J. A new delay system approach to network-based control[J]. Automatica, 2008, 44(1): 39-52.