第46卷第5期 中国科学技术大学学报 Vol. 46, No. 5
2016年5月 JOURNAL OF UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA May 2016

Article ID: 0253-2778(2016)05-0392-06

$L \to l l' l' \nu_l \nu_L$ in the SM and beyond

FLORES-TLALPA A.[1], LÓPEZ-CASTRO G.[2], ROIG P.[2]

(1. *Instituto de Física, Universidad Nacional Autónoma de México, Apartado Postal* 20-364, 01000 *México D. F., México*;
2. *Departamento de Física, Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, Apartado Postal* 14-740, 07000 *México D. F., México*)

Abstract: The $L \to l l' l' \nu_l \nu_L$ decays ($L=\tau, \mu$; $l, l'=\mu, e$) in the Standard Model (SM) and in the effective field theory (EFT) description of the weak charged current are studied at low energy, both for polarized and unpolarized L, keeping for the first time l, l' mass dependence. The discrepancy is clarified between two previous SM calculations improving their precision. The recent 3.5σ anomaly found in $\tau \to e\gamma\bar{\nu}_e\nu_\tau$ could be checked using our precise prediction for the $\tau \to eee\bar{\nu}_e\nu_\tau$ decays, which shall be measured analysing already existing data from the first generation B-factories. It is shown how measurements of the di-l' mass distribution (with appropriate cuts) and T-asymmetries are able to reveal the corresponding lepton flavor violating (LFV) processes without neutrinos in the final state.

Key words: decays of muons; decays of taus; leptonic interactions; Michel parameters; T-violation; lepton flavor violation

CLC number: O572.3 **Document code**: A doi: 10.3969/j.issn.0253-2778.2016.05.006

Citation: FLORES-TLALPA A, LÓPEZ-CASTRO G, ROIG P. $L \to l l' l' \nu_l \nu_L$ in the SM and beyond[J]. Journal of University of Science and Technology of China, 2016, 46(5): 392-397.

标准模型的和超出标准模型的 $L \to l l' l' \nu_l \nu_L$

FLORES-TLALPA A.[1], LÓPEZ-CASTRO G.[2], ROIG P.[2]

(1. 墨西哥国立自治大学物理研究所，墨西哥城 20-364，01000，墨西哥；
2. 墨西哥国立理工学院高等研究中心物理系，墨西哥城 14-740，07000，墨西哥)

摘要：研究了在低能情况下标准模型中的和有效场理论描述的保持 l, l' 质量的依赖性的弱电流过程，包括极化的和非极化的 L，$L \to l l' l' \nu_l \nu_L$ 衰变. 澄清了之前标准模型关于衰变率的两个计算的差异并提高了计算精度. 最近在 $\tau \to e\gamma\bar{v}_e v_\tau$ 过程中发现的 3.5σ 反常可以通过我们对第一代 B 工厂已有数据分析测量的 $\tau \to eee\bar{v}_e\nu_\tau$ 衰变过程的精确预测来检验. 它揭示了如何通过对 di-l' 质量分布（使用合适的截断条件）和非对称 T 矩阵的测量显示出关联的末态无中微子轻子味破缺过程.

关键词：μ 衰变；τ 衰变；轻子相互作用；Michel 参数；T 破缺；轻子味破缺

Received: 2015-12-20; **Revised**: 2016-04-20

Foundation item: Supported by Conacyt, México.

Biography: FLORES-TLALPA A., PhD. Research field: high energy particle physics. E-mail: alain@fisica.unam.mx

Corresponding author: ROIG P., PhD/Investigador Cinvestav 3-A. E-mail: proig@fis.cinvestav.mx

0 Introduction

(Multi) lepton lepton decays are interesting for a plethora of reasons. Historically, muon decays were essential in determining the V-A character of the weak charged interactions and the precise study of deviations from this Lorentz structure motivated the development of the Bouchiat-Michel-Kinoshita-Sirlin parameters[1-3] which now place stringent limits on the corresponding New Physics effects.

Radiative lepton lepton decays, $L \to l\nu_l\nu_L\gamma$, carry information about the l spin state which gives access to the Michel-like parameter $\bar{\eta}$[4-6]. Recent and ongoing experiments measuring the radiative muon and tau lepton decays[7-8] will reach up to 1% sensitivity on $\bar{\eta}$ extracted from $\tau \to l\nu_l\nu_\tau\gamma$ decays at Belle-II. In our case, where the photon undergoes lepton pair conversion, the corresponding Michel-like parameters[9] could be measured up to 3% precision.

The recent measurement by BaBar of the branching ratio for the $\tau \to e\gamma\bar{\nu}_e\nu_\tau$ decays[10] disagrees at 3.5σ with the SM computation at NLO[11], while agreement at the one sigma level is found for the muon mode (which should show a larger deviation). An obvious way to test this anomaly shall be possible through Belle's forthcoming analysis of the $\tau \to l\nu_l\nu_\tau\gamma$ decays. An indirect way to verify it can be obtained by confronting our precise SM prediction of the $\tau \to 3e\bar{\nu}_e\nu_\tau$ decays[12] with BaBar and Belle measurements (both first generation B-factories shall be able to measure the corresponding BR, $\sim 4\times 10^{-5}$).

Another important motivation for our study comes from the fact that the considered processes are the most important irreducible backgrounds in searches for the LFV $L \to ll'l'$ decays. Therefore, we aim to provide precise predictions of the SM five-lepton L decays so that the TAUOLA Monte Carlo generator[13-14] can include our matrix elements to help these searches using BaBar and Belle(-II) data.

Finally, let us also mention that a possible explanation for the MiniBooNe and LSND anomalies was given[15] by a long-living sterile neutrino that could be easily searched for in the considered decays, due to its characteristic signature of a displaced vertex of several cm.

For all these reasons, we have performed the first SM computation of the $L \to ll'l'\nu_l\nu_L$ decays keeping daughter lepton masses (as required by the precise B-factories data) for the (un)polarized L cases and generalize the Michel-like formalism developed in Ref. [9] accordingly, including L polarization for the first time. We have also made the corresponding computations in the most general EFT of the weak interactions at low energies and discussed in detail how to overcome the backgrounds from these SM processes in searches for LFV $L \to ll'l'$ decays.

1 SM description

At leading order, the considered decays proceed through photon emission (followed by lepton pair conversion) from the charged leptons in the well-known $L^- \to l^-\bar{\nu}_l\nu_L$ processes. Our notation for momenta is

$$L^-(Q) \to l^-(p_1)l'^-(p_2)l'^+(p_3)\bar{\nu}_l(p_4)\nu_L(p_5) \quad (1)$$

The masses of the charged leptons will be denoted by M, m_1, m ($m_2 = m_3 = m$), where M stands for the mass of the decaying lepton. The two diagrams in the $l \neq l'$ case double in the identical particle case giving rise to two extra terms that are obtained from the original ones by (antisymmetric) exchange of p_1 and p_2.

Since neutrinos are massless, the decay matrix element can be written as (both for the unpolarized and polarized L cases)

$$\overline{|\mathcal{M}|^2} = \frac{1}{2}\sum_{\text{pols}} |\mathcal{M}|^2 = \mathcal{T}_{\alpha\beta} p_4^\alpha p_5^\beta \quad (2)$$

where $\mathcal{T}^s_{\alpha\beta}$ will denote the polarized case, with s referring to the L helicity.

Being five-body decays, the corresponding squared (unpolarized) amplitude $\overline{|\mathcal{M}|^2}$ depends upon 8 independent kinematical variables that we choose following the invariants proposed in Ref. [16]. Since precise numerical integrations in the multi-particles case are far from trivial, instead of integrating over phase space as in Ref. [17] (employing the factorized expression Eq. (2) to integrate neutrinos phase space first and then integrating over the remaining variables), our integration following the optimized variables in Ref. [16] allows for a completely independent evaluation of the observables.

Analytic results for the matrix elements in the (un)polarized case can be found in the appendices of our paper[12].

Branching ratios for the different modes are shown in Tab. 1, where our predictions are compared to previous results and available experimental data. The quoted errors arise from VEGAS[20] integration, which was used in our numerical calculations and in Ref. [17]. This comparison shows good agreement between our results and those in Ref. [17], while sizable discrepancies are found with the results in Ref. [18]. Moreover, the closer agreement (at the one σ level) with Ref. [17] in the modes with two or three electrons than in the modes with two or three muons (about three σ) could be attributed to the different tau mass values used in both works (since the heavier the daughter particles, the larger the effect). Remaining tiny differences can be explained because of the overall effect of different tau lifetimes employed in these analyses.

The normalized differential decay width (with respect to the decaying lepton decay width) versus $q^2 = m^2_{l'^+l'^-}$ is plotted in Fig. 1 for the considered channels. All of them peak at low $m^2_{l'^+l'^-}$ values given the low virtuality of the photon close to threshold.

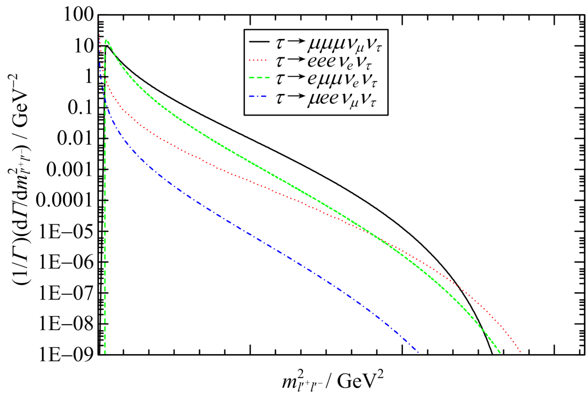

Fig. 1 Differential decay width (normalized to the partial decay width) versus $q^2 = m^2_{l'^+l'^-}$

Inclusion of L spin effects is straightforward. After integrating over neutrinos phase space, the differential decay width reads

$$\mathcal{T}^s_{\alpha\beta} I^{\alpha\beta}(P) = e^4 G_F^2 [F - L\vec{\boldsymbol{p}}_1 \cdot \vec{\boldsymbol{s}} - G_1 \vec{\boldsymbol{p}}_2 \cdot \vec{\boldsymbol{s}} - G_2 \vec{\boldsymbol{p}}_3 \cdot \vec{\boldsymbol{s}}] \tag{3}$$

where the generalized form factors F, L, G_1 and G_2 depend only upon scalar products of charged leptons momenta.

The most convenient writing of the differential decay width is in terms of the normalized charged daughter lepton energies and solid angles:

$$\frac{d\Gamma_5}{dx_1 d\Omega_1 dx_2 d\Omega_2 dx_3 d\Omega_3} = \frac{M^2 |\vec{\boldsymbol{p}}_1||\vec{\boldsymbol{p}}_2||\vec{\boldsymbol{p}}_3|}{3 \cdot 2^{21} \pi^{10}} \mathcal{T}^s_{\alpha\beta} I^{\alpha\beta}(P) \tag{4}$$

Tab. 1 Branching ratios for the five-lepton decays of taus and muons

channel	Ref. [17]	Ref. [18]	this work	PDG[19]
$BR(\tau^- \to e^- e^+ e^- \bar{\nu}_e \nu_\tau) \times 10^5$	4.15±0.06	4.457±0.006	4.21±0.01	2.8±1.5
$BR(\tau^- \to e^- \mu^+ \mu^- \bar{\nu}_e \nu_\tau) \times 10^7$	1.257±0.003	1.347±0.002	1.247±0.001	—
$BR(\tau^- \to \mu^- e^+ e^- \bar{\nu}_\mu \nu_\tau) \times 10^5$	1.97±0.02	2.089±0.003	1.984±0.004	<3.6
$BR(\tau^- \to \mu^- \mu^+ \mu^- \bar{\nu}_\mu \nu_\tau) \times 10^7$	1.190±0.002	1.276±0.004	1.183±0.001	—
$BR(\mu^- \to e^- e^+ e^- \bar{\nu}_e \nu_\mu) \times 10^5$	3.60±0.02	3.605±0.005	3.597±0.002	3.4±0.4

【Note】 For comparison some of the previous calculations are shown in the middle columns. When available, experimental data are consistent with the SM predictions, even though their errors are still rather large.

with $x_i=2E_i/M$ ($i=1,2,3$). In our paper[12], we have derived the first formulation in terms of Michel-like parameters for these decays in the polarized L case keeping charged leptons masses throughout (which are first kept in our work in the unpolarized case).

2 EFT analysis

The precise predictions given in Tab. 1 allow for a stringent test of the V-A character of the charged weak interactions. With this purpose, we have made an EFT analysis (valid at scales much smaller than that of electroweak symmetry breaking, as it is the case in L decays) of these processes.

The most general lepton-flavour conserving Lagrangian describing four-lepton interactions (local and without derivatives) is[1-3,6,9,21-22]

$$\mathscr{L}=-\frac{4G_{ll'}}{\sqrt{2}}\sum_{i,\lambda,\rho}g^{i}_{\lambda\rho}[\overline{l'_\lambda}\Gamma^{i}(\nu_{l'})_{\xi}][\overline{(\nu_l)_\kappa}\Gamma_i l_\rho] \quad (5)$$

The Lorentz structure of the weak currents is indicated by means of $i=S,V,T$; $\Gamma^S=I$, $\Gamma^V=\gamma^\mu$, $\Gamma^T=\sigma^{\mu\nu}/\sqrt{2}$ and $\lambda,\rho=L,R$ stand for the charged lepton's helicity (analogously ξ and κ for neutrinos). The 10 independent complex coefficients (4 scalar, 4 vector and the two tensor couplings g^T_{LR} and g^T_{RL}) give rise to 19 independent real couplings (removing an unphysical global phase).

$G_{ll'}$ in Eq. (5) is fixed by the total decay width[6]

$$1=\frac{1}{4}\sum_{\lambda,\rho}|g^S_{\lambda\rho}|^2+\sum_{\lambda,\rho}|g^V_{\lambda\rho}|^2+3(|g^T_{RL}|^2+|g^T_{LR}|^2) \quad (6)$$

In the Standard Model, $|g^V_{LL}|=1$ is the only non-vanishing coupling. Moreover, in the limit of massless left-handed neutrinos, only g^S_{RR} survives, leaving the interference between these two contributions as the most promising signature of New Physics affecting these decays. In Ref. [12] we have developed the Michel-like parameter formalism that will allow to scrutinize the Lorentz structure of the charged weak current in these decays.

The presence of the tau polarization vector could a priori allow for six spin-dependent generalized form factors in such a way that the Eq. (3) would generalize to

$$\mathscr{T}_{\alpha\beta}I^{\alpha\beta}(P)=e^4|G_{ll'}|^2[F-L\vec{\boldsymbol{p}}_1\cdot\vec{\boldsymbol{s}}-G_1\vec{\boldsymbol{p}}_2\cdot\vec{\boldsymbol{s}}-G_2\vec{\boldsymbol{p}}_3\cdot\vec{\boldsymbol{s}}+H_1\vec{\boldsymbol{s}}\cdot(\vec{\boldsymbol{p}}_1\times\vec{\boldsymbol{p}}_2)+H_2\vec{\boldsymbol{s}}\cdot(\vec{\boldsymbol{p}}_2\times\vec{\boldsymbol{p}}_3)+H_3\vec{\boldsymbol{s}}\cdot(\vec{\boldsymbol{p}}_1\times\vec{\boldsymbol{p}}_3)] \quad (7)$$

However, since CP violation (through T violation, in this case) effects are suppressed (in the case of Dirac neutrinos) by a lepton analog of the Jarlskog[23] invariant involving, in particular, the product $(m^2_{\nu_1}-m^2_{\nu_2})(m^2_{\nu_1}-m^2_{\nu_3})(m^2_{\nu_2}-m^2_{\nu_3})$; again neutrino masses make this effect unmeasurable and the only non-vanishing spin-dependent form factors in this limit are L, G_1 and G_2. Their corresponding expressions can be found in the appendices of our paper[12].

On the contrary, since CP violation effects are not suppressed in the LFV $L\to l'l^+l^-$ decays[24-25], dedicated searches of T violating effects in processes with a lepton decaying into three charged leptons could unveil LFV.

3 LFV $L\to ll'l'$ decays

The considered five-lepton lepton decays constitute the most difficult irreducible background in the searches for LFV $L\to ll'l'$ processes, particularly in the configuration where both neutrinos carry away so little energy that they cannot be detected. It is thus interesting to compare our predictions for different observables to the expected signal of the LFV processes in order to gain insight for the experimental searches of the latter.

In this spirit, we have considered the three benchmark scenarios proposed in Ref. [26] for dipole, scalar and vector interactions originating the LFV processes. We have considered the differential decay width as a function of one of the lepton energies and of the same- and opposite-sign lepton pair. It is concluded that the distributions in $m^2_{l'l'}$ are best suited to split signal from background with a moderate cut between 0.5 and 1 GeV^2 depending on the considered tau decay channel[12]. In the case of muon decays, huge statistics and an exquisite control of SM backgrounds will be needed to compensate the tiny signal to background ratio given in Tab. 2.

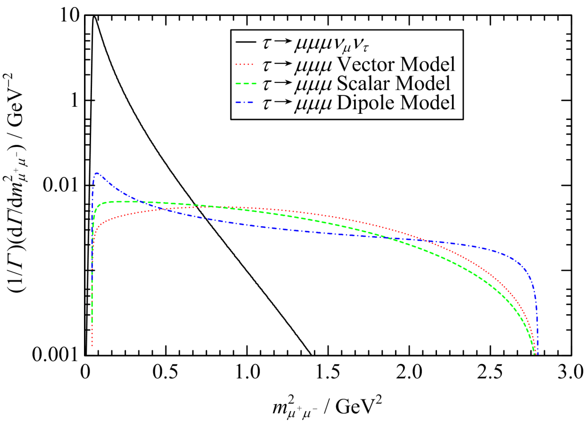

Fig. 2 Comparison of opposite-sign lepton-pair invariant mass distribution in five-body and LFV decays of tau leptons. Three benchmark scenarios of New Physics in $\tau\to 3\mu$ are used, according to Ref. [26], and S/B∼0.1 (about the current upper limit) is assumed

Tab. 2 Current and expected UL and signal to background ratios in LFV $L^-\to l^-\ l'^+\ l'^-$ searches

channel	current upper limit (UL)[19,27]	S/B (UL)	expected UL[28]
$BR(\tau^-\to e^-\ e^+e^-)$	1.4×10^{-8}	$\sim3\times10^{-4}$	$\sim1\times10^{-9}$
$BR(\tau^-\to e^-\ \mu^+\mu^-)$	1.6×10^{-8}	~0.1	$\sim1\times10^{-9}$
$BR(\tau^-\to \mu^-\ e^+e^-)$	1.1×10^{-8}	$\sim6\times10^{-4}$	$\sim1\times10^{-9}$
$BR(\tau^-\to \mu^-\ \mu^+\mu^-)$	1.2×10^{-8}	~0.1	$\sim1\times10^{-9}$
$BR(\mu^-\to e^-\ e^+e^-)$	1.0×10^{-12}	$\sim3\times10^{-8}$	$\sim1\times10^{-16}$

4 Conclusion

We have made the first analysis of the five-lepton lepton decays keeping daughter masses for the (un)polarized parent lepton cases. Our results agree with one of the earlier references in the literature, substantially improving the precision. We have derived analytic expressions for the Michel-like parameters useful for accurate tests of the V-A character of the charged weak interactions including decaying lepton spin effects. Our precise results would also allow to test the 3.5σ anomaly found by BaBar in $\tau\to e\gamma\bar{\nu}_e\nu_\tau$ decays by analyzing (with first generation B-factories data) the $\tau\to 3e\bar{\nu}_e\nu_\tau$ processes. We have also extended our computations to an EFT description of these at low energies and show that, even in this case, T-violation is suppressed by the tiny neutrino masses. Therefore, a wise way of searching for LFV $L\to ll'l'$ is to measure non-vanishing T-odd correlations involving the charged leptons, which would point to the beyond SM process immediately. Finally, we have shown that reasonable cuts in $m_{l'l'}$ are very efficient in splitting signal from background in these New Physics searches.

Acknowledgements We thank the Organizing Committees and Secretariat of PhiPsi15 for the very interesting and fruitful Hefei conference. P. R. acknowledges finnancial support from Dpto. de Física del Cinvestav for attending this workshop. This work was partly funded by Conacyt and A. F. T. was supported in part by DGAPA-UNAM. We appreciate very much discussions with Denis Epifanov and Martín González-Alonso concerning this research.

References

[1] MICHEL L. Interaction between four half spin particles and the decay of the μ-meson[J]. Proceedings of the Physical Society A, 1950, 63: 514-531.

[2] BOUCHIAT C, MICHEL L. Theory of μ-meson decay with the hypothesis of nonconservation of parity [J]. Physical Review, 1957, 106: 170-172.

[3] KINOSHITA T, SIRLIN A. Muon decay with parity nonconserving interactions and radiative corrections in the two-component theory[J]. Physical Review, 1957, 107: 593-599.

[4] PRATT R H. Time-reversal invariance and radiative muon decay[J]. Phys Rev, 1958, 111: 649-651.

[5] EICHENBERGER W, ENGFER R, VAN DER SCHAAF A. Measurement of the parameter $\bar{\eta}$ in the radiative decay of the muon as a test of the V-A structure of the weak interaction[J]. Nuclear Physics A, 1984, 412: 523-533.

[6] FETSCHER W, GERBER H J, JOHNSON K F. Muon decay: Complete determination of the interaction and comparison with the standard model[J]. Physics

Letters B, 1986, 173: 102-106.

[7] POCANIC D. New results in rare allowed muon and pion decays [J]. International Journal of Modern Physics: Conference Series, 2014, 35: 1460437.

[8] ABDESSELLAM A, ADACHI I, ADAMCZYK K, et al (Belle Collaboration). Study of Michel parameters in leptonic τ decays at Belle[DB/OL]. arXiv:1409.4969 [hep-ex].

[9] KERSCH A, KRAUS N, ENGFER R. Analysis of the rare allowed muon decay $\mu^+\rightarrow e^+ v_e \bar{v}_\mu e^+ e^-$ [J]. Nuclear Physics B, 1988, 485: 606-620.

[10] LEES J P, POIREAU V, TISSERAND V, et al (BaBar Collaboration). Measurement of the branching fractions of the radiative leptonic τ decays $\tau\rightarrow e\gamma\nu\bar{v}$ and $\tau\rightarrow\mu\gamma\nu\bar{v}$ at BABAR[J]. Physical Review D, 2015, 91: 051103(R).

[11] FAEL M, MERCOLLI L, PASSERA M. Raditive μ and τ leptonic decays at NLO[J]. Journal of High Energy Physics, 2015, 1507:153 and their contribution to these proceedings.

[12] FLORES-TLALPA A, LÓPEZ-CASTRO G, ROIG P. Five-body leptonic decays of muon and tau leptons[DB/OL]. arXiv: 1508.01822 [hep-ph].

[13] JADACH S, KÜHN J H, WAS Z. TAUOLA: A library of Monte Carlo programs to simulate decays of polarized τ leptons [J]. Computer Physics Communications, 1990, 64: 275-299.

[14] JADACH S, WAS Z, DECKER R, et al. The tau decay library TAUOLA: Version 2.4[J]. Computer Physics Communications, 1993, 76: 361-380.

[15] DIB C, HELO J C, HIRSCH M, et al. Heavy sterile neutrinos in tau decays and the MiniBooNE anomaly [J]. Physical Review D, 2012, 85: 011301(R).

[16] KUMAR R. Covariant phase-space calculations of n-body decay and production processes [J]. Physical Review, 1969, 185: 1 865-1 875.

[17] DICUS D A, VEGA R. Standard model decays of tau into three charged leptons [J]. Physics Letters B, 1994, 338: 341-348.

[18] ALAM M S, KIM I J, LING Z, et al (CLEO Collaboration). Tau decays into three charged leptons and two neutrinos[J]. Physical Review Letters, 1996, 76: 2 637-2 641.

[19] OLIVE K A, AGASHE K, AMSLER C, et al (Particle Data Group). Review of particle physics[J]. Chinese Physics C, 2014, 38: 090001.

[20] LEPAGE G P. A new algorithm for adaptive multidimensional integration[J]. Journal of Computer Physics, 1978, 27: 192-203.

[21] SCHECK F. Muon physics [J]. Physics Reports, 1978, 44: 187-248.

[22] PICH A, SILVA J P. Constraining new interactions with leptonic τ decays[J]. Physical Review D, 1995, 52: 4 006-4 018.

[23] JARLSKOG C. Commutator of the quark mass matrices in the standard electroweak model and a measure of maximal CP Nonconservation [J]. Physical Review Letters, 1985, 55: 1039-1042.

[24] OKADA Y, OKUMURA K I, SHIMIZU Y. $\mu\rightarrow e\gamma$ and $\mu\rightarrow 3e$ processes with polarized muons and supersymmetric grand unified theories [J]. Physical Review D, 2000, 61: 094001.

[25] KITANO R, OKADA Y. P and T odd asymmetries in lepton flavor violating tau decays[J]. Physical Review D, 2001, 63: 113003.

[26] CELIS A, CIRIGLIANO V, PASSEMAR E. Model-discriminating power of lepton flavor violating τ decays [J]. Physical Review D, 2014, 89: 095014.

[27] AMHIS Y, BANERJEE Sw, BEN-HAIM E, et al (Heavy Flavor Averaging Group (HFAG) Collaboration). Averages of b-hadron, c-hadron, and τ-lepton properties as of summer 2014 [DB/OL]. arXiv:1412.7515 [hep-ex].

[28] HAYASAKA K, INAMI K, MIYAZAKI Y, et al. Search for lepton flavor violating τ decays into three leptons with 719 million produced $\tau^+\tau^-$ pairs [J]. Physics Letters B, 2010, 687: 139-143.