

A resolution of the puzzle of low V_{us} values from inclusive flavor-breaking sum rule analyses of hadronic τ decay data

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Abstract: Continuum and lattice methods are used to investigate systematic issues in the sum rule determination of V_{us} using inclusive hadronic τ decay data. Results for V_{us} employing assumptions for $D>4$ OPE contributions used in previous conventional implementations of this approach are shown to display unphysical dependence on the sum rule weight, ω , and choice of upper limit, s_0 , of the relevant experimental spectral integrals. Continuum and lattice results suggest a new implementation of the sum rule approach with not just $|V_{us}|$, but also $D>4$ effective condensates fit to data. Lattice results are also shown to provide a quantitative assessment of truncation uncertainties for the slowly converging $D=2$ OPE series. The new sum rule implementation yields $|V_{us}|$ results free of unphysical s_0 - and ω -dependences and ~ 0.0020 higher than that obtained using the conventional implementation. With preliminary new experimental results for the $K\pi$ branching fraction, the resulting $|V_{us}|$ is in excellent agreement with that based on K_{l3} , and compatible within errors with expectations from three-family unitarity.

Key words: V_{us} ; lattice; sum rules

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τ 强衰变数据单举味破缺求和规则分析中低 V_{us} 值疑难的一个解决方案

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摘要: 在应用单举 τ 强衰变数据由求和规则来确定 V_{us} 中, 连续和格点方法用于研究系统问题. 采用此方法先前常规使用的 $D>4$ OPE 贡献的假设得到的 V_{us} 表明对求和权重 ω 和相关实验谱积分的上限选择 s_0 的非物理依赖. 连续的和格点的结果表明这是一个不仅使用 $|V_{us}|$ 并且使用 $D>4$ 有效凝聚的拟合数据的求和规则方法. 格点结果也提供了一个对缓慢收敛的 $D=2$ OPE 序列的缩短不确定性的定量评估. 新求和规则的应用

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得到的 $|V_{us}|$ 结果没有非物理的 s_0 和 $\tau\omega$ 依赖性并且比常规方法得到的值要高 ~ 0.002 。利用初步的新 $K\pi$ 分支比实验结果得到的 $|V_{us}|$ 与基于 K_{l3} 的结果高度一致, 并且与三代归一性的预测结果相比在误差范围内符合。

关键词: V_{us} ; 格点; 求和规则

0 Introduction

The conventional τ decay determination of $|V_{us}|$ employs finite-energy sum rules (FESRs) and flavor-breaking (FB) combinations of inclusive hadronic τ decay data^[1-3]. With $\Pi_{V/A;ij}^{(J)}(s)$ the $J=0,1$ components of flavor $ij=ud,us$, vector (V) or axial vector (A) current 2-point functions, $\rho_{V/A;ij}^{(J)}(s)$ the corresponding spectral functions, and $\Delta\Pi_\tau \equiv [\Pi_{V+A;ud}^{(0+1)} - \Pi_{V+A;us}^{(0+1)}]$, the FESR relation

$$\int_0^{s_0} \omega(s) \Delta\rho_\tau(s) ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} \omega(s) \Delta\Pi_\tau(s) ds \quad (1)$$

is valid for any s_0 and any analytic $\omega(s)$. The spectral function of $\Delta\Pi_\tau$, $\Delta\rho_\tau$, is experimentally accessible in terms of the differential distribution, $dR_{V/A;ij}/ds$, of $R_{V/A;ij} \equiv \Gamma[\tau^- \rightarrow \nu_\tau \text{ hadrons}_{V/A;ij}(\gamma)] / \Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)]$. Explicitly^[4]

$$\frac{dR_{V/A;ij}}{ds} = c_\tau^{EW} |V_{ij}|^2 [\omega_\tau(s) \rho_{V/A;ij}^{(0+1)}(y_\tau) - \omega_L(y_\tau) \rho_{V/A;ij}^{(0)}(s)] \quad (2)$$

with $y_\tau = s/m_\tau^2$, $\omega_\tau(y) = (1-y)^2(1+2y)$, $\omega_L(y) = 2y(1-y)^2$, c_τ^{EW} a known constant, and V_{ij} the flavor ij CKM matrix element. The RHS of Eq. (1) is treated using the OPE (operator product expansion).

One uses the $J=0+1$ FESR Eq. (1), rather than the analogue involving the spectral function combination in Eq. (2), because of the very bad behavior of the integrated $J=0$, $D=2$ OPE series^[5-6]. $\rho_{V/A;ud}^{(0+1)}(s)$ is obtained after subtracting phenomenologically determined $J=0$ contributions from $dR_{V/A;ud}/ds$. This subtraction is dominated by the accurately known, non-chirally-suppressed π and \bar{K} pole terms. Continuum $\rho_{V/A;ud}^{(0)}$ contributions are $\propto(m_d \mp m_u)^2$ and numerically negligible. Small, but not totally negligible, $(m_s \mp m_u)^2$ -suppressed continuum $\rho_{V/A;us}^{(0)}$ contributions are determined

using highly constrained dispersive and sum rule methods^[7-10]. With $|V_{ud}|$ fixed^[11], $\Delta\rho_\tau(s)$ is determined by experimental data and $|V_{us}|$. $|V_{us}|$ is then obtained using the OPE on the RHS and data on the LHS of Eq. (1).

From the distribution $dR_{V+A;ud,us}^{(0+1)}/ds$, obtained after subtracting $J=0$ contributions, re-weighted $J=0+1$ versions,

$$R_{V+A;ij}^{w}(s_0) \equiv \int_0^{s_0} ds \frac{\omega(s)}{\omega_\tau(s)} \frac{dR_{V+A;ij}^{(0+1)}(s)}{ds},$$

of $R_{V+A;ud,us}$, may be constructed for any ω and $s_0 \leq m_\tau^2$. With $\delta R_{V+A}^{w, \text{OPE}}(s_0)$ the OPE representation of

$$\delta R_{V+A}^{w}(s_0) \equiv \frac{R_{V+A;ud}^{w}(s_0)}{|V_{ud}|^2} - \frac{R_{V+A;us}^{w}(s_0)}{|V_{us}|^2},$$

one then has

$$|V_{us}| = \sqrt{R_{V+A;us}^{w}(s_0) / \left[\frac{R_{V+A;ud}^{w}(s_0)}{|V_{ud}|^2} - \delta R_{V+A}^{w, \text{OPE}}(s_0) \right]} \quad (3)$$

The resulting $|V_{us}|$ should be independent of s_0 and the choice of weight, ω , provided all experimental data, and any assumptions employed in evaluating $\delta R_{V+A}^{w, \text{OPE}}(s_0)$, are reliable. Since integrated $D=2k+2$ OPE contributions scale as $1/s_0^k$, problems with assumptions about higher D non-perturbative contributions, e. g., will produce an unphysical s_0 -dependence in $|V_{us}|$.

The conventional implementation of Eq. (3)^[1-3] employs $\omega = \omega_\tau$ and $s_0 = m_\tau^2$. With this choice, the spectral integrals $R_{V+A;ud,us}^{w_\tau}(m_\tau^2)$ are determinable from inclusive non-strange and strange hadronic τ branching fractions, but assumptions about higher dimension $D=6,8$ OPE contributions, in principle present for a degree 3 weight like ω_τ , are unavoidable. Using a single ω and single s_0 precludes subjecting these assumptions to ω - and s_0 -independence tests. It is a long-standing puzzle that this implementation produces inclusive $\tau |V_{us}|$ determinations $> 3\sigma$

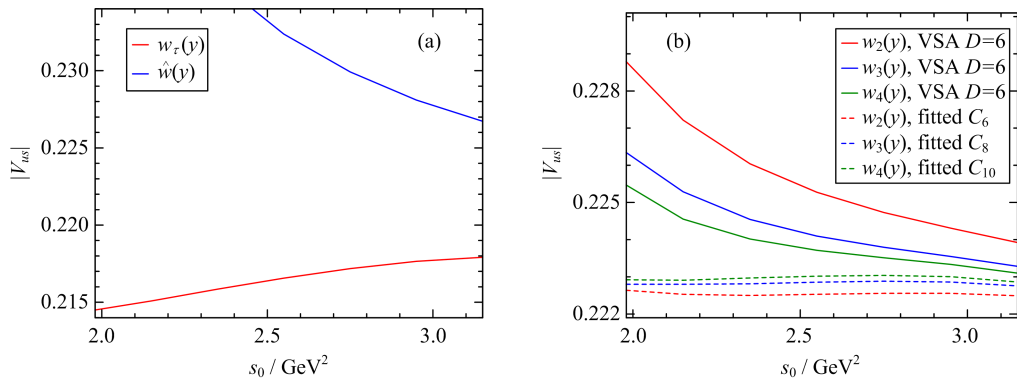


Fig. 1 (a) $|V_{us}|$ from the w_τ and \hat{w} FESRs with standard^[1-3] OPE treatment (including CIPT for the $D = 2$ series). (b) Comparison of conventional implementation results with those obtained using central fitted $C_{6,8,10}$ values and the FOPT $D = 2$ prescription favored by lattice results, for the weights $w_{2,3,4}$ defined in the text

below 3-family-unitarity expectations (the most recent version, $|V_{us}| = 0.2176(21)$ ^[12-13], e. g., lies 3.6σ below the current unitarity expectation, $|V_{us}| = 0.2258(9)$ ^[11]). Tests of the conventional implementation, however, show sizeable s_0 - and w -dependence^[14-17] (see also, e. g., the left panel, and solid lines in the right panel, of Fig. 1), indicating the existence of systematic problems in the conventional implementation. The dashed lines in the right panel show the results of the alternate implementation discussed below.

Two obvious theoretical systematic issues exist which might account for the observed w - and s_0 -instabilities. The first concerns the treatment of $D=6, 8$ OPE contributions. Both the conventional implementation and generalized versions just mentioned^[14-17], estimate $D=6$ contributions using the vacuum saturation approximation (VSA) and neglect $D = 8$ contributions. The VSA $D = 6$ estimate is very small due to significant cancellations, both in the individual ud and us $V+A$ sums and in the subsequent FB difference of these sums. With sizeable channel-dependent VSA breaking observed in the flavor ud V and A channels^[18-19], such strong cancellations make the VSA estimate potentially quite unreliable. The second possibility concerns the slow convergence of the $D=2$ OPE series for $\Delta\Pi_\tau$. With $\bar{a} = \alpha_s(Q^2)/\pi$, and $m_s(Q^2)$, $\alpha_s(Q^2)$ the running strange quark mass and coupling in the \overline{MS} scheme, one has, to

four loops^[20-21] (neglecting $O(m_{u,d}^2/m_s^2)$ corrections)

$$[\Delta\Pi_\tau(Q^2)]_{D=2}^{\text{OPE}} = \frac{3}{2\pi^2} \frac{m_s(Q^2)}{Q^2} \cdot$$

$$\left[1 + \frac{7}{3}\bar{a} + 19.93\bar{a}^2 + 208.75\bar{a}^3 + \dots \right] \quad (4)$$

Since $\bar{a}(m_\tau^2) \simeq 0.1$, convergence at the spacelike point on $|s| = s_0$ is marginal at best, raising questions concerning the choice of truncation order and truncation error estimates for the corresponding integrated series. The $D = 2$ convergence/truncation issue is also evident in the significant difference (increasing from ~ 0.0010 to ~ 0.0020 between 3- and 5-loop truncation order) in $|V_{us}|$ results obtained using alternate (fixed-order (FOPT) and contour-improved (CIPT)) prescriptions (prescriptions differing only by terms beyond the common truncation order) for the truncated integrated $D=2$ series^[14-17].

In what follows, we first investigate the treatment of the $D=2$ OPE series using lattice data for $\Delta\Pi_\tau$, then test the $D=6, 8$ assumptions of the conventional implementation by comparing FESR results for a judiciously chosen pair of weights, $w_\tau(y)$ and $\hat{w}(y) = (1-y)^3$, $y = s/s_0$. Results obtained employing an alternate implementation of the FB FESR approach suggested by these investigations are then presented.

1 Lattice and continuum investigations of the OPE representation of $\Delta\Pi_\tau$

Data for $\Delta\Pi_\tau(Q^2)$ can be generated over a

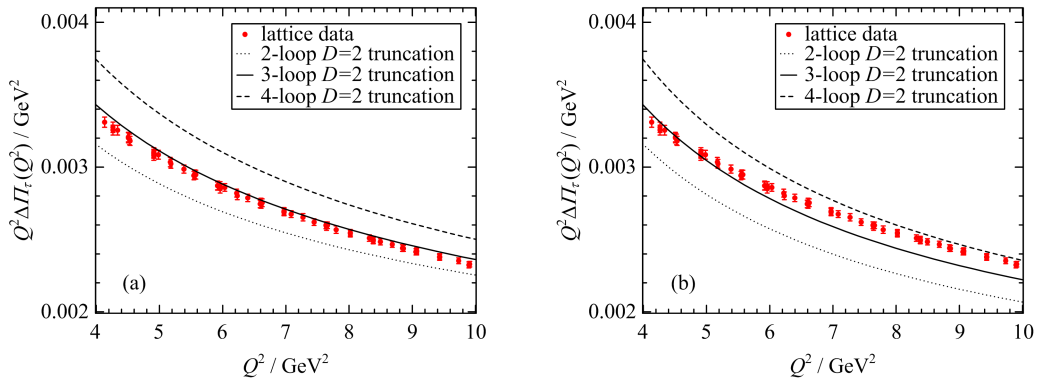


Fig. 2 Comparison of lattice data and OPE $D = 2 + 4$ expectations for $Q^2 \Delta \Pi_\tau(Q^2)$,

for various truncation orders and either the fixed-scale treatment (a) or local-scale treatment (b) of the $D = 2$ series

wide range of Euclidean Q^2 using the lattice. The (tight) cylinder cut which must be applied to avoid lattice artifacts at higher Q^2 has been determined, for the ensemble employed here, in a recent analysis aimed at using lattice current-current two-point function data to determine α_s ^[22]. We first consider data at high enough Q^2 that $[\Delta \Pi_\tau]_{\text{OPE}}$ will be safely dominated by the leading $D = 2$ and 4 contributions. The latter are determined by light and strange quark masses and condensates and hence known. We use FLAG results for physical quark masses^[23] and GMOR for the light condensate. $\langle \bar{s}s \rangle$ then follows from $\langle \bar{s}s \rangle / \langle \bar{u}u \rangle$ (the HPQCD physical- m_q version of this ratio^[24] is easily translated to the m_q of the ensemble employed using NLO ChPT^[25]). We then consider various combinations of truncation order and log-summation schemes for the $D = 2$ OPE series, investigating whether a choice exists which produces a good match between the resulting $D = 2 + 4$ OPE sum and the lattice data in the high- Q^2 region.

For this high- Q^2 study, we employ the RBC/UKQCD $n_f = 2 + 1$, $32^3 \times 64$, $1/a = 2.38$ GeV, $m_\pi \sim 300$ MeV domain wall fermion ensemble^[26]. We find that 3-loop $D = 2$ truncation with fixed-scale choice (the analogue of the FOPT FESR prescription) provides an excellent OPE-lattice match over a wide range of Q^2 , extending from near ~ 10 GeV² down to just above ~ 4 GeV². The comparison is shown, for fixed scale choice $\mu^2 = 4$ GeV², in the left panel of Fig. 2, where, for ease

of display, we plot results for the product $Q^2 \Delta \Pi_\tau(Q^2)$ (this removes a factor of $1/Q^2$ present in the $D = 2$ OPE contributions). The right panel of Fig. 2 shows the analogous comparison for the alternate local-scale ($\mu^2 = Q^2$) choice (analogous to the CIPT FESR prescription). It is clear that the Q^2 dependence of the lattice data prefers the fixed-scale treatment of the $D = 2$ series.

The lattice data also provides us with the possibility of investigating the reliability of conventional methods for estimating the theoretical error to be associated with the truncated OPE. This is of particular relevance given the very slow convergence of the $D = 2$ series, which might raise doubts about the suitability of such conventional estimates in the case of the $D = 2$ truncation uncertainty. Fig. 3 shows the $D = 2 + 4$ OPE error

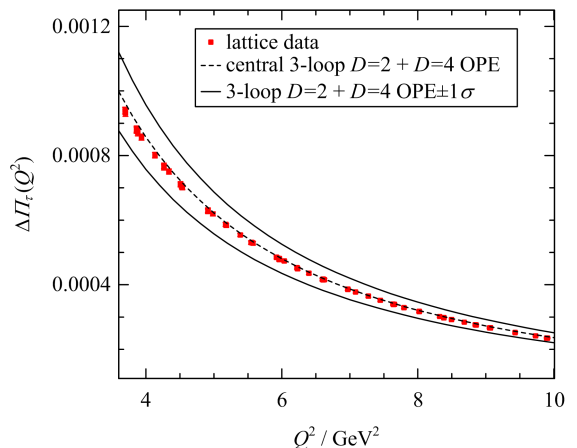


Fig. 3 Lattice data and the $D = 2 + 4$ OPE sum, with conventional OPE error estimates, for the 3-loop-truncated, fixed-scale $D = 2$ treatment

band obtained using the 3-loop-truncated, fixed-scale $D=2$ OPE treatment and such conventional OPE error estimates, taking into account uncertainties in the input OPE parameters and using the magnitude of the last term kept to estimate the $D=2$ series truncation uncertainty. One sees that, despite the very slow convergence of the $D=2$ series, the resulting conventionally determined error turns out to be extremely conservative.

We now switch our attention to lower Q^2 , the goal being to use lattice data to test the assumptions about $D>4$ contributions underlying the standard implementation, namely that $D>6$ contributions are safely negligible and $D=6$ contributions can be reasonably well approximated using the VSA. Fig. 4 shows the comparison of lattice data for $\Delta\Pi_\tau(Q^2)$ in the region below ~ 4 GeV^2 with two versions of the truncated OPE. The dashed line represents the 3-loop-truncated, fixed-scale $D=2+4$ OPE sum discussed above, which provides an excellent match to the lattice data at higher scales, while the solid line shows the result of supplementing the $D=2+4$ sum with the VSA estimate for the $D=6$ contribution. The results show clear evidence for the onset of $D>4$ contributions below $Q^2 \sim 4$ GeV^2 significantly larger than those obtained using the VSA estimate for $D=6$ and neglecting $D>6$ contributions. The VSA $D=6$ estimate is not only far too small in magnitude to bring the low- Q^2 truncated OPE into agreement with the lattice data but, in fact, moves the truncated OPE sum slightly in the wrong direction. Unfortunately, the Euclidean lattice data provides no means of selectively isolating contributions of different $D>4$ in this lower Q^2 region. Further investigation of the higher D question thus requires continuum FESR methods.

Our continuum FESR studies employ the $D=2, 4$ OPE treatment favored by lattice data, detailed above. Spectral integral input is as follows: $\pi_{\mu 2}$, $K_{\mu 2}$ and Standard Model expectations for the π and K pole contributions, recent ALEPH

data for the continuum ud $V+A$ distribution^[27], BaBar^[28] and Belle^[29] results for the $K^- \pi^0$ and $\bar{K}^0 \pi^-$ distributions, BaBar results^[30] for the $K^- \pi^+ \pi^-$ distribution, Belle results^[31] for the $\bar{K}^0 \pi^- \pi^0$ distribution and 1999 ALEPH results^[32] for the combined distribution of those strange modes not remeasured by the B-factory experiments.

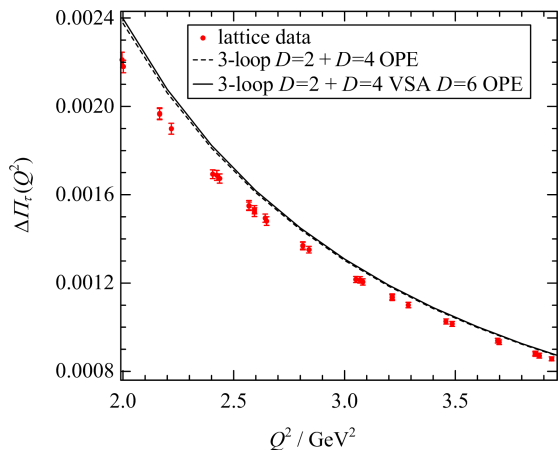


Fig. 4 Comparison of lower- Q^2 lattice data with fixed-scale $D=2$ OPE based expectations, one employing just the sum of $D=2$ and 4 contributions, the other supplementing this with the estimated $D=6$ contribution obtained using the VSA

The BaBar and Belle exclusive mode distributions are unit-normalized and must have their overall scales fixed using experimental branching fractions. In the results quoted below, HFAG strange exclusive mode branching fractions have been used, with the exception of the $K^- \pi^0$ mode, for which the updated version from the recent BaBar Adametz thesis^[33] has been employed. A corresponding (very) small rescaling is applied to the continuum ud $V+A$ distribution to restore unitarity.

Neglecting α_s -suppressed logarithmic corrections, $D>4$ OPE contributions to $\Delta\Pi_\tau(Q^2)$ can be written $\sum_{D>4} C_D/Q^D$ with C_D an effective dimension D condensate. The degree 3 weights $w_\tau(y) = 1 - 3y^2 + 2y^3$ and $\hat{w}(y) = 1 - 3y + 3y^2 - y^3$ generate integrated OPE contributions up to $D=8$ only. The integrated $D=6, 8$ results,

$$-\frac{3C_6}{s_0^2} - \frac{2C_8}{s_0^3} \text{ for } w_\tau \text{ and } \frac{3C_6}{s_0^2} + \frac{C_8}{s_0^3} \text{ for } \hat{w} \quad (5)$$

have $D=6$ contributions identical in magnitude but opposite in sign, and a \hat{w} $D=8$ contribution similarly opposite in sign but half in magnitude that of w_τ . Were the assumptions of the conventional implementation to be correct, with $D=6, 8$ contributions numerically negligible in the w_τ FESR, this will necessarily also be the case for the \hat{w} FESR. The two FESRs should then produce results for $|V_{us}|$ which not only agree, but are both s_0 -independent. In contrast, if the $D=6$ and/or 8 contributions to the w_τ FESR are not, in fact, negligible, the results for $|V_{us}|$ from the two FESRs, obtained assuming they are, should show s_0 -instabilities of opposite sign, decreasing in magnitude with increasing s_0 for both. The left panel of Fig. 1 shows the latter scenario to be the one actually realized. The sizeable s_0 - and weight-choice dependences demonstrate unambiguously the breakdown of the assumptions underlying the conventional implementation. The 3σ low $|V_{us}|$ results obtained employing them are thus afflicted with significant previously unquantified systematic uncertainties.

2 An alternate implementation of the FB FESR approach

With previously employed approaches to estimating $D>4$ effective OPE condensates shown to be untenable, our only option is to fit these condensates to data. This can be done only by exploiting the fact that integrated OPE contributions of different D scale differently with s_0 , and hence requires working with FESRs involving a range of values of the variable s_0 . This precludes determining the required spectral integrals solely in terms of hadronic τ decay branching fractions.

Considering FESRs based on different weights provides further tests of possible theoretical systematics. To suppress duality violating contributions, we restrict our attention to weights having at least a double zero at $s=s_0$. The weights $w_N(y) = 1 - \frac{N}{N-1}y + \frac{1}{N-1}y^N$, $N \geq 2$ ^[34] are

particularly convenient in this regard since the corresponding integrated OPE involves a single $D>4$ contribution (with $D=2N+2$). With $D=2+4$ OPE contributions under control (as discussed above) this leaves $|V_{us}|$ and the effective condensate C_{2N+2} as the only parameters to be determined. These are obtained through a fit to the set of w_N -weighted spectral integrals in the chosen s_0 fit window. Further tests of the analysis are provided by verifying (i) that the $|V_{us}|$ obtained from the different w_N FESRs are in good agreement and (ii) that the fitted C_D are physically plausible, in the sense of showing FB cancellation relative to the results of Ref. [18-19] for the corresponding effective flavor ud channel condensates. We have analyzed the w_N FESRs for $N=2, 3, 4$ and verified that the results pass these self-consistency tests.

In the right panel of Fig. 1 we display, in dashed/dotted lines, the results which follow from taking as input the central value for the condensate, C_{2N+2} , obtained from the w_N FESR analysis, and solving Eq. (3) for $|V_{us}|$ as a function of s_0 . The results are displayed for each of the w_2, w_3 and w_4 FESR cases. The results make clear (i) the underlying excellent match between the fitted OPE and spectral integral sets, (ii) the excellent agreement between results for $|V_{us}|$ obtained from the different w_N FESR analyses, and (iii) the dramatic decrease in the s_0 - and weight-dependence of the results for $|V_{us}|$ produced by using $D>4$ OPE effective condensates fit to data in place of those based on the assumptions of the conventional implementation. One also sees that, as expected, the fitted $|V_{us}|$ lie between the s_0 -unstable results produced by the conventional implementation of the w_τ and \hat{w} FESRs, and are ~ 0.0020 higher than the results of the conventional w_τ implementation.

In Tab. 1, we give the error budgets for the $|V_{us}|$ determinations based on the w_2, w_3 and w_4 FESRs, using the 3-loop, fixed-scale treatment of the $D=2$ OPE series favored by lattice data. The

errors in the first half are those associated with input uncertainties on the theory side, with “long corr” labelling those associated with the sum rule/dispersive determinations of the small, doubly-chirally-suppressed continuum us V and A channel $J = 0$ subtractions. Combining these errors in quadrature yields a total theory error of 0.000 4 for each of the three cases. The errors listed in the second half of the table are those induced by the errors and covariances of the flavor ud and us V+A distributions. The experimental and total errors are both strongly dominated by the uncertainty on the us V+A spectral integrals.

Tab. 1 Error budgets for the ω_2 , ω_3 and ω_4 determinations of $|V_{us}|$, using the 3-loop-truncated, fixed-scale treatment of the $D = 2$ OPE series

error source	$\delta V_{us} $ (ω_2 FESR)	$\delta V_{us} $ (ω_3 FESR)	$\delta V_{us} $ (ω_4 FESR)
$\delta\alpha_s$	0.000 01	0.000 04	0.000 04
$\delta m_s(2 \text{ GeV})$	0.000 17	0.000 19	0.000 19
$\delta\langle m_{\bar{s}s} \rangle$	0.000 35	0.000 35	0.000 35
$\delta(\text{long corr})$	0.000 09	0.000 09	0.000 09
experimental (ud)	0.000 27	0.000 28	0.000 28
experimental (us)	0.002 26	0.002 27	0.002 27

The excellent s_0 -stability and agreement between the results from the different ω_N FESRs allows us to arrive at a final result for $|V_{us}|$ obtained by performing a combined fit to the ω_2 , ω_3 and ω_4 FESRs. We find

$$|V_{us}| = 0.2228(23)_{\text{exp}}(5)_{\text{th}} \quad (6)$$

This is in excellent agreement with the results, $0.223\,5(4)_{\text{exp}}(9)_{\text{th}}$ and $0.223\,1(4)_{\text{exp}}(7)_{\text{th}}$, obtained using the 2014 FlaviaNet experimental K_{l3} update^[36] and most recent $n_f = 2 + 1$ ^[37] and $n_f = 2 + 1 + 1$ ^[38] lattice results for $f_+(0)$. It is also compatible within errors with (j) the results, $0.225\,1(3)_{\text{exp}}(9)_{\text{th}}$ and $0.022\,50(3)_{\text{exp}}(7)_{\text{th}}$ obtained using the 2014 update for the experimental ratio $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ ^[36] and the most recent $n_f = 2 + 1$ ^[39]

and $n_f = 2 + 1 + 1$ ^[40] lattice determinations of f_K/f_π and (j) the expectations of 3-family unitarity.^①

It is worth noting that, among the methods mentioned above, the one having the smallest theoretical error is, in fact, the FB FESR determination. This error is, moreover, as we have seen, a very conservative one. At present the experimental error on the FB FESR determination (resulting almost entirely from uncertainties in the us exclusive mode distributions) is larger than those of the competing methods. We note, however, that the us spectral integral error is currently dominated by the uncertainty on the branching fraction normalizations for the exclusive strange modes, and hence systematically improvable through improvements in these branching fraction values in the near future. In the longer term, it will be important to complete the analysis of the exclusive mode us distributions not yet remeasured by the B-factory experiments and to finalize the analyses of the covariances of those unit normalized exclusive distributions which are not yet complete at present.

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References

- [1] GAMIZ E, PRADES J, JAMIN M, et al. Determination of m_s and $|V_{us}|$ from hadronic τ decays[J]. JHEP, 2003(1): 060.
- [2] GAMIZ E, JAMIN M, PICH A, et al. V_{us} and m_s from hadronic τ decays[J]. Phys Rev Lett, 2005, 94: 011803.
- [3] GAMIZ E, JAMIN M, PICH A, et al. Theoretical progress on the V_{us} determination from tau decays[J]. Proceedings of Science, 2007: PoS(KAON)008.

① An analogous analysis with $K\pi$ normalization not employing the $B[K^-\pi^0\nu_\tau]$ update of Ref. [33] yields $|V_{us}| = 0.220\,0(23)_{\text{exp}}(5)_{\text{th}}$. Of the 0.002 4 difference between this result and the conventional implementation result^[12-13] noted above, 0.000 5 results from the use of $K_{\mu 2}$ for the K pole contribution; the remainder is due to presence of the $D=6, 8$ contributions not correctly accounted for by the assumptions of the conventional implementation. Note that the normalization of the two-mode $K\pi$ sum produced by the $B[K^-\pi^0\nu_\tau]$ update is in good agreement with the results of the dispersive study of $K\pi$ detailed in Ref. [35].

- [4] TSAI Y S. Decay correlations of heavy leptons in $e^+ + e^- \rightarrow l^+ + l^-$ [J]. Phys Rev D, 1971, 4: 2821-2837 (Erratum: Phys Rev D, 1976, 13: 771).
- [5] MALTMAN K. Problems with extracting m_s from flavor breaking in hadronic τ decays [J]. Phys Rev D, 1998, 58: 093015.
- [6] MALTMAN K, KAMBOR J. Longitudinal contributions to hadronic τ decay [J]. Phys Rev D, 2001, 64: 093014.
- [7] JAMIN M, OLLER J A, PICH A. S-wave $K\pi$ scattering in chiral perturbation theory with resonances [J]. Nucl Phys B, 2000, 587: 331-362.
- [8] JAMIN M, OLLER J A, PICH A. Strangeness-changing scalar form factors [J]. Nucl Phys B, 2002, 622: 279-308.
- [9] JAMIN M, OLLER J A, PICH A. Scalar $K\pi$ form factor and light quark masses [J]. Phys Rev D, 2006, 74: 074009.
- [10] MALTMAN K, KAMBOR J. Decay constants, light quark masses, and quark mass bounds from light quark pseudoscalar sum rules [J]. Phys Rev D, 2002, 65: 074013.
- [11] HARDY J C, TOWNER I S. Parametrization of the statistical rate function for select superallowed transitions [J]. Phys Rev C, 2015, 91: 015501.
- [12] LUSIANI A. Determination of $|V_{us}|$ from the tau lepton branching fractions [DB/OL]. arXiv: 1411.4526 [hep-ex].
- [13] PASSEMAR E. Determinations of $|V_{us}|$ from hadronic tau decays: A theory perspective [C]// the 8th International Workshop on the CKM Unitarity Triangle (CKM 2014), Vienna, Austria, September 8-12, 2014.
- [14] MALTMAN K, WOLFE C E. Joint extraction of m_s and V_{us} from hadronic τ decays [J]. Phys Lett B, 2007, 650: 27-32.
- [15] MALTMAN K, WOLFE C E, BANERJEE S, et al. Status of the hadronic τ determination of V_{us} [J]. Int J Mod Phys A, 2008, 23: 3191-3195.
- [16] MALTMAN K, WOLFE C E, BANERJEE S, et al. Status of the hadronic τ decay determination of $|V_{us}|$ [J]. Nucl Phys Proc Suppl, 2009, 189: 175-180.
- [17] MALTMAN K. A critical look at V_{us} determinations from hadronic τ decay data [J]. Nucl Phys Proc Suppl, 2011, 218: 146-151.
- [18] BOITO D, GOLTERMAN M, JAMIN M, et al. Updated determination of α_s from τ decays [J]. Phys Rev D, 2012, 85: 093015.
- [19] BOITO D, GOLTERMAN M, MALTMAN K, et al. Strong coupling from the revised ALEPH data for hadronic τ decays [J]. Phys Rev D, 2015, 91: 034003.
- [20] CHETYRKIN K G, KWIATKOWSKI A. Mass corrections to the tau decay rate [J]. Z Phys C, 1993, 59: 525-531
- [21] BAIKOV PA, CHETYRKIN K G, KÜHN J H. Strange-quark mass from tau-lepton decays with $O(\alpha_s^3)$ accuracy [J]. Phys Rev Lett, 2005, 95: 012003.
- [22] HUDSPITH R J, LEWIS R, MALTMAN K, et al. Determining the QCD coupling from lattice vacuum polarization [DB/OL]. arXiv:1510.04890 [hep-lat].
- [23] AOKI S, AOKI Y, BERNARD C, et al. Review of lattice results concerning low-energy particle physics [J]. Eur Phys J C, 2014, 74: 2890.
- [24] MCNEILE C, BAZAVOV A, DAVIES C T H, et al. Direct determination of the strange and light quark condensates from full lattice QCD [J]. Phys Rev D, 2013, 87: 034503.
- [25] GASSER J, LEUTWYLER H. Chiral perturbation theory: Expansions in the mass of the strange quark [J]. Nucl Phys B, 1985, 250: 465-516.
- [26] AOKI Y, ARTHUR R, BLUM T, et al (RBC and UKQCD Collaborations). Continuum limit physics from 2+1 flavor domain wall QCD [J]. Phys Rev D, 2011, 83: 074508.
- [27] DAVIER M, HOECKER A, MALAESCU B, et al. Update of the ALEPH non-strange spectral functions from hadronic $\bar{\tau}$ decays [J]. Eur Phys J C, 2014, 74: 2803.
- [28] AUBERT B, BONA M, BOUTIGNY D, et al (BaBar Collaboration). Measurement of the $\tau^- \rightarrow K^- \pi^0 \nu_\tau$ branching fraction [J]. Phys Rev D, 2007, 76: 051104(R).
- [29] EPIFANOV D, ADACHI I, AIHARA H, et al (Belle Collaboration). Study of $\tau^- \rightarrow K_S \pi^- \nu_\tau$ decay at Belle [J]. Phys Lett, B, 2007, 654: 65-73.
- [30] NUGENT I M (on behalf of the BaBar Collaboration). Invariant mass spectra of $\tau^- \rightarrow h^- h^- h^+ \nu_\tau$ decays [J]. Nucl Phys Proc Suppl, 2014, 253-255: 38-41.
- [31] RYU S (for the Belle Collaboration). Measurement of the branching fractions and mass spectra for τ lepton decays including K_S^0 at Belle [J]. Nucl Phys Proc Suppl, 2014, 253-255: 33-37.
- [32] BARATE R, DECAMP D, GHEZ P, et al (ALEPH Collaboration). Study of τ decays involving kaons, spectral functions and determination of the strange quark mass [J]. Eur Phys J C, 1999, 11: 599-648.
- [33] ADAMETZ A. Measurement of τ decays into charged hadron accompanied by neutral π mesons and determination of the CKM matrix element $|V_{us}|$ [D]. Heidelberg, Germany: University of Heidelberg.

- [34] MALTMAN K, YAVIN T. $\alpha_s(M_Z^2)$ from hadronic τ decays [J]. Phys Rev D, 2008, 78: 094020.
- [35] ANTONELLI M, CIRIGLIANO V, LUSIANI A, et al. Predicting the tau strange branching ratios and implications for V_{ts} [J]. JHEP, 2013, 1310: 070.
- [36] MOULSON M. Experimental determination of V_{ts} from kaon decays[J]. arXiv:1411.5252 [hep-ex].
- [37] BOYLE P A, CHRIST N H, FLYNN J M, et al (the RBC/UKQCD Collaboration). The kaon semileptonic form factor in $N_f=2+1$ domain wall lattice QCD with physical light quark masses [J]. JHEP, 2015, 1506:164.
- [38] PRIMER T, TOUSSAINT D, EL-KHADRA A, et al (FNAL and MILC Collaborations). Kaon and D meson semileptonic form factors from lattice QCD [J]. Proceedings of Science, 2014: PoS (LATTICE2014)374.
- [39] BLUM T, BOYLE PA, CHRIST N H, et al (RBC/UKQCD Collaboration). Domain wall QCD with physical quark masses[J]. arXiv: 1411.7017 [hep-lat].
- [40] BAZAVOV A, BERNARD C, KOMIJANI J, et al (FNAL and MILC Collaborations). Charmed and light pseudoscalar meson decay constants from four-flavor lattice QCD with physical light quarks[J] Phys Rev D, 2014, 90: 074509.

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- [9] BECIREVIC D, YAOUANC A L, OYANGUREN A, et al. Insight into $D/B \rightarrow \pi l \nu_l$ decay using the pole models[DB/OL]. arXiv:1407.1019 [hep-ph].
- [10] DEL AMO SANCHEZ P, LEES J P, POIREAU V, et al (BaBar Collaboration). Observation of new resonances decaying to $D\pi$ and $D^* \pi$ in inclusive $e^+ e^-$ collisions near $\sqrt{s} = 10.58$ GeV[J]. Phys Rev D, 2010, 82: 111101(R).
- [11] LEES J P, POIREAU V, TISSERAND V, et al. Measurement of the $D^*(2010)^+$ natural line width and the $D^*(2010)^+ - D^0$ mass difference[J]. Phys Rev D, 2013, 88(5): 052003 (Erratum; Phys Rev D 2013, 88(7): 079902).
- [12] GODFREY S, ISGUR N. Mesons in a relativized quark model with chromodynamics[J]. Phys Rev D, 1985, 32:189-231.
- [13] OYANGUREN A. Measurement of the $D^0 \rightarrow \pi^- e^+ \nu$ branching fraction, form factor and implications for V_{cb} [C]// 7th International Workshop on Charm Physics (Charm 2015). Detroit, MI, USA, May 18-22, 2015.
- [14] LEES J P, POIREAU V, TISSERAND V, et al (BaBar Collaboration). Branching fraction and form-factor shape measurements of exclusive charmless semileptonic B decays, and determination of $|V_{cb}|$ [J]. Phys Rev D, 2012, 86: 092004.
- [15] COLANGELO P, DE FAZIO F, GIANNUZZI F, et al. New meson spectroscopy with open charm and beauty[J]. Phys Rev D, 2012, 86: 054024.
- [16] BURDMAN G, KAMBOR J. Dispersive approach to semileptonic form-factors in heavy to light meson decays[J]. Phys Rev D, 1997, 55: 2 817-2 826.