HRS optics optimization method for Coulomb sum rule（CSR）experiment in JLab Hall-A

YAN Xinhu[1]，YE Yunxiu[2]，LYU Haijiang[1]，JIANG Fengjian[1]，
ZHU Pengjia[2]，SHI Ying[1]

(1. Department of Physics，Huangshan University，Huangshan 245041，China；
2. Department of Modern Physics，University of Science and Technology of China，Hefei 230026，China)

Abstract：In order to study the acceptance and calculate cross sections for CSR experiment，one needs to obtain target variables on the target plane of the spectrometer. Target variables can be reconstructed from focal plane variables with optics. In principle，the optics is the mathematical expression of the magnetic field of HRS. The optics optimization method which minimizes χ^2 of $TgVar_{data} - TgVar_{theory}$ to determine the better optics matrix elements for CSR experiment was studied. The optimization results show a good agreement between the data and theoretical values.

Key words：Coulomb sum rule；optics；acceptance；target coordinate

JLab Hall-A 库仑求和规则实验高精度谱仪光学性能优化方法

闫新虎[1]，叶云秀[2]，吕海江[1]，蒋峰建[1]，朱鹏佳[2]，石　瑛[1]

(1. 黄山学院物理系，安徽黄山 245041；2. 中国科学技术大学近代物理系，安徽合肥 230026)

摘要：在库仑求和实验中，为了研究接收度和截面计算，需要在高精度谱仪的靶平面上获得一些靶变量. 这些靶变量可以通过谱仪的光学性能从聚集平面上的变量反推重建出来. 原则上，谱仪的光学性能就是高精度谱仪内部磁场的数学表达，可以用光学矩阵来表示. 研究了谱仪光学性能优化方法，就是对变量 χ^2（$TgVar_{data} - TgVar_{theory}$）进行最小化计算. 通过这个计算过程，可以为库仑求和实验获得更好的光学矩阵数据. 从最后的优化结果来看，实际数据和理论值符合得很好.

关键词：库仑求和规则；光学；接收度；靶坐标系

0 Introduction

There are two high resolution spectrometers (HRS) in Hall A at Jefferson Lab, both with QQDQ (Q: quadrapole, D: dipole) configurations[1]. The HRSs have focusing properties that are point-to-point in the dispersive direction. The scattered particles passing through the HRS are bent and focused on the VDC (vertical drift chamber) plane. All signals are translated into physical variables on the VDC plane, also called "focal plane variables". However, in order to study the acceptance and calculate cross sections, one needs to obtain "target variable" on the target plane. Target variables can be reconstructed from focal plane variables with "optics". In principle, the optics is the mathematical expression (i. e. optics matrix)[2] of magnetic field of HRS. The optics matrix elements allow the reconstruction of the interaction vertex from the coordinates of the detected particles on the focal plane. Data obtained with a set of foil targets (which define a set of well-defined interaction points along the beam) and a sieve-slit collimator were used to determine the optical matrix elements. For Coulomb sum rule (CSR)[3-7] experiment, the nominal momentum range for optics runs for both arms is $0.4\sim2.0$ GeV/c. The optimization method to determine the optics matrix elements will be studied as follows.

1 HRS optics system

1.1 Optics optimization process

During a normal calibration procedure, we can obtain the theoretical and experimental values from one measurement and then try to change calibration constants to make theoretical values as close to experimental values as possible. For example, we know V_{theory} and V_{data}, assuming $V_{data} = C \cdot V_{theory}$. With optimizing C, we finally get a linear fit for V_{theory} vs V_{data}. And C is the result we want to get.

It is the same idea for optics optimization.

What we have are target variables, such as Z_{react}, ϕ_{tg}, θ_{tg}, y_{tg}, calculated from sieve slit and survey information. We can define these variables as $TgVar_{theory}$. The experimental values for target variables are reconstructed from focal plane variables (FpVar) detected by VDC with the help of optics matrix (OP). In general, the relation between them is $TgVar_{data} = OP \cdot FpVar$. By changing OP and getting smallest χ^2 of ($TgVar_{data} - TgVar_{theory}$), we can get new optics matrix (OP) for better optics optimization.

1.2 Target and focal plane coordinate

Target coordinate system is one of the coordinates of Hall-A at Jefferson Lab used to define the target variables. Fig. 1 shows the coordinates for electrons scattered from a thin foil target. The target coordinate center is shown in the figure as a black cross. L is the distance from Hall center to the sieve slit plane. The black line tagged with z_{tg} is along the central line of the spectrometer and the one with y_{tg} is perpendicular to it. Θ_0 is the setting central angle of the spectrometer. ϕ_{tg}, y_{tg} and z_{react} are defined in the figure[8]. D is the horizontal displacement of the spectrometer axis from its ideal position. And D is also the D_y defined later in Eq. (1). Note that x_{tg}, θ_{tg}, D_x and x_{sieve} are vertically down (into the page).
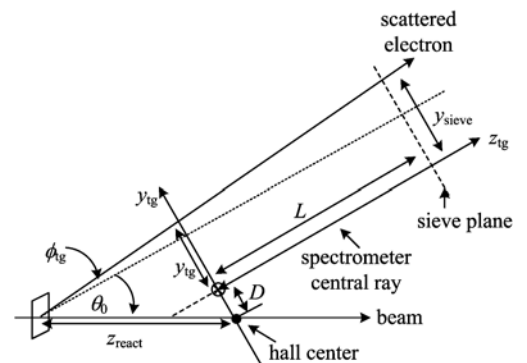

Fig. 1 Target coordinate system schematic

The HRS sieve slit hole pattern is shown in Fig. 2. The distance between holes and the diameter of different holes are defined in the figure.
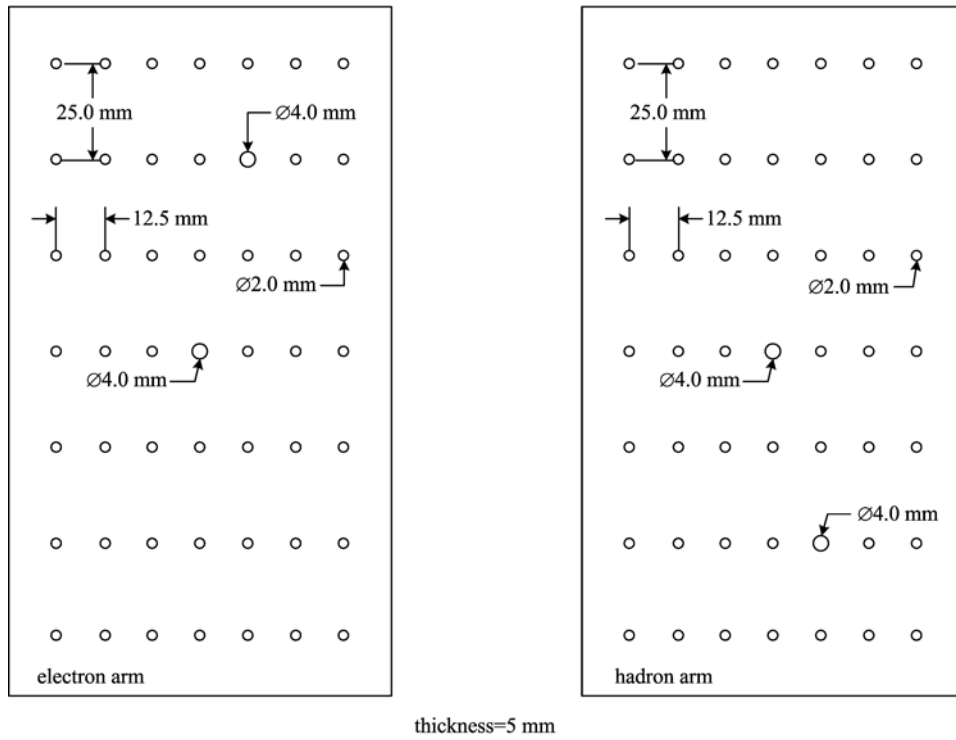
Fig. 2　Sieve slit structure in Hall-A

Tab. 1　L and sieve slit survey

	location	Z/mm	X/mm	Y/mm	yaw angle/degree	3D dist/mm
	left arm sieve slit	357.72	1 109.13	−0.12	72.124	1 165.39
2006-07-03	left arm 6 Msr	335.89	1 038.08	0.05	72.070	1 091.08
	right arm sieve slit	994.57	−657.87	−0.15	−33.483	1 192.46
	right arm 6Msr	933.01	−615.17	−0.18	−33.398	1 117.56
2008-02-08	left arm sieve slit	948.05	677.59	2.05	35.554	1 165.30
	left arm 6Msr	888.31	633.35	2.53	35.488	1 090.98

Survey information on the distance from the Hall center to the front surface of the sieve slits, L, is shown in Tab. 1. Sieve survey information is in Fig. 2 and Tabs. 1，2. The survey informations of Z_{react}，D_x，D_y of the left and right arm were also obtained. The distance between each optics foil is 4 cm.

In Tab. 1，the coordinates are relative to the ideal Hall-A target and beamline，with ＋Z along the beam，＋X to the beam left and ＋Y up. Measurements are to the upstream face of the collimators. In Tab. 2，the first sets of coordinates are to the center of the slits relative to the spectrometer center line. The second set is relative to the Hall-A target and beam line as reported in the DT A1102. A ＋X is to the beam left，A ＋Z is downstream and a ＋Y is up.

Tab. 2　Sieve slit offset

location	Z/mm	X/mm	Y/mm
relative to the spectrometer center line			
left (electron slit)	974	1.48	2.76
right (hadron slit)	1 002	−1.31	−2.67
relative to the Hall A beam line			
left (electron slit)	949	213.93	2.12
right (hadron slit)	977	−215.76	−3.19

1.3　TgVar$_{\text{theory}}$ and TgVar$_{\text{data}}$ calculation

The Formulas to calculate TgVar$_{\text{theory}}$ are defined as follows.

$$
\left.
\begin{aligned}
\phi_{\mathrm{tg}} &= \frac{y_{\mathrm{sieve}} + D_y - x_{\mathrm{beam}}\cos(\theta_0) + z_{\mathrm{react}}\sin(\theta_0)}{L - z_{\mathrm{react}}\cos(\theta_0) - x_{\mathrm{beam}}\sin(\theta_0)}, \\
\theta_{\mathrm{tg}} &= \frac{x_{\mathrm{sieve}} + D_x - y_{\mathrm{beam}}}{L - z_{\mathrm{react}}\cos(\theta_0) - x_{\mathrm{beam}}\sin(\theta_0)}, \\
y_{\mathrm{tg}} &= y_{\mathrm{sieve}} - L\phi_{\mathrm{tg}}, \\
x_{\mathrm{tg}} &= x_{\mathrm{sieve}} - L\theta_{\mathrm{tg}}, \\
\theta_{\mathrm{scat}} &= \arccos\left[\frac{\cos(\theta_0) - \phi_{\mathrm{tg}}\sin(\theta_0)}{\sqrt{(1 + \theta_{\mathrm{tg}}^2 + \phi_{\mathrm{tg}}^2)}}\right], \\
p(M,\theta) &= E' = \frac{E}{1 + E/M(1 - \cos(\theta))}, \\
dp_{\mathrm{kin}} &= dp - \frac{p(M,\theta_{\mathrm{scat}}) - p(M,\theta_0)}{p_0}
\end{aligned}
\right\}
\tag{1}
$$

Making use of the survey values, Fig. 2 and Eq. (1), we can calculate $\mathrm{TgVar}_{\mathrm{theory}}$.

VDC detector and focal plane coordinate system is shown in Figs. 3 and 4, respectively.
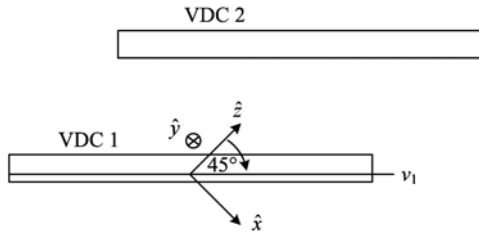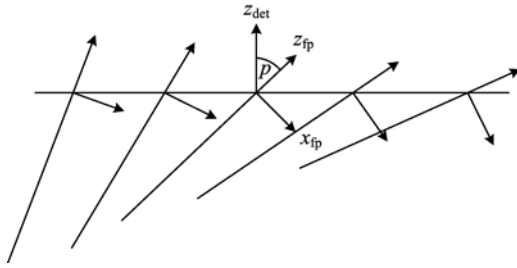

Fig. 3　VDC detector coordinate system

Fig. 4　Focal plane coordinate system

The formulas to calculate $\mathrm{TgVar}_{\mathrm{data}}$ are defined as follows.

$$
\left.
\begin{aligned}
\delta &= \sum_{j,k,l} D_{jkl}\,\theta_{\mathrm{fp}}^j\, y_{\mathrm{fp}}^k\, \phi_{\mathrm{fp}}^l, \\
\theta_{\mathrm{tg}} &= \sum_{j,k,l} T_{jkl}\,\theta_{\mathrm{fp}}^j\, y_{\mathrm{fp}}^k\, \phi_{\mathrm{fp}}^l, \\
y_{\mathrm{tg}} &= \sum_{j,k,l} Y_{jkl}\,\theta_{\mathrm{fp}}^j\, y_{\mathrm{fp}}^k\, \phi_{\mathrm{fp}}^l, \\
\phi_{\mathrm{tg}} &= \sum_{j,k,l} P_{jkl}\,\theta_{\mathrm{fp}}^j\, y_{\mathrm{fp}}^k\, \phi_{\mathrm{fp}}^l
\end{aligned}
\right\}
\tag{2}
$$

where

$$
D_{jkl} = \sum_{i=0}^m C_{ijkl}^D\, x_{\mathrm{fp}}^i \tag{3}
$$

So after obtaining the δ, θ_{tg}, y_{tg} and ϕ_{tg} value from theory and data, we can extract the calibration coefficients (D_{jkl}, T_{jkl}, Y_{jkl}, P_{jkl}) with χ^2 minimization method for VDC and get better optics conditions.

2　The optics optimization results

After the optimization calculation procedure, we obtained some results. Fig. 5 shows y_{tg} vs ϕ_{tg} distribution before optics calibration, from which we can see the peak location or the center of events cluster can not match well with the theoretical line or center initially.
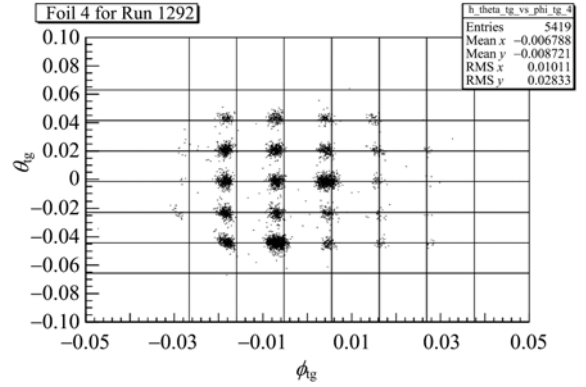

Fig. 5　y_{tg} vs ϕ_{tg} distribution before optics calibration

Fig. 6 shows y_{tg} vs ϕ_{tg} distribution after optics calibration, from which we can see that the peak location or the center of events cluster are aligned well to the theoretical line or center after the optics calibration well.
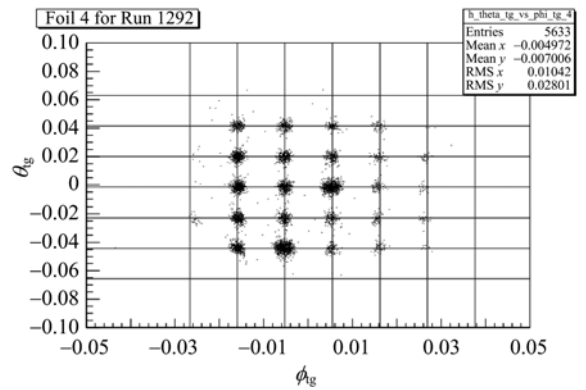

Fig. 6　y_{tg} vs ϕ_{tg} distribution after optics calibration

3　Conclusion

In summary，in order to study the acceptance and calculate cross sections，one needs to obtain "target variables" on the target plane. Target variables can be reconstructed from focal plane variables with "optics". In principle，the optics is the mathematical expression(i. e. optics matrix) of magnetic field of HRS. We studied the optimization method which minimzes χ^2 of $(TgVar_{data} - TgVar_{theory})$ to determine the better optics matrix elements for CSR experiment. From the optimization results，we can see the data and theoretical values matched very well.

References

[1] Alcorna J，Andersonb B D，Aniolc K A. Basic instrumentation for Hall A at Jefferson Lab[J]. NIM A，2004，522：294-346.

[2] Brown K L. SLAC Report 75 (1970)[Z]. revised edition，1982.

[3] Chen J P，Meziani Z E，Beck D，et al. Longitudinal and transverse response functions in ^{56}Fe(e，e') at momentum transfer near 1 GeV/c[J]. Phys Rev Lett，1991,66：1 283-2 186.

[4] Meziani Z E，Barreau P，Bernheim M，et al. Coulomb sum rule for ^{40}Ca，^{48}Ca，and ^{56}Fe for $|q| \leqslant 500$ MeV/c [J]. Phys Rev Lett，1984，52：2 130-2 133.

[5] Meziani Z E，Barreau P，Bernheim M，et al. Transverse response functions in deep-inelastic electron scattering for ^{40}Ca，^{48}Ca，and ^{56}Fe[J]. Phys Rev Lett，1985，54：1 233-1 236.

[6] Deady M，Williamson C F，Zimmerman P D，et al. Deep inelastic separated response functions from ^{40}Ca and ^{48}Ca[J]. Phys Rev C，1986，33：1 897-1 904.

[7] Blatchley C C，LeRose J J，Pruet O E，et al. Quasi-elastic electron scattering from ^{238}U[J]. Phys Rev C，1986,34：1 234-1 247.

[8] Liyanage N. Optics calibration of the Hall a high-resolution spectrometers using the new optimizer[R]. Newport News，VA：Jefferson Lab，2002：JLab-TN-02-012.

[6] Ge G，Miao Y，Yao Z. Optimal frequency hopping sequences：auto- and cross-correlation properties[J]. IEEE Trans Inform Theory，2009，55(2)：867-879.

[7] Nyberg K. Perfect nonlinear S-boxes[C]// Advances in cryptology- EUROCRYPT '91 (Brighton，1991). Berlin/ Heidelberg：Springer，1991，547：378-386.

[8] Wang Q，Zhou Y. Sets of zero-difference balanced functions and their applications[DB/OL]. arXiv：1208. 1878.

[9] Yan S. Elementary Number Theory[M]. Berlin/ Heidelberg：Springer，2002.

[10] Feng T. A new construction of perfect nonlinear functions using Galois rings[J]. J Comb Designs，2009，17(3)：229-239.

[11] Zha Z，Kyureghyan G M，Wang X. Perfect nonlinear binomials and their semifields[J]. Finite Fields and Their Applications，2009，15(2)：125-133.

[12] Zhou Z，Tang X，Wu D，et al. Some new classes of zero-difference balanced functions[J]. IEEE Trans Inform Theory，2012，58(1)：139-145.