第45卷第12期 中国科学技术大学学报 Vol. 45, No. 12
2015年12月 JOURNAL OF UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA Dec. 2015

Article ID: 0253-2778(2015)12-0994-09

Novel double Casoratian solutions to a negative order four-potential isospectral Ablowitz-Ladik equation

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Abstract: A matrix method for constructing double Casoratian entries was applied to a negative order four-potential isospectral Ablowitz-Ladik equation. Novel double Casoratian solutions to it were obtained by allowing the general matrix to be some special cases, including Matveev solutions and mixed solutions.

Key words: a negative order four-potential isospectral Ablowitz-Ladik equation; double Casoratian determinant; Matveev solution; mixed solution

CLC number: O175.29 **Document code**: A doi: 10.3969/j.issn.0253-2778.2015.12.006

2010 Mathematics Subject Classification: 35Q51

Citation: Xue Yimin, Chen Shouting. Novel double Casoratian solutions to a negative order four-potential isospectral Ablowitz-Ladik equation[J]. Journal of University of Science and Technology of China, 2015, 45(12): 994-1002.

负向等谱 4 位势 Ablowitz-Ladik 方程的新双 Casorati 解

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摘要：利用构造双 Casorati 行列式元素的矩阵方法研究了负向等谱 4 位势 Ablowitz-Ladik 方程. 通过将矩阵取成一些特殊的形式，导出该方程新的双 Casorati 解，即 Matveev 解和混合解.

关键词：负向等谱 4 位势 Ablowitz-Ladik 方程；双 Casorati 行列式；Matveev 解；混合解

0 Introduction

As a well-known method to search for the multi-soliton solutions to the nonlinear evolution equations, Wronskian technique is considered a powerful tool and efficient direct approach to obtaining exact solutions to the equations possessing bilinear forms[1-3]. In recent decades, Wronskian technique has received considerable attention to its application owing to its obvious

Received: 2014-10-21; **Revised**: 2015-01-18

Foundation item: National Natural Science Foundation of China (11301454, 11271168), the Natural Science Foundation for Colleges and Universities in Jiangsu Province (13KJD110009), the Jiangsu Qing Lan Project for Excellent Young Teachers in University (2014) and XZIT (XKY 2013202).

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advantages[4-7]. Exact soliton solutions in terms of the Wronskian technique can be verified by the direct substitution into the soliton equations. As a generalization of Wronskian, double Wronskian technique has always been used to derive the solutions in the double Wronski determinant form[8-10]. Double Casoratian is the discrete version of the double Wronskian[5-6].

In 2007, Chen et al. put forward a matrix method for constructing double Casoratian entries which satisfy an arbitrary matrix equation and obtained the generalized double Wronskian solutions of the AKNS equation[11]. By letting the spectral matrix be a triangular case, Jordan case with zero on the main diagonal and other special cases, the soliton solutions, rational solutions, Matveev solutions, Complexitons and interaction solutions were derived. The matrix approach has been applied to a class of soliton equations[12-15], such as the general nonlinear Schrödinger equation with derivative, a first-order four-potential isospectral Ablowitz-Ladik (AL) equation, and so on. Recently, the soliton solution and rational-like solution in terms of the method for a negative order four-potential AL equation have been expressed[21].

In this paper, we consider a negative order four-potential isospectral AL equation and obtain its novel double Casoratian solutions. The paper is organized as follows. In Section 1, we recall the generalized double Casoratian solutions. In Section 2, soliton and rational-like solutions are presented. In Section 3, we work out the Matveev solutions. In Section 4, we construct the mixed solutions between the rational-like and Matveev solutions. A conclusion is given in the last section.

1 The generalized double Casoratian solutions

A negative order four-potential isospectral AL equation reads[16]

$$\left.\begin{aligned} Q_{n,t} &= (1-Q_nR_n)S_{n-1}, \\ R_{n,t} &= -(1-Q_nR_n)T_n, \\ S_{n,t} &= (1-S_nT_n)Q_n, \\ T_{n,t} &= -(1-S_nT_n)R_{n+1} \end{aligned}\right\} \tag{1}$$

where the functions of variable $n\{Q_n, R_n, S_n, T_n\}$ are four potentials. Its spectral problem is[17]

$$\left.\begin{aligned} E\Phi_n &= U_n\Phi_n, \\ U_n &= \begin{pmatrix} z^2+S_nR_n & zQ_n+z^{-1}S_n \\ zT_n+z^{-1}R_n & z^{-2}+T_nQ_n \end{pmatrix}, \\ \Phi_n &= \begin{pmatrix} \phi_{1n} \\ \phi_{2n} \end{pmatrix} \end{aligned}\right\} \tag{2}$$

where z is spectral parameter and E is the shift operator defined as $E^k v(n) = v(n+k)$, $k\in Z$. Usually for convenience, we write $v(n) = v_n$ without any confusion.

The auxiliary problem is[16]

$$\left.\begin{aligned} &\Phi_{n,t} = V_n\Phi_n, \\ &V_n^{(-1)} = \\ &\begin{pmatrix} -\frac{1}{2}R_nS_{n-1}+\frac{1}{2}z^{-2} & -S_{n-1}z^{-1} \\ -R_nz^{-1} & \frac{1}{2}R_nS_{n-1}-\frac{1}{2}z^{-2} \end{pmatrix} \end{aligned}\right\} \tag{3}$$

Through the dependent variable transformation[18]

$$\left.\begin{aligned} Q_n = \frac{g_n}{f_n},\ R_n = \frac{h_n}{f_n}, \\ S_n = \frac{G_n}{F_n},\ T_n = \frac{H_n}{F_n} \end{aligned}\right\} \tag{4}$$

the bilinear form of Eq. (1) is given by

$$D_t g_n \cdot f_n = F_n G_{n-1} \tag{5a}$$

$$D_t h_n \cdot f_n = -F_{n-1} H_n \tag{5b}$$

$$D_t G_n \cdot F_n = f_{n+1} g_n \tag{5c}$$

$$D_t H_n \cdot F_n = -f_n h_{n+1} \tag{5d}$$

$$f_n^2 - g_nh_n = F_nF_{n-1} \tag{5e}$$

$$F_n^2 - G_nH_n = f_{n+1} f_n \tag{5f}$$

where D is the well-known Hirota bilinear operator defined by

$$D_t^m D_x^n f \cdot g = (\partial_t - \partial_{t'})^m (\partial_x - \partial_{x'})^n f(t,x) \times g(t',x') \mid_{t'=t,\ x'=x} \tag{6}$$

We consider the following matrix equations

$$E\Phi_n = A\Phi_n,\ E^{-1}\Psi_n = A\Psi_n \tag{7a}$$

$$\Phi_{n,t} = \frac{1}{2}E^{-1}\Phi_n,\ \Psi_{n,t} = -\frac{1}{2}E\Psi_n \tag{7b}$$

where $A=(a_{ij})$ is an $(m+p+2)\times(m+p+2)$ arbitrarily invertible, real matrix independent of n and t and

$$\left.\begin{aligned} \Phi_n &= (\phi_{1n}, \phi_{2n}, \cdots, \phi_{m+p+2,n})^T, \\ \Psi_n &= (\psi_{1n}, \psi_{2n}, \cdots, \psi_{m+p+2,n})^T \end{aligned}\right\} \tag{8}$$

The double Casorati determinant is a discrete version of a double Wronskian defined as[19]

$$\mathrm{Cas}^{m+1,p+1}(\Phi_n;\ \Psi_n)=$$
$$|\ \Phi_n,\ E\Phi_n,\cdots,E^m\Phi_n;\ \Psi_n,\ E\Psi_n,\cdots,\ E^p\Psi_n\ |=$$
$$|\ \hat{m};\hat{p}\ | \tag{9}$$

Besides, $|\tilde{l};\hat{k}|$ and $|\tilde{l};\tilde{k}|$ can be expressed as[20]

$$|\ \tilde{l};\ \hat{k}\ |=|\ E\Phi_n,\ E^2\Phi_n,\cdots,E^l\Phi_n;\ \Psi_n,\ E\Psi_n,\cdots,\ E^k\Psi_n\ | \tag{10a}$$

$$|\ \tilde{l};\ \tilde{k}\ |=|\ E\Phi_n,\ E^2\Phi_n,\cdots,E^l\Phi_n;\ E\Psi_n,\ E^2\Psi_n,\cdots,\ E^k\Psi_n\ | \tag{10b}$$

In order to obtain the novel double Casoratian solution to Eq. (1), the following lemma is needed.

Lemma 1.1 Suppose that M is an $N\times(N-2)$ matrix and a, b, c, d are N-order column vectors, then

$$|\ M,a,b\ |\ |\ M,c,d\ |-|\ M,a,c\ |\ |\ M,b,d\ |+|\ M,a,d\ |\ |\ M,b,c\ |=0 \tag{11}$$

Employing the double Casoratian technique, we have Theorem 1.1[15].

Theorem 1.1 Eq. (5) has the following double Casoratian solution:

$$\left.\begin{aligned}
&f_n=|\ \hat{m};\ \hat{p}\ |,\ g_n=|\ \widehat{m+1};\ \widehat{p-1}\ |,\\
&h_n=-|\ \widehat{m-1};\ \widehat{p+1}\ |,\\
&F_n=|\ A\ |^{\frac{1}{2}}\ |\ \hat{m};\ \widetilde{p+1}\ |,\\
&G_n=|\ A\ |^{\frac{1}{2}}\ |\ \widehat{m+1};\ \tilde{p}\ |,\\
&H_n=-|\ A\ |^{\frac{1}{2}}\ |\ \widehat{m-1};\ \widetilde{p+2}\ |
\end{aligned}\right\} \tag{12}$$

which entries satisfy (7).

Hence, we obtain the corresponding solution of Eq. (1):

$$\left.\begin{aligned}
&Q_n=\frac{|\ \widehat{m+1};\ \widehat{p-1}\ |}{|\ \hat{m};\ \hat{p}\ |},\\
&R_n=-\frac{|\ \widehat{m-1};\ \widehat{p+1}\ |}{|\ \hat{m};\ \hat{p}\ |}
\end{aligned}\right\} \tag{13a}$$

$$\left.\begin{aligned}
&S_n=\frac{|\ \widehat{m+1};\ \tilde{p}\ |}{|\ \hat{m};\ \widetilde{p+1}\ |},\\
&T_n=-\frac{|\ \widehat{m-1};\ \widetilde{p+2}\ |}{|\ \hat{m};\ \widetilde{p+1}\ |}
\end{aligned}\right\} \tag{13b}$$

2 Soliton solutions and rational-like solutions

From (7), we derive the general solution[21]

$$\Phi_n=A^n e^{\frac{1}{2}A^{-1}t}C,\ \Psi_n=A^{-n}e^{-\frac{1}{2}A^{-1}t}D \tag{14}$$

where $C=(c_1,c_2,\cdots,c_{m+p+2})^T$ and $D=(d_1,d_2,\cdots,d_{m+p+2})^T$ are real constant vectors. Let $A=e^{\frac{1}{2}B}$, then (14) can be rewritten as

$$\Phi_n=e^{\frac{1}{2}nB+\frac{1}{2}e^{-\frac{1}{2}B}t}C,\ \Psi_n=e^{-\frac{1}{2}nB-\frac{1}{2}e^{-\frac{1}{2}B}t}D \tag{15}$$

(15) can be expressed as a power series in B:

$$\Phi_n=e^{\frac{1}{2}nB+\frac{1}{2}e^{-\frac{1}{2}B}t}C=\sum_{r=0}^{\infty}\frac{t^r}{2^r r!}\sum_{s=0}^{\infty}\Big[\sum_{l=0}^{s}\frac{(-1)^l r^l n^{s-l}}{2^s l!(s-l)!}\Big]B^sC \tag{16a}$$

$$\Psi_n=e^{-\frac{1}{2}nB-\frac{1}{2}e^{-\frac{1}{2}B}t}D=\sum_{r=0}^{\infty}\frac{(-1)^r t^r}{2^r r!}\sum_{s=0}^{\infty}\Big[\sum_{l=0}^{s}\frac{(-1)^s r^l n^{s-l}}{2^s l!(s-l)!}\Big]B^sD \tag{16b}$$

If

$$B=\begin{pmatrix}k_1 & & & 0\\ & k_2 & & \\ & & \ddots & \\ 0 & & & k_{m+p+2}\end{pmatrix},\ k_i\neq k_j(i\neq j) \tag{17}$$

we can obtain soliton solutions of Eq. (1)[18], where

$$\left.\begin{aligned}
&\phi_{jn}=e^{\frac{1}{2}k_j n+\frac{1}{2}e^{-\frac{1}{2}k_j t}}c_j,\\
&\psi_{jn}=e^{-\frac{1}{2}k_j n-\frac{1}{2}e^{-\frac{1}{2}k_j t}}d_j
\end{aligned}\right\} \tag{18}$$

If

$$B=\begin{pmatrix}0 & & & 0\\ 1 & 0 & & \\ & \ddots & \ddots & \\ 0 & & 1 & 0\end{pmatrix}_{(m+p+2)\times(m+p+2)} \tag{19}$$

it is obvious that $B^{m+p+2}=0$. Thus we can calculate the rational-like solutions with double Casoratian form of Eq. (1). Taking $c_1=d_1=1$, $c_k=d_k=0$ ($k=2,3,\cdots,m+p+2$), then the components of Φ_n and Ψ_n are

$$\phi_{jn}=\sum_{r=0}^{\infty}\frac{t^r}{2^r r!}\sum_{l=0}^{j-1}\frac{(-1)^l r^l n^{j-1-l}}{2^{j-1}l!(j-1-l)!} \tag{20a}$$

$$\psi_{jn}=\sum_{r=0}^{\infty}\frac{(-1)^r t^r}{2^r r!}\sum_{l=0}^{j-1}\frac{(-1)^{j-1}r^l n^{j-1-l}}{2^{j-1}l!(j-1-l)!} \tag{20b}$$

For example, set $m=p=0$, we obtain

$$\phi_{1n} = \sum_{r=0}^{\infty} \frac{t^r}{2^r r!} = e^{\frac{1}{2}t} \tag{21a}$$

$$\psi_{1n} = \sum_{r=0}^{\infty} \frac{(-1)^r t^r}{2^r r!} = e^{-\frac{1}{2}t} \tag{21b}$$

$$\phi_{2n} = \sum_{r=0}^{\infty} \frac{t^r}{2^r r!}\left(\frac{n}{2} - \frac{r}{2}\right) = \left(\frac{n}{2} - \frac{t}{4}\right)e^{\frac{1}{2}t} \tag{21c}$$

$$\psi_{2n} = \sum_{r=0}^{\infty} \frac{(-1)^r t^r}{2^r r!}\left(-\frac{n}{2} - \frac{r}{2}\right) = -\left(\frac{n}{2} - \frac{t}{4}\right)e^{-\frac{1}{2}t} \tag{21d}$$

3 Matveev solutions

Let B be a $(m+p+2)\times(m+p+2)$ Jordan matrix:

$$B = \begin{pmatrix} J(k_1) & & & 0 \\ & J(k_2) & & \\ & & \ddots & \\ 0 & & & J(k_s) \end{pmatrix} \tag{22}$$

Without loss of generality, we observe some Jordan block (dropping the subscript of k)

$$J(k) = \begin{pmatrix} k & & & 0 \\ 1 & k & & \\ & \ddots & \ddots & \\ 0 & & 1 & k \end{pmatrix}_{l\times l} = kI_l + Y_l \tag{23}$$

where I_l denotes an $l\times l$ unite matrix and $Y_l = \begin{pmatrix} 0 & & & 0 \\ 1 & 0 & & \\ & \ddots & \ddots & \\ 0 & & 1 & 0 \end{pmatrix}_{l\times l}$. It is obvious that

$$J^s(k) = (kI_l + Y_l)^s = \left(I_l + Y_l\partial_k + \frac{1}{2!}Y_l^2\partial_k^2 + \cdots + \frac{1}{j!}Y_l^j\partial_k^j + \cdots + \frac{1}{s!}Y_l^s\partial_k^s\right)k^s \tag{24}$$

Hence, for an arbitrary positive integer s, we have

$$\left.\begin{aligned} &J^s(k) = T(k)k^s, \\ &T(k) = \\ &\begin{pmatrix} 1 & & & & & 0 \\ \partial_k & 1 & & & & \\ \frac{1}{2}\partial_k^2 & \partial_k & 1 & & & \\ \frac{1}{6}\partial_k^3 & \frac{1}{2}\partial_k^2 & \partial_k & 1 & & \\ & \ddots & \ddots & \ddots & \ddots & \\ \frac{1}{(l-1)!}\partial_k^{l-1} & \cdots & \frac{1}{6}\partial_k^3 & \frac{1}{2}\partial_k^2 & \partial_k & 1 \end{pmatrix}_{l\times l} \end{aligned}\right\} \tag{25}$$

Substituting $J(k)$ for matrix B in (16) gives

$$\left.\begin{aligned} \Phi_n(k) &= T(k)e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t}C, \\ \Psi_n(k) &= T(k)e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t}D \end{aligned}\right\} \tag{26}$$

whose components are

$$\phi_{jn}(k) = \left(c_1\frac{1}{(j-1)!}\partial_k^{j-1} + \cdots + c_{j-1}\partial_k + c_j\right)e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t} \tag{27a}$$

$$\psi_{jn}(k) = \left(d_1\frac{1}{(j-1)!}\partial_k^{j-1} + \cdots + d_{j-1}\partial_k + d_j\right)e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t} \tag{27b}$$

$j=1,2,\cdots,l$. Particularly, let

$c_1 = d_1 = 1$, $c_j = d_j = 0$ $(j = 2,3,\cdots,l)$,

(27) can be reduced to

$$\phi_{jn}(k) = \frac{1}{(j-1)!}\partial_k^{j-1}e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t} \tag{28a}$$

$$\psi_{jn}(k) = \frac{1}{(j-1)!}\partial_k^{j-1}e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t} \tag{28b}$$

Thus, the Matveev solutions to Eq. (1) are worked out, where

$$\Phi_n = (\phi_{1n}(k_1),\cdots,\phi_{l_1 n}(k_1);\cdots;\phi_{1n}(k_s),\cdots,\phi_{l_s n}(k_s))^T \tag{29a}$$

$$\Psi_n = (\psi_{1n}(k_1),\cdots,\psi_{l_1 n}(k_1);\cdots;\psi_{1n}(k_s),\cdots,\psi_{l_s n}(k_s))^T \tag{29b}$$

$l_1 + l_2 + \cdots + l_s = m + p + 2$.

Setting

$$\left.\begin{aligned} \Phi_n &= (\phi_{1n}(k),\phi_{2n}(k))^T, \\ \Psi_n &= (\psi_{1n}(k),\psi_{2n}(k))^T \end{aligned}\right\} \tag{30}$$

in (29), where $\phi_{jn}(k)$ and $\psi_{jn}(k)$ are defined by (28), we can derive

$$\left.\begin{aligned} \phi_{1n}(k) &= e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t}, \\ \psi_{1n}(k) &= e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t} \end{aligned}\right\} \tag{31a}$$

$$\phi_{2n}(k) = \partial_k e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t} = \left(\frac{n}{2} - \frac{t}{4}e^{-\frac{1}{2}k}\right)e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t} \tag{31b}$$

$$\psi_{2n}(k) = \partial_k e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t} = -\left(\frac{n}{2} - \frac{t}{4}e^{-\frac{1}{2}k}\right)e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t} \tag{31c}$$

Substituting (31) into (12) yields

$$f_n=\begin{vmatrix}\phi_{1n}(k) & \psi_{1n}(k)\\ \phi_{2n}(k) & \psi_{2n}(k)\end{vmatrix}=\begin{vmatrix}e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t} & e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t}\\ \left(\frac{n}{2}-\frac{t}{4}e^{-\frac{1}{2}k}\right)e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t} & -\left(\frac{n}{2}-\frac{t}{4}e^{-\frac{1}{2}k}\right)e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t}\end{vmatrix}=-n+\frac{t}{2}e^{-\frac{1}{2}k} \tag{32a}$$

$$g_n=\begin{vmatrix}\phi_{1n}(k) & \phi_{1,n+1}(k)\\ \phi_{2n}(k) & \phi_{2,n+1}(k)\end{vmatrix}=\begin{vmatrix}e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t} & e^{\frac{1}{2}(n+1)k+\frac{1}{2}e^{-\frac{1}{2}k}t}\\ \left(\frac{n}{2}-\frac{t}{4}e^{-\frac{1}{2}k}\right)e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t} & \left(\frac{n+1}{2}-\frac{t}{4}e^{-\frac{1}{2}k}\right)e^{\frac{1}{2}(n+1)k+\frac{1}{2}e^{-\frac{1}{2}k}t}\end{vmatrix}=\frac{1}{2}e^{(n+\frac{1}{2})k+e^{-\frac{1}{2}k}t} \tag{32b}$$

$$h_n=-\begin{vmatrix}\psi_{1n}(k) & \psi_{1,n+1}(k)\\ \psi_{2n}(k) & \psi_{2,n+1}(k)\end{vmatrix}=-\begin{vmatrix}e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t} & e^{-\frac{1}{2}t}\\ -\left(\frac{n}{2}-\frac{t}{4}e^{-\frac{1}{2}k}\right)e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t} & -\left(\frac{n+1}{2}-\frac{t}{4}e^{-\frac{1}{2}k}\right)e^{-\frac{1}{2}(n+1)k-\frac{1}{2}e^{-\frac{1}{2}k}t}\end{vmatrix}=\frac{1}{2}e^{-(n+\frac{1}{2})k-e^{-\frac{1}{2}k}t} \tag{32c}$$

Similarly, a direct calculation gives rise to

$$F_n=\left(-n-\frac{1}{2}+\frac{t}{2}e^{-\frac{1}{2}k}\right)|A|^{\frac{1}{2}}e^{-\frac{1}{2}k} \tag{33a}$$

$$G_n=\frac{1}{2}|A|^{\frac{1}{2}}e^{(n+\frac{1}{2})k+e^{-\frac{1}{2}k}t} \tag{33b}$$

$$H_n=\frac{1}{2}|A|^{\frac{1}{2}}e^{-(n+\frac{3}{2})k-e^{-\frac{1}{2}k}t} \tag{33c}$$

Thus, substituting (32) and (33) into (4), we obtain the Matveev solution of Eq. (1):

$$Q_n=-\frac{e^{\frac{1}{2}k}}{2n-te^{-\frac{1}{2}k}}e^{nk+e^{-\frac{1}{2}k}t} \tag{34a}$$

$$R_n=-\frac{e^{-\frac{1}{2}k}}{2n-te^{-\frac{1}{2}k}}e^{-nk-e^{-\frac{1}{2}k}t} \tag{34b}$$

$$S_n=-\frac{e^{k}}{2n+1-te^{-\frac{1}{2}k}}e^{nk+e^{-\frac{1}{2}k}t} \tag{34c}$$

$$T_n=-\frac{e^{-k}}{2n+1-te^{-\frac{1}{2}k}}e^{-nk-e^{-\frac{1}{2}k}t} \tag{34d}$$

Now, taking

$$\left.\begin{aligned}\Phi_n&=(\phi_{1n}(k),\phi_{2n}(k),\phi_{3n}(k))^{\mathrm{T}},\\ \Psi_n&=(\psi_{1n}(k),\psi_{2n}(k),\psi_{2n}(k))^{\mathrm{T}}\end{aligned}\right\} \tag{35}$$

We work out

$$\phi_{3n}(k)=\frac{1}{2}\partial_k^2 e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t}=\left(\frac{n^2}{8}-\frac{nt}{8}e^{-\frac{1}{2}k}+\frac{t}{16}e^{-\frac{1}{2}k}+\frac{t^2}{32}e^{-k}\right)e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t} \tag{36a}$$

$$\psi_{3n}(k)=\frac{1}{2}\partial_k^2 e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t}=\left(\frac{n^2}{8}-\frac{nt}{8}e^{-\frac{1}{2}k}-\frac{t}{16}e^{-\frac{1}{2}k}+\frac{t^2}{32}e^{-k}\right)e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t} \tag{36b}$$

When setting $(m, p)=(1, 0)$, we derive

$$Q_n=\frac{2e^{\frac{3}{2}k}}{(4n^2+2n)e^{\frac{1}{2}k}-(4nt+2t)+t^2e^{-\frac{1}{2}k}}e^{nk+e^{-\frac{1}{2}k}t} \tag{37a}$$

$$R_n=\frac{(4n^2+2n)e^{-\frac{1}{2}k}-4nte^{-k}+t^2e^{-\frac{3}{2}k}}{(4n^2+2n)e^{\frac{1}{2}k}-(4nt+2t)+t^2e^{-\frac{1}{2}k}}e^{-nk-e^{-\frac{1}{2}k}t} \tag{37b}$$

$$S_n=\frac{2e^{\frac{3}{2}k}}{4n^2+6n+2-(4nt+4t)e^{-\frac{1}{2}k}+t^2e^{-k}}e^{nk+e^{-\frac{1}{2}k}t} \tag{37c}$$

$$T_n=\frac{(4n^2+6n+2)e^{-\frac{3}{2}k}-(4nt+2t)e^{-2k}+t^2e^{-\frac{5}{2}k}}{4n^2+6n+2-(4nt+4t)e^{-\frac{1}{2}k}+t^2e^{-k}}\cdot e^{-nk-e^{-\frac{1}{2}k}t} \tag{37d}$$

Taking $(m, p)=(0, 1)$ gives

$$Q_n=\frac{(4n^2+2n)e^{\frac{1}{2}k}-(4nt+2t)+t^2e^{-\frac{1}{2}k}}{-(4n^2+2n)e^{-\frac{1}{2}k}+4nte^{-k}-t^2e^{-\frac{3}{2}k}}e^{nk+e^{-\frac{1}{2}k}t} \tag{38a}$$

$$R_n=\frac{2e^{-\frac{3}{2}k}}{-(4n^2+2n)e^{-\frac{1}{2}k}+4nte^{-k}-t^2e^{-\frac{3}{2}k}}e^{-nk-e^{-\frac{1}{2}k}t} \tag{38b}$$

$$S_n=\frac{(4n^2+6n+2)-(4nt+4t)e^{-\frac{1}{2}k}+t^2e^{-k}}{-(4n^2+6n+2)e^{-\frac{3}{2}k}+(4nt+2t)e^{-2k}-t^2e^{-\frac{5}{2}k}}\cdot$$

$$e^{nk+e^{-\frac{1}{2}k}t} \tag{38c}$$

$$T_n = \frac{2e^{-3k}}{-(4n^2+6n+2)e^{-\frac{3}{2}k}+(4nt+2t)e^{-2k}-t^2e^{-\frac{5}{2}k}} \cdot e^{-nk-e^{-\frac{1}{2}k}t} \tag{38d}$$

It is easy to verify that the above Q_n, R_n, S_n, T_n solve Eq. (1), respectively.

4 Mixed solutions

In this section, we would like to construct the mixed solutions in the double Casoratian form of Eq. (1). We assume that B is a paradiagonal matrix constituting of the matrices (19) (B_r) and (22) (B_m), i. e.

$$B = \begin{pmatrix} B_r & 0 \\ 0 & B_m \end{pmatrix} \tag{39}$$

then Φ_n and Ψ_n defined as (7) are described by

$$\left.\begin{aligned} \Phi_n &= (\Phi_{nr}{}^{T}, \Phi_{nm}{}^{T})^{T}, \\ \Psi_n &= (\Psi_{nr}{}^{T}, \Psi_{nm}{}^{T})^{T} \end{aligned}\right\} \tag{40}$$

where

$$E\Phi_{nz} = A_z\Phi_{nz},\ E^{-1}\Psi_{nz} = A_z\Psi_{nz} \tag{41a}$$

$$\Phi_{nz,t} = \frac{1}{2}E^{-1}\Phi_{nz},\ \Psi_{nz,t} = -\frac{1}{2}E\Psi_{nz},\ z=r,\ m \tag{41b}$$

It is obvious that the double Casorati determinant (12) constructed by (40) is a solution to Eq. (5) and this type of solutions is called the mixed solution.

Letting

$$\left.\begin{aligned} \Phi_n &= (\phi_{1,nr},\ \phi_{2,nr},\ \Phi_{nm})^{T}, \\ \Psi_n &= (\psi_{1,nr},\ \psi_{2,nr},\ \Psi_{nm})^{T} \end{aligned}\right\} \tag{42}$$

and $(m,\ p)=(1,\ 0)$, we have

$$f_n = \begin{vmatrix} \phi_{1,nr} & \phi_{1,(n+1)r} & \psi_{1,nr} \\ \phi_{2,nr} & \phi_{2,(n+1)r} & \psi_{2,nr} \\ \phi_{nm} & \phi_{(n+1)m} & \psi_{nm} \end{vmatrix} = \begin{vmatrix} e^{\frac{1}{2}t} & e^{\frac{1}{2}t} & e^{-\frac{1}{2}t} \\ \left(\frac{n}{2}-\frac{t}{4}\right)e^{\frac{1}{2}t} & \left(\frac{n+1}{2}-\frac{t}{4}\right)e^{\frac{1}{2}t} & -\left(\frac{n}{2}-\frac{t}{4}\right)e^{-\frac{1}{2}t} \\ e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t} & e^{\frac{1}{2}(n+1)k+\frac{1}{2}e^{-\frac{1}{2}k}t} & e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t} \end{vmatrix} =$$

$$\left(n-\frac{t}{2}\right)e^{\frac{1}{2}nk+\frac{1}{2}k+\frac{1}{2}e^{-\frac{1}{2}k}t}-\left(n-\frac{t}{2}+\frac{1}{2}\right)e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t}+\frac{1}{2}e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t+t} \tag{43a}$$

$$g_n = \begin{vmatrix} \phi_{1,nr} & \phi_{1,(n+1)r} & \phi_{1,(n+2)r} \\ \phi_{2,nr} & \phi_{2,(n+1)r} & \phi_{2,(n+2)r} \\ \phi_{nm} & \phi_{(n+1)m} & \phi_{(n+2)m} \end{vmatrix} = \begin{vmatrix} e^{\frac{1}{2}t} & e^{\frac{1}{2}t} & e^{\frac{1}{2}t} \\ \left(\frac{n}{2}-\frac{t}{4}\right)e^{\frac{1}{2}t} & \left(\frac{n+1}{2}-\frac{t}{4}\right)e^{\frac{1}{2}t} & \left(\frac{n}{2}+1-\frac{t}{4}\right)e^{\frac{1}{2}t} \\ e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t} & e^{\frac{1}{2}(n+1)k+\frac{1}{2}e^{-\frac{1}{2}k}t} & e^{\frac{1}{2}(n+2)k+\frac{1}{2}e^{-\frac{1}{2}k}t} \end{vmatrix} =$$

$$\left(-e^{\frac{1}{2}k}+\frac{1}{2}e^{k}+\frac{1}{2}\right)e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t+t} \tag{43b}$$

$$h_n = -\begin{vmatrix} \phi_{1,nr} & \psi_{1,nr} & \psi_{1,(n+1)r} \\ \phi_{2,nr} & \psi_{2,nr} & \psi_{2,(n+1)r} \\ \phi_{nm} & \psi_{nm} & \psi_{(n+1)m} \end{vmatrix} = \begin{vmatrix} e^{\frac{1}{2}t} & e^{-\frac{1}{2}t} & e^{-\frac{1}{2}t} \\ \left(\frac{n}{2}-\frac{t}{4}\right)e^{\frac{1}{2}t} & -\left(\frac{n}{2}-\frac{t}{4}\right)e^{-\frac{1}{2}t} & -\left(\frac{n+1}{2}-\frac{t}{4}\right)e^{-\frac{1}{2}t} \\ e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t} & e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t} & e^{-\frac{1}{2}(n+1)k-\frac{1}{2}e^{-\frac{1}{2}k}t} \end{vmatrix} =$$

$$\frac{1}{2}e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t-t}+\left[\left(n-\frac{t}{2}\right)e^{-\frac{1}{2}k}-\left(n-\frac{t}{2}+\frac{1}{2}\right)\right]e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t} \tag{43c}$$

Similarly, we obtain

$$F_n = |A|^{\frac{1}{2}} \cdot \left\{\left[\left(n-\frac{t}{2}+\frac{1}{2}\right)e^{\frac{1}{2}k}-\left(n-\frac{t}{2}+1\right)\right]e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t}+\frac{1}{2}e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t-\frac{1}{2}k+t}\right\} \tag{44a}$$

$$G_n = |A|^{\frac{1}{2}} \cdot \left(-e^{\frac{1}{2}k}+\frac{1}{2}e^{k}+\frac{1}{2}\right)e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t+t} \tag{44b}$$

$$H_n = |A|^{\frac{1}{2}} \cdot \left\{\frac{1}{2}e^{\frac{1}{2}nk+\frac{1}{2}e^{-\frac{1}{2}k}t-t}+\left[-\left(n-\frac{t}{2}+1\right)e^{-\frac{1}{2}k}+\left(n-\frac{t}{2}+\frac{1}{2}\right)e^{-k}\right]e^{-\frac{1}{2}nk-\frac{1}{2}e^{-\frac{1}{2}k}t}\right\} \tag{44c}$$

Making use of transformation (4), we can obtain the following mixed solution between the rational-like solution and Matveev solution for Eq. (1)

$$Q_n=\frac{e^{k+t}-2e^{\frac{1}{2}k+t}+e^{t}}{-(2n-t+1)+(2n-t)e^{\frac{1}{2}k}+e^{-nk-e^{-\frac{1}{2}k}t+t}} \tag{45a}$$

$$R_n=\frac{\left[-(2n-t+1)+(2n-t)e^{-\frac{1}{2}k}\right]e^{-nk-e^{-\frac{1}{2}k}t}+e^{-t}}{-(2n-t+1)+(2n-t)e^{\frac{1}{2}k}+e^{-nk-e^{-\frac{1}{2}k}t+t}} \tag{45b}$$

$$S_n=\frac{-2e^{\frac{1}{2}k+t}+e^{k+t}+e^{t}}{(2n-t+1)e^{\frac{1}{2}k}-(2n-t+2)+e^{-nk-e^{-\frac{1}{2}k}t-\frac{1}{2}k+t}} \tag{45c}$$

$$T_n=\frac{\left[-(2n-t+2)e^{-\frac{1}{2}k}+(2n-t+1)e^{-k}\right]e^{-nk-e^{-\frac{1}{2}k}t}+e^{-t}}{(2n-t+1)e^{\frac{1}{2}k}-(2n-t+2)+e^{-nk-e^{-\frac{1}{2}k}t-\frac{1}{2}k+t}} \tag{45d}$$

When $(m, p)=(0, 1)$, we derive

$$Q_n=\frac{\left[-(2n-t+1)+(2n-t)e^{\frac{1}{2}k}\right]e^{nk+e^{-\frac{1}{2}k}t}+e^{t}}{(2n-t+1)-(2n-t)e^{-\frac{1}{2}k}-e^{nk+e^{-\frac{1}{2}k}t-t}} \tag{46a}$$

$$R_n=\frac{e^{-k-t}-2e^{-\frac{1}{2}k-t}+e^{-t}}{(2n-t+1)-(2n-t)e^{-\frac{1}{2}k}-e^{nk+e^{-\frac{1}{2}k}t-t}} \tag{46b}$$

$$S_n=\frac{\left[-(2n-t+2)+(2n-t+1)e^{\frac{1}{2}k}\right]e^{nk+e^{-\frac{1}{2}k}t}+e^{-\frac{1}{2}k+t}}{(2n-t+2)e^{-\frac{1}{2}k}-(2n-t+1)e^{-k}-e^{nk+e^{-\frac{1}{2}k}t-t}} \tag{46c}$$

$$T_n=\frac{e^{-\frac{1}{2}k-t}-2e^{-k-t}+e^{-\frac{3}{2}k-t}}{(2n-t+2)e^{-\frac{1}{2}k}-(2n-t+1)e^{-k}-e^{nk+e^{-\frac{1}{2}k}t-t}} \tag{46d}$$

Choosing

$$\left.\begin{aligned}\Phi_n&=(\phi_{1,nr},\ \phi_{2,nr},\ \phi_{1,nm},\ \phi_{2,nm})^{\mathrm{T}},\\ \Psi_n&=(\psi_{1,nr},\ \psi_{2,nr},\ \psi_{1,nm},\ \psi_{2,nm})^{\mathrm{T}}\end{aligned}\right\} \tag{47}$$

and $(m, p)=(1, 1)$, we get

$$\begin{aligned}f_n=&-\frac{1}{4}e^{nk+e^{-\frac{1}{2}k}t+\frac{1}{2}k-t}+\\&\left(n^2+n-\frac{nt}{2}-\frac{t}{4}+\frac{1}{4}\right)e^{\frac{1}{2}k}+\\&\left(n^2+n+\frac{nt}{2}+\frac{t}{4}-\frac{t^2}{2}+\frac{1}{4}\right)e^{-\frac{1}{2}k}+\\&\left(-\frac{nt}{2}+\frac{t^2}{4}-\frac{t}{4}\right)e^{-k}+\\&\left(-2n^2-2n+\frac{nt}{2}+\frac{t^2}{4}+\frac{t}{4}\right)-\\&\frac{1}{4}e^{-nk-e^{-\frac{1}{2}k}t-\frac{1}{2}k+t}\end{aligned} \tag{48a}$$

$$\begin{aligned}g_n=&\left[\left(\frac{t}{4}-\frac{n}{2}\right)e^{\frac{3}{2}k}+\left(n-\frac{t}{2}+\frac{1}{2}\right)e^{k}-\right.\\&\left.\left(\frac{n}{2}-\frac{t}{4}+\frac{1}{2}\right)e^{\frac{1}{2}k}\right]e^{nk+e^{-\frac{1}{2}k}t}-\\&\left(\frac{n}{2}+\frac{1}{2}\right)e^{k+t}+\left(n+\frac{t}{4}+\frac{1}{2}\right)e^{\frac{1}{2}k+t}+\\&\frac{t}{4}e^{-\frac{1}{2}k+t}-\left(\frac{n}{2}+\frac{t}{2}\right)e^{t}\end{aligned} \tag{48b}$$

$$\begin{aligned}h_n=&\left[\left(\frac{t}{4}-\frac{n}{2}\right)e^{-\frac{3}{2}k}+\left(n-\frac{t}{2}+\frac{1}{2}\right)e^{-k}-\right.\\&\left.\left(\frac{n}{2}-\frac{t}{4}+\frac{1}{2}\right)e^{-\frac{1}{2}k}\right]e^{-nk-e^{-\frac{1}{2}k}t}+\\&\left(n+\frac{t}{4}+\frac{1}{2}\right)e^{-\frac{1}{2}k-t}-\left(\frac{n}{2}+\frac{t}{2}+\frac{1}{2}\right)e^{-k-t}+\\&\frac{t}{4}e^{-\frac{3}{2}k-t}-\frac{n}{2}e^{-t}\end{aligned} \tag{48c}$$

$$\begin{aligned}F_n=&|A|^{\frac{1}{2}}\cdot\left[-\frac{1}{4}e^{nk+e^{-\frac{1}{2}k}t+\frac{1}{2}k-t}+\right.\\&\left(-2n^2-4n+\frac{nt}{2}+\frac{t^2}{4}+\frac{t}{2}-\frac{3}{2}\right)e^{-\frac{1}{2}k}+\\&\left(n^2+2n+\frac{nt}{2}-\frac{t^2}{2}+\frac{t}{2}+1\right)e^{-k}+\\&\left(-\frac{nt}{2}+\frac{t^2}{4}-\frac{t}{2}\right)e^{-\frac{3}{2}k}+\\&\left.\left(n^2+2n-\frac{nt}{2}-\frac{t}{2}+1\right)-\frac{1}{4}e^{-nk-e^{-\frac{1}{2}k}t-\frac{3}{2}k+t}\right]\end{aligned} \tag{48d}$$

$$\begin{aligned}G_n=&|A|^{\frac{1}{2}}\cdot\left\{\left[\left(\frac{t}{4}-\frac{n}{2}-\frac{3}{4}\right)e^{\frac{1}{2}k}+\right.\right.\\&\left.\left(n-\frac{t}{2}+1\right)e^{k}-\left(\frac{n}{2}-\frac{t}{4}+\frac{1}{4}\right)e^{\frac{3}{2}k}\right]e^{nk+e^{-\frac{1}{2}k}t}+\\&\left(n+\frac{t}{4}+1\right)e^{t}-\left(\frac{n}{2}+\frac{3}{4}\right)e^{\frac{1}{2}k+t}-\\&\left.\left(\frac{n}{2}+\frac{t}{2}+\frac{1}{4}\right)e^{-\frac{1}{2}k+t}+\frac{t}{4}e^{-k+t}\right\}\end{aligned} \tag{48e}$$

$$
\begin{aligned}
H_n = & |A|^{\frac{1}{2}} \cdot \left\{ \left[\left(\frac{t}{4} - \frac{n}{2} - \frac{3}{4} \right) e^{-\frac{3}{2}k} + \right.\right. \\
& \left(n - \frac{t}{2} + 1 \right) e^{-2k} - \\
& \left.\left(\frac{n}{2} - \frac{t}{4} + \frac{1}{4} \right) e^{-\frac{5}{2}k} \right] e^{-nk - e^{-\frac{1}{2}k} t} - \\
& \left(\frac{n}{2} + \frac{1}{4} \right) e^{-\frac{1}{2}k - t} + \left(n + \frac{t}{4} + 1 \right) e^{-k-t} - \\
& \left.\left(\frac{n}{2} + \frac{t}{2} + \frac{3}{4} \right) e^{-\frac{3}{2}k - t} + \frac{t}{4} e^{-2k-t} \right\} \qquad (48f)
\end{aligned}
$$

Again making use of (4), we obtain the mixed solution to Eq. (1) between the rational-like and the Matveev solutions. Besides, we can also construct the mixed solution under the conditions of $(m, p) = (2, 0)$ and $(m, p) = (0, 2)$. Furthermore, we can also deduce the other mixed solutions between the rational-like cases and Matveev cases.

5 Conclusion

In this paper, the double Wronskian technique and the matrix method have been applied to a negative order four-potential isospectral Ablowitz-Ladik equation possessing bilinear form. By expanding the general solutions to the matrix equation satisfied by double Casoratian entries as the series of the arbitrary matrix B, Soliton solutions, rational-like solutions, Matveev solutions and mixed solutions in double Caosoratian form for the equation have been presented. By taking B to be Jordan matrices and the combination of different Jordan form matrices with respect to rational-like cases and Matveev cases, we have constructed novel Matveev solutions and mixed solutions. Furthermore, some explicit solutions, such as complexitons and interaction solutions, can also be derived by letting the general matrix be some other special cases.

References

[1] Freeman N C, Nimmo J J C. Soliton solutions of the KdV and KP equations: The Wronskian technique[J]. Physics Letters A, 1983, 95(1): 1-3.

[2] Nimmo J J C, Freeman N C. A method of obtainning the soliton solution of the Boussinesq equation in terms of a Wronskain[J]. Physics Letters A, 1983, 95(1): 4-6.

[3] Nimmo J J C. Soliton solutions of three differential-difference equations in Wronskian form[J]. Physics Letters A, 1983, 99(6-7): 281-286.

[4] Satsuma J. A Wronskian representation of n-soliton solutions of nonlinear evolution equations[J]. Journal of the Physical Society of Japan, 1979, 46(1): 359-360.

[5] Hirota R, Ito M, Kato F. Two-dimensional Toda lattice equations[J]. Progress of Theoretical Physics Supplement, 1988, 94(94): 42-58.

[6] Hirota R, Ohta Y, Satsuma J. Solutions of the KP equation and the two dimensional Toda equations[J]. Journal of the Physical Society of Japan, 1988, 57(6): 1 901-1 904.

[7] Zhang D J. Notes on solutions in Wronskian form to soliton equations: KdV-type [DB/OL]. arXiv: nlin/0603008.

[8] Darboux G. Lecons Surla Théorie Générale des Surfaces, Volume Ⅱ[M]. 3rd edition. New York: Chelsea Publishing Company, 1972.

[9] Nimmo J J C, Freeman N C. A bilinear Bäcklund transformation for the nonlinear Schrödinger equation [J]. Physics Letters A, 1983, 99(6-7): 279-281.

[10] Liu Q M. DoubleWronskian solutions of the AKNS and the classical Boussinesq hierarchies[J]. Journal of the Physical Society of Japan, 1990, 59(10): 3 520-3 527.

[11] Chen D Y, Zhang D J, Bi J B. New double Wronskian solutions of the AKNS equation[J]. Science in China Series A: Mathematics, 2008, 51(1): 55-69.

[12] Zhai W, Chen D Y. Rational solutions of the general nonlinear Schrödinger equation with derivative [J]. Physics Letters A, 2008, 372(23): 4 217-4 221.

[13] Yao Y Q, Zhang D J, Chen D Y. The double Wronskian solutions to the Kadomtset-Petviashvili equation[J]. Modern Physics Letters B, 2008, 22(9): 621-641.

[14] Chen S T, Zhang J B, Chen D Y. Generalized double Casoratian solutions to the four-potential isospectral Ablowitz-Ladik equation [J]. Communications in Nonlinear Science and Numerical Simulation, 2013, 18(11): 2 949-2 959.

[15] Chen S T, Li Q. Double Casoratian solutions of a negative order isospectral four-potential Ablowitz-Ladik equation [J]. Journal of Jiangsu Normal University: Natural Science Edition, 2013, 31(4): 11-17.

[16] Zhang D J, Chen S T. Symmetries for the Ablowitz-Ladik hierarchy: Ⅰ. Four-potential case[J]. Studies in

Applied Mathematics, 2010, 125(4): 393-418.
[17] Ablowitz M J, Ladik J F. Nonlinear differential-difference equations [J]. Journal of Mathematical Physics, 1975, 16(3): 598-603.
[18] Chen S T, Zhu X M, Li Q, Chen D Y. N-Soliton solutions for the four-potential isospectral Ablowitz-Ladik equation[J]. Chinese Physics Letters, 2011, 28(6): 060202.
[19] Gegenhasi, Hu X B, Levi D. On a discrete Davey-Stewartson system [J]. Inverse Problems, 2006, 22(5): 1 677-1 688.
[20] Wu H, Zhang D J. Mixed rational-soliton solutions of two differential-difference equations in Casorati determinant form [J]. Journal of Physics A: Mathematical and General, 2003, 36 (17): 4 867-4 873.
[21] Chen Y. Rational-like solutions for a negative order isospectral four-potential Ablowitz-Ladik equation[J]. Journal of Jiangsu Normal University (Natural Science Edition), 2014, 32(4): 36-39.

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[6] Agarwal A K. An analogue of Euler's identity and new combinatorial properties of n-colour compositions[J]. J Comput Appl Math, 2003, 160(1-2): 9-15.
[7] Narang G, Agarwal A K. Lattice paths and n-colour compositions[J]. Discrete Math, 2008, 308 (9): 1 732-1 740.
[8] Guo Y H. Some n-color compositions[J]. Journal of Integer Sequence, 2012, 15: Article 12.1.2.
[9] Narang G, Agarwal A K. n-Colour self-inverse compositions[J]. Proc Indian Acad Sci Math Sci, 2006, 116(3): 257-266.
[10] Guo Y H. n-Colour even compositions [J]. Ars Combina, 2013, 109(2): 425-432.
[11] Guo Y H. n-Colour even self-inverse compositions[J]. Proc Indian Acad Sci Math Sci, 2010, 120(1):27-33.
[12] Shapcott C. C-color compositions and palindromes[J]. Fibonacci Quart, 2012, 50(4): 297-303.
[13] Hoggatt V E, Bicknell M. Palindromic composition [J]. Fibonacci Quart, 1975, 13(4): 350-356.
[14] MacMahon P A. Combinatory Analysis, Vol. Ⅰ and Ⅱ[M]. New York: AMS Chelsea Publishing, 2001.