第45卷第12期 中国科学技术大学学报 Vol. 45, No. 12
2015年12月 JOURNAL OF UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA Dec. 2015

Article ID: 0253-2778(2015)12-0983-06

Boundedness of multilinear oscillatory singular integral on weighted Morrey spaces

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Abstract: Using the boundedness result of the multilinear oscillatory integral operators on weighted Lebesgue spaces, based on the extrapolation theory, some inequality techniques and decomposition of space, some boundedness of two classes of multilinear oscillatory singular integrals on the weighted Morrey spaces $L^{p,\kappa}(\omega)$ were obtained.

Key words: multilinear oscillatory integral; weighted Morrey spaces; weight

CLC number: O174.2 **Document code**: A doi: 10.3969/j.issn.0253-2778.2015.12.004

2010 Mathematics Subject Classification: 42B20; 42B35

Citation: Pan Yali, Li Changwen, Zhu Xiaolin. Boundedness of multilinear oscillatory singular integral on weighted Morrey spaces [J]. Journal of University of Science and Technology of China, 2015, 45(12): 983-988.

多线性振荡积分算子在加权 Morrey 空间的有界性

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摘要:利用多线性振荡积分算子在加权 Lebesgue 空间的有界性结论,依据外推原理、一些不等式和空间分解理论,得到了两种多线性振荡积分算子在加权 Morrey 空间 $L^{p,\kappa}(\omega)$中的有界性结论.

关键词:多线性振荡积分;加权 Morrey 空间;权

0 Introduction

The classical Morrey spaces were introduced by Morrey in Ref. [1] to investigate the local behavior of solutions to second order elliptic partial differential equations. On the other hand, Muckenhoupt et al.[2] introduced the classical A_p weighted theory. The study about weighted estimates for some operators on weighted spaces has become increasingly important. In 2009,

Received: 2013-12-11; **Revised**: 2014-05-10

Foundation item: Supported by the National Natural Science Foundation of China (11501233), China Postdoctoral Science Foundation (2015M572327), Humanities and Social Sciences Program of the Ministry of Education (15YJC630053), Natural Science Foundation of Anhui Province (1408085MA08; 1508085SMA204), Natural Science Foundation of the Education Department of Anhui Province (KJ2015A335; KJ2015A270).

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Komori and Shirai[3] first defined the weighted Morrey spaces $L^{p,\kappa}(\omega)$ which could be viewed as an extension of weighted Lebesgue spaces. They studied the boundedness of the fractional integral operator, the Hardy-Littlewood maximal operator and the Calderón-Zygmund singular integral operator on $L^{p,\kappa}(\omega)$. For the boundedness of some operators on these spaces, we refer the readers to Refs. [4-16].

Recently, some people studied the boundedness of two classes of oscillatory singular integrals on weighted Morrey spaces. Let K be a standard Calderón-Zygmund kernel, that is, K is C^1 on $\mathbb{R}^n$ away from the origin and has mean value zero on the unit sphere centered at the origin. The first class of oscillatory singular integral operator T is defined by

$$Tf(x) = \text{p. v.} \int_{\mathbb{R}^n} e^{iP(x,y)} K(x-y) f(y) dy \quad (1)$$

where $P(x,y)$ is a real-valued polynomial defined on $\mathbb{R}^n \times \mathbb{R}^n$.

The second class of oscillatory integral operator T_λ is defined by

$$T_\lambda f(x) = \text{p. v.} \int_{\mathbb{R}^n} e^{i\lambda\Phi(x,y)} K(x-y) \varphi(x,y) f(y) dy \quad (2)$$

where $\lambda \in \mathbb{R}$, $\varphi \in C_0^\infty(\mathbb{R}^n \times \mathbb{R}^n)$, the space of infinitely differentiable functions on $\mathbb{R}^n \times \mathbb{R}^n$ with compact support, and Φ is a real-analytic function or a real-$C^\infty(\mathbb{R}^n \times \mathbb{R}^n)$ function satisfying that for any $(x_0, y_0) \in \text{supp } \varphi$, there exists (j_0, k_0), $1 \leqslant j_0$, $k_0 \leqslant n$, such that $\partial^2 \Phi(x_0, y_0)/\partial x_{j_0} \partial y_{k_0}$ does not vanish up to the infinite order. These operators have arisen in the study of singular integrals supported on lower dimensional varieties, and the singular Radon transform. Their results are stated as follows.

Theorem A[17] Let $\omega \in A_1$, $0 < \kappa < 1$, and $K(x,y)$ be a standard Calderón-Zygmund kernel, then there exists a constant $C > 0$ independent of the coefficients of $P(x,y)$ such that

$$\sup_{\lambda > 0} \lambda \omega(\{x \in B: |Tf(x)| > \lambda\}) \leqslant C \| f \|_{L^{1,\kappa}(\omega)} \omega(B)^\kappa.$$

Theorem B[18] Let $\lambda \in \mathbb{R}$, $\varphi \in C_0^\infty(\mathbb{R}^n \times \mathbb{R}^n)$ and Φ is a real-$C^\infty(\mathbb{R}^n \times \mathbb{R}^n)$ function satisfying that for any $(x_0, y_0) \in \text{supp } \varphi$, there exists (j_0, k_0), $1 \leqslant j_0, k_0 \leqslant n$, such that $\partial^2 \Phi(x_0, y_0)/\partial x_{j_0} \partial y_{k_0}$ does not vanish up to the infinite order. Assume K is a standard Calderón-Zygmund kernel and T_λ is defined as in (2). Then for any $1 < p < \infty$, $0 < \kappa < 1$, and $\omega \in A_p$, T_λ is bounded on $L^{p,\kappa}(\omega)$.

The purpose of this paper is to study two classes of multilinear operators which are closely related to the operator T defined by (1) and T_λ defined by (2).

Let $m \in \mathbb{N}$ and $m \geqslant 2$. Let Ω be homogeneous of degree zero, belonging to the space $Lip_1(S^{n-1})$ and satisfying the following conditions

$$\left. \begin{aligned} &\int_{S^{n-1}} \Omega(\theta) \theta^\alpha d\theta = 0 \quad \text{for} \\ &\alpha \in (N \cup \{0\})^n \text{ and } |\alpha| = m \end{aligned} \right\} \quad (3)$$

Let A have derivatives of order m in $BMO(\mathbb{R}^n)$ and let $R_m(A; x, y)$ denote the m-th order Taylor series remainder of A at x about y, that is,

$$R_m(A;\ x,\ y) = A(x) - \sum_{|\alpha| \leqslant m-1} \frac{1}{\alpha!} D^\alpha A(y)(x-y)^\alpha.$$

The first operator we will consider here is defined by

$$T^A f(x) = \text{p. v.} \int_{\mathbb{R}^n} e^{iP(x,y)} \frac{\Omega(x-y)}{|x-y|^{n+m}} Q_{m+1}(A; x, y) f(y) dy \quad (4)$$

where

$$Q_{m+1}(A; x, y) = R_m(A;\ x,\ y) - \sum_{|\alpha| = m} \frac{1}{\alpha!} D^\alpha A(y)(x-y)^\alpha.$$

Hu and Yang[19] considered the boundedness of the operator T^A on $H^1(\mathbb{R}^n)$ when $P(x,y) = P(x-y)$. They proved that T^A is bounded from $H^1(\mathbb{R}^n)$ to $L^1(\mathbb{R}^n)$. In 2000, Hu et al. [20] proved that T^A is also bounded from weighted Hardy space $H^1_\omega(\mathbb{R}^n)$ to the weighted Lebesgue space $L^1_\omega(\mathbb{R}^n)$ for $\omega \in A_1(\mathbb{R}^n)$ and from the weighted Herz-type Hardy space to the weighted Herz space.

The second operator we will consider is defined by

$$T_\lambda^A f(x) = \text{p. v.}\int_{\mathbb{R}^n} e^{i\lambda\Phi(x,y)}\varphi(x,y)\cdot \frac{\Omega(x-y)}{|x-y|^{n+m}}Q_{m+1}(A;x,y)f(y)\,dy \qquad (5)$$

where $\lambda\in\mathbb{R}$, $\varphi\in C_0^\infty(\mathbb{R}^n\times\mathbb{R}^n)$, $\Phi\in C^\infty(\mathbb{R}^n\times\mathbb{R}^n)$ and $\Omega, Q_{m+1}(A;x,y)$ be as above.

In 2002, Wu and Yang[21] proved that T_λ^A is bounded from weighted Hardy space $H_\omega^1(\mathbb{R}^n)$ to the weighted Lebesgue space $L_\omega^1(\mathbb{R}^n)$ for $\omega\in A_1(\mathbb{R}^n)$ and from the weighted Herz-type Hardy space to the weighted Herz space.

Our main results in this paper are formulated as follows.

Theorem 0.1 Let $1<p<\infty$, $0<\kappa<1$, and $\omega\in A_p$, T^A be defined as in (4). $P(x,y)$ is a real-valued polynomial on $\mathbb{R}^n\times\mathbb{R}^n$ with $\nabla_y P(0,y)=0$. Then T^A is bounded on $L^{p,\kappa}(\omega)$.

Theorem 0.2 Let $\lambda\in\mathbb{R}$, $\varphi\in C_0^\infty(\mathbb{R}^n\times\mathbb{R}^n)$, $\Phi\in C^\infty(\mathbb{R}^n\times\mathbb{R}^n)$ satisfy $\nabla_y\varphi(0,y)=0$ and T_λ^A be defined as in (5). Then for any $1<p<\infty$, $0<\kappa<1$, and $\omega\in A_p$, T_λ^A is bounded on $L^{p,\kappa}(\omega)$.

1 Notations and preliminary lemmas

We begin this section with some properties of A_p weight classes which play an important role in the proofs of our results.

Let $B=B(x_0,r)$ be the ball with the center x_0 and radius r. Given a ball B and $\lambda>0$, λB denotes the ball with the same center as B whose radius is λ times that of B.

A weight ω is a locally integrable function on $\mathbb{R}^n$, which takes values in $(0,\infty)$ almost everywhere. For a given weight function ω, we denote the Lebesgue measure of B by $|B|$ and the weighted measure of E by $\omega(E)$, i.e. $\omega(E)=\int_E\omega(x)\,dx$. Given a weight ω, we say that ω satisfies the doubling condition if there exists a constant $D>0$ such that for any ball B, we have $\omega(2B)\leqslant D\omega(B)$.

We say $\omega\in A_p$ with $1<p<\infty$, if there exists a constant $C>0$, such that

$$\left(\frac{1}{|B|}\int_B\omega(x)\,dx\right)\left(\frac{1}{|B|}\int_B\omega(x)^{-\frac{1}{(p-1)}}\,dx\right)^{p-1}\leqslant C$$

for every ball $B\subseteq\mathbb{R}^n$. When $p=1$, $\omega\in A_1$ if there exists $C>0$, such that

$$\frac{1}{|B|}\int_B\omega(x)\,dx\leqslant C\operatorname*{essinf}_{x\in B}\omega(x)$$

for almost every $x\in\mathbb{R}^n$. We define $A_\infty=\bigcup_{p\geqslant 1}A_p$. A weight function ω is said to belong to the reverse Hölder class RH_r if there exist two constants $r>0$ and $C>0$ such that the following reverse Hölder inequality holds

$$\left(\frac{1}{|B|}\int_B\omega(x)^r\,dx\right)^{1/r}\leqslant C\left(\frac{1}{|B|}\int_B\omega(x)\,dx\right)$$

for every ball $B\subseteq\mathbb{R}^n$.

It is well known that if $\omega\in A_p$ with $1\leqslant p<\infty$, then there exists $r>1$ such that $\omega\in RH_r$.

Lemma 1.1[22] Let $\omega\in A_p$, $p\geqslant 1$, and $r>0$. Then for any ball B and $\lambda>1$,

$$\omega(2B)\leqslant C\omega(B),$$
$$\omega(\lambda B)\leqslant C\lambda^{np}\omega(B),$$

where C does not depend on B nor on λ.

Lemma 1.2[23] Let $\omega\in RH_r$ with $r>1$. Then there exists a constant C such that

$$\frac{\omega(E)}{\omega(B)}\leqslant C\left(\frac{|E|}{|B|}\right)^{(r-1)/r}$$

for any measurable subset E of a ball B.

The weighted Morrey spaces was defined as follows.

Definition 1.1[3] Let $1\leqslant p<\infty$, $0<\kappa<1$ and ω be a weight function. Then the weighted Morrey space is defined by

$$L^{p,\kappa}(\omega)=\{f\in L_{loc}^p(\omega):\|f\|_{L^{p,\kappa}(\omega)}<\infty\},$$

where

$$\|f\|_{L^{p,\kappa}(\omega)}=\sup_B\left(\frac{1}{\omega(B)^\kappa}\int_B|f(x)|^p\omega(x)\,dx\right)^{1/p}$$

and the supremum is taken over all balls B in $\mathbb{R}^n$.

Our argument is based heavily on the following results.

Lemma 1.3[20] Assume T^A is defined as in (4). Then for any $1<p<\infty$, and $\omega\in A_p$, we have

$$\|T^A f\|_{L_\omega^p}\leqslant C(m,n,p,\deg P,A_p(\omega))\cdot\sum_{|\alpha|=m}\|D^\alpha A\|_{BMO}\|f\|_{L_\omega^p},$$

where $A_p(\omega)$ denotes the $A_p(\mathbb{R}^n)$-constant of ω.

Lemma 1.4[21] Let $\lambda\in\mathbb{R}$, $\varphi\in C_0^\infty(\mathbb{R}^n\times\mathbb{R}^n)$ and Φ is a real-$C^\infty(\mathbb{R}^n\times\mathbb{R}^n)$ function satisfying that for any $(x_0, y_0) \in \operatorname{supp} \varphi$, there exists (j_0, k_0), $1 \leqslant j_0, k_0 \leqslant n$, such that $\partial^2 \Phi(x_0, y_0)/\partial x_{j_0} \partial y_{k_0}$ does not vanish up to the infinite order. Assume T_λ^A is defined as in (5). Then for any $1 < p < \infty$, $\omega \in A_p$, T_λ^A is bounded on L_ω^p, that is, for all $f \in L_\omega^p$,

$$\| T_\lambda^A f \|_{L_\omega^p} \leqslant C \sum_{|\alpha|=m} \| D^\alpha A \|_{BMO} \| f \|_{L_\omega^p},$$

where C is a constant independent of λ and A, but may depend on $A_p(\omega)$.

Lemma 1.5[24] Let $b(x)$ be a function on $\mathbb{R}^n$ with m-th order derivatives in $L_{loc}^q(\mathbb{R}^n)$ for some $q > n$. Then

$$| R_m(b; x, y) | \leqslant C_{m,n} | x - y |^m \cdot \sum_{|\alpha|=m} \left(\frac{1}{|\widetilde{Q}(x, y)|} \int_{\widetilde{Q}(x,y)} | D^\alpha b(z) |^q dz \right)^{1/q},$$

where $\widetilde{Q}(x, y)$ is the cube centered at x and having diameter $5\sqrt{n}|x - y|$.

In the following the letter C will denote a constant which may vary at each occurrence.

2 Proof of theorems

Proof of Theorem 0.1

It is sufficient to prove that there exists a constant $C > 0$ such that

$$\frac{1}{\omega(B)^\kappa} \int_B | T^A f(x) |^p \omega(x) dx \leqslant C \| f \|_{L^{p,\kappa}(\omega)}^p.$$

Fix a ball $B = B(x_0, r_B)$ and decompose $f = f_1 + f_2$, with $f_1 = f\chi_{2B}$ and $f_2 = f\chi_{(2B)^c}$. Then we have

$$\frac{1}{\omega(B)^\kappa} \int_B | T^A f(x) |^p \omega(x) dx \leqslant C\left\{ \frac{1}{\omega(B)^\kappa} \int_B | T^A f_1(x) |^p \omega(x) dx + \frac{1}{\omega(B)^\kappa} \int_B | T^A f_2(x) |^p \omega(x) dx \right\} = C\{ I_1 + I_2 \}.$$

Using Lemma 1.1 and Lemma 1.3, we get

$$I_1 \leqslant C \frac{1}{\omega(B)^\kappa} \int_{2B} | f(x) |^p \omega(x) dx \leqslant C \| f \|_{L^{p,\kappa}(\omega)}^p \cdot \frac{\omega(2B)^\kappa}{\omega(B)^\kappa} \leqslant C \| f \|_{L^{p,\kappa}(\omega)}^p.$$

We now estimate I_2. Let us consider the operator $\widetilde{T}^A$ defined by

$$\widetilde{T}^A f(x) = \text{p. v.} \int_{\mathbb{R}^n} e^{iP(x,y)} \frac{\Omega(x - y)}{| x - y |^{n+m}} R_{m+1}(A; x, y) f(y) dy.$$

Then we can write

$$| T^A f_2(x) | = | \widetilde{T}^A f_2(x) - \sum_{|\alpha|=m} [D^\alpha A, T_\alpha] f_2(x) | \leqslant | \widetilde{T}^A f_2(x) | + \sum_{|\alpha|=m} | [D^\alpha A, T_\alpha] f_2(x) |,$$

where T_α is the type of (1) with the kernel K replaced by $K_\alpha(x) = \Omega(x) x^\alpha / | x |^{n+m}$ which is a Calderón-Zygmund kernel. The boundedness of T_α on weighted Morrey spaces can be seen in Ref. [17]. By Lemma 1.5 we can write

$$| \widetilde{T}^A f_2(x) | \leqslant \left| \int_{(2B)^c} e^{iP(x,y)} \frac{\Omega(x - y)}{| x - y |^{n+m}} R_{m+1}(A; x, y) f_2(y) dy \right| \leqslant C \sum_{|\alpha|=m} \| D^\alpha A \|_{BMO} \int_{\mathbb{R}^n} \frac{| f_2(y) |}{| x - y |^n} dy.$$

Noting that for $x \in B$ and $y \in (2B)^c$, $| x_0 - y | < C| x - y|$. We have

$$| \widetilde{T}^A f_2(x) | \leqslant C \sum_{|\alpha|=m} \| D^\alpha A \|_{BMO} \int_{|x_0 - y| > 2r} \frac{| f_2(y) |}{| x_0 - y |^n} dy \leqslant C \sum_{|\alpha|=m} \| D^\alpha A \|_{BMO} \sum_{j=1}^{\infty} \int_{2^j r < |x_0 - y| < 2^{j+1} r} \frac{| f(y) |}{| x_0 - y |^n} dy \leqslant C \sum_{|\alpha|=m} \| D^\alpha A \|_{BMO} \sum_{j=1}^{\infty} \frac{1}{| 2^j B |} \int_{2^{j+1} B} | f(y) | dy.$$

Hölder's inequality and the A_p condition imply that

$$\int_{2^{j+1}B} | f(y) | dy \leqslant \left(\int_{2^{j+1}B} | f(y) |^p \omega(y) dy \right)^{\frac{1}{p}} \left(\int_{2^{j+1}B} \omega(y)^{-\frac{p'}{p}} dy \right)^{\frac{1}{p'}} \leqslant C \| f \|_{L^{p,\kappa}(\omega)} \omega(2^{j+1}B)^{\frac{\kappa}{p}} \left(\int_{2^{j+1}B} \omega(y)^{1-p'} dy \right)^{1-\frac{1}{p}} \leqslant C \| f \|_{L^{p,\kappa}(\omega)} | 2^{j+1}B | \omega(2^{j+1}B)^{\frac{\kappa-1}{p}}.$$

By Lemma 1.2, we have

$$\frac{1}{\omega(B)^\kappa} \int_B | \widetilde{T}^A f_2(x) |^p \omega(x) dx \leqslant C \| f \|_{L^{p,\kappa}(\omega)}^p \left(\sum_{j=1}^{\infty} \frac{\omega(B)^{\frac{1-\kappa}{p}}}{\omega(2^{j+1}B)^{\frac{1-\kappa}{p}}} \right)^p \leqslant C \| f \|_{L^{p,\kappa}(\omega)}^p \tag{7}$$

On the other hand by the boundedness of the commutators on the weighted Morrey spaces (see in Ref. [12]), we have

$$\frac{1}{\omega(B)^{\kappa}}\int_{B} |[D^{\alpha}A, T_{\alpha}]f_2(x)|^p \omega(x)\,dx \leqslant C\| f \|^p_{L^{p,\kappa}(\omega)} \quad (8)$$

By (7) and (8), we can obtain

$$I_2 \leqslant C\| f \|^p_{L^{p,\kappa}(\omega)}.$$

The proof of Theorem 0.1 is completed. □

Proof of Theorem 0.2

Similar to the proof of Theorem 0.1, we decompose $f = f_1 + f_2$, with $f_1 = f\chi_{2B}$ and $f_2 = f\chi_{(2B)^c}$. Then we have

$$\frac{1}{\omega(B)^{\kappa}}\int_{B} | T^A_{\lambda} f(x) |^p \omega(x)\,dx \leqslant C\left\{ \frac{1}{\omega(B)^{\kappa}}\int_{B} | T^A_{\lambda} f_1(x) |^p \omega(x)\,dx + \frac{1}{\omega(B)^{\kappa}}\int_{B} | T^A_{\lambda} f_2(x) |^p \omega(x)\,dx \right\} = C\{ J_1 + J_2 \}.$$

Using Lemma 1.1 and Lemma 1.4, we get

$$J_1 \leqslant C\frac{1}{\omega(B)^{\kappa}}\int_{2B} | f(x) |^p \omega(x)\,dx \leqslant C\| f \|^p_{L^{p,\kappa}(\omega)} \cdot \frac{\omega(2B)^{\kappa}}{\omega(B)^{\kappa}} \leqslant C\| f \|^p_{L^{p,\kappa}(\omega)}.$$

We now estimate J_2. By Lemma 1.5, we have $|R_m(A;x,y)| \leqslant C|x-y|^m$; in addition,

$$| Q_{m+1}(A;x,y) | \leqslant C| x-y |^m (1 + \sum_{|\alpha|=m} | D^{\alpha}A(x) |).$$

Then we write

$$| T^A_{\lambda} f_2(x) | = \left| \int_{(2B)^c} e^{i\lambda\Phi(x,y)} \varphi(x,y) \frac{\Omega(x-y)}{| x-y |^{n+m}} \cdot Q_{m+1}(A;x,y) f_2(y)\,dy \right| \leqslant C\left| \int_{\mathbb{R}^n} e^{i\lambda\Phi(x,y)} \varphi(x,y) \frac{\Omega(x-y)}{| x-y |^{n}} \cdot (1 + \sum_{|\alpha|=m} | D^{\alpha}A(x) |) f_2(y)\,dy \right| \leqslant C\sum_{|\alpha|=m} \| D^{\alpha}A \|_{BMO} | T_{\lambda} f_2(x) |.$$

By Theorem B, we have

$$\| T_{\lambda} f_2 \|_{L^{p,\kappa}(\omega)} \leqslant C\| f \|_{L^{p,\kappa}(\omega)}.$$

Then, we get

$$J_2 \leqslant C\| f \|^p_{L^{p,\kappa}(\omega)}.$$

Thus, we finish the proof of Theorem 0.2. □

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