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Biharmonic submanifolds with mean parallel curvature on $M^m(c)\times R$

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Abstract: Let M^n be an n-dimensional submanifold with parallel mean curvature H of product space form $M^m(c)\times R$, where $M^m(c)$ is a space form with constant sectional curvature c. By using the method of Simons inequality, a series of results are obtained.

Key words: biharmonic submanifold; with parallel mean curvature; product space form

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$M^m(c)\times R$ 中具有平行平均曲率的2-调和子流形

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摘要:令 M^n 为 n 维子流形,其乘积的平均曲率 H 为 $M^m(c)\times R$,其中,$M^m(c)$具是截面曲率 c 为常数的空间型. 通过利用 Simons 不等式,得到了一系列结果.

关键词:2-调和子流形;具有平行平均曲率;乘积空间型

0 Introduction

A harmonic map ψ between two Riemannian manifolds (M, g) and (N, h) is defined as a critical point of the energy function

$$E(\psi)=\frac{1}{2}\int_M |\mathrm{d}\psi|^2 \mathrm{d}V_g.$$

In 1964, Eells and Sampson[1] suggested a natural generalization of harmonic map is a critical point of the bienergy function

$$E_2(\psi)=\frac{1}{2}\int_{M^n} |\tau(\psi)|^2 \mathrm{d}V_g,$$

where $\tau(\psi)=\mathrm{tr}\,\nabla \mathrm{d}\psi$ is the tension field that vanishes for harmonic maps. The Euler-Lagrange equation for the bienergy functional is given by $\tau_2(\psi)=0$ which was derived by Jiang[2-3].

$$\tau_2(\psi)=\Delta\tau(\psi)-\mathrm{tr}(R^n(\mathrm{d}\psi,\tau(\psi))\mathrm{d}\psi),$$

where $\tau_2(\psi)$ is the bitension field of ψ. $\Delta=\mathrm{tr}(\nabla^{\psi})^2=\mathrm{tr}(\nabla^{\psi}\nabla^{\psi}-\nabla^{\psi}_{\nabla})$ is the rough Laplacian defined on sections of $\psi^{-1}(TN)$ and R^n is the curvature tensor of N, given by $R^n(X,Y)Z=[\nabla^n_X,\nabla^n_Y]Z-\nabla^n_{[X,Y]}Z$.

Biharmonic submanifolds of different ambient spaces have been intensively studied in the last decade, in particular for real space forms[4-10],

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complex space forms[11-14] or Sasaician space forms[15]. Naturally, the next step has been the study of biharmonic submanifolds of product spaces form of constant sectional curvature. This subject has already been started by several authors. In Ref. [16], Ou and Wang studied the biharmonicity of constant mean curvature surfaces in product space $S^2\times\mathbb{R}$. Abresh and Rohsenberg in Refs. [17], [18] and Alencar, do Carmo and Tribuzy in Ref. [19] studied the case of constant mean curvature surfaces in product spaces of type $M^2\times\mathbb{R}$, where M^2 is a simply connected surfaces with constant sectional curvature c. In the very recent paper, Fetcu, Oniciuc and Rosenberg[20] proved a gap theorem for the mean curvature of certain complete proper-biharmonic parallel mean curvature submanifolds. In Ref. [21], Roth proved a necessary and sufficient condition for biharmonic submanifolds in product spaces. This paper is devoted to the study of biharmonic submanifolds in a product space form. We find a Simons type integral inequality for submanifolds with parallel mean curvature vector as well as some relevant conclusions.

Theorem 0. 1 Let M^n be the n-dimensional biharmonic submanifold in product space form $M^m(c)\times\mathbb{R}$ $(c<0)$. If the mean curvature vector of M^n is parallel, then M^n is minimal.

Theorem 0. 2 Let M^n be the n-dimensional biharmonic submanifold in product space form $M^m(c)\times\mathbb{R}$ $(c>0)$. If the mean curvature vector of M^n is parallel and $S<(n-|T|^2)c$, then M^n is minimal.

Theorem 0. 3 Let M^n be the n-dimensional biharmonic submanifold in product space form $M^m(c)\times\mathbb{R}$ $(c\geqslant 0)$. If the mean curvature vector h of M^n is parallel and $S_h\neq(n-|T|^2)c$, then M^n is minimal. Where S_h is the square of the second fundamental form of M^n with respect to h.

Theorem 0. 4 Let M^n be the n-dimensional compact biharmonic submanifold in product space form $M^m(c)\times\mathbb{R}$ delete $(c\geqslant 0)$. Then we have

$$\int_{M^n}\Big\{(2n^2+n-|T|^2)H^2c+S\Big[\frac{3}{2}S+HS^{\frac{1}{2}}-2nc+(1+2n)c|T|^2\Big]\Big\}\mathrm{d}V_{M^n}\geqslant 0.$$

Theorem 0. 5 Let M^n be the n-dimensional compact biharmonic submanifold in product space form $M^m(c)\times\mathbb{R}$ $(c<0)$. If the mean curvature vector of M^n is parallel and $S\leqslant\frac{2}{3}[2nc-(2n+1)c|T|^2]$, then M^n is totally geodesic, or $S=\frac{2}{3}[2nc-(2n+1)c|T|^2]$.

1 Preliminaries

Let M^n be an n-dimensional connected submanifold immersed in $\widetilde{M}=M^m(c)\times\mathbb{R}$, where $M^m(c)\times\mathbb{R}$ is the Riemannina product of $M^m(c)$ and $\mathbb{R}$ with the standard metric $\langle\,,\rangle$. Let p be the codimensional of M^n in $\widetilde{M}$, i. e. $p=m+1-n$. We choose a local orthonormal frame field $\{e_1,\cdots,e_{n+p}\}$, the dual frame are $\{\omega_1,\cdots,\omega_{n+p}\}$, such that $\{e_1,\cdots,e_n\}$ are tangent to M^n, and $\{e_{n+1},\cdots,e_{n+p}\}$ are normal to M^n. We use the following convention on the range of indices unless otherwise stated:

$$1\leqslant A,B,\cdots\leqslant n+p;\ 1\leqslant i,j,k,\cdots\leqslant n;$$
$$n+1\leqslant\alpha,\beta,\gamma,\cdots\leqslant n+p.$$

Hence, the second fundamental form II and the mean curvature H of M^n are defined, respectively, by

$$\mathrm{II}=\sum_{\alpha,i,j}h^\alpha_{ij}\omega_i\otimes\omega_j\otimes e_\alpha,\ H=|h|=\frac{1}{n}\sum_i h^\alpha_{ii}.$$

We denote by t the coordinate on $\mathbb{R}$ and hence $\partial_t=\frac{\partial}{\partial t}$ is the unit vector field in the tangent bundle $\widetilde{TM}$ which is tangent to the $\mathbb{R}$-direction. We can decompose ∂_t as[22-24].

$$\partial_t=\sum_i T_ie_i+\sum_\alpha\eta_\alpha.$$

Using the formulas of Gauss and Weingarten, we get

$$T_{i,j}=\sum_\alpha h^\alpha_{ij}\eta_\alpha,\ \eta_{\alpha,i}=-\sum_j h^\alpha_{ij}T_j.$$

As it is well-known, the Gauss equation of

M^n is given by

$$R_{ijkl}=c(\delta_{ik}\delta_{jl}-\delta_{il}\delta_{jk}+T_jT_k\delta_{il}+T_iT_l\delta_{jk}-T_iT_k\delta_{jl}-T_jT_l\delta_{ik})+\sum_{\alpha}(h^{\alpha}_{ik}h^{\alpha}_{jl}-h^{\alpha}_{il}h^{\alpha}_{jk}) \tag{1}$$

The squared norm of the second fundamental form is given by $S=\sum_{\alpha,i,j}(h^{\alpha}_{ij})^2$, and R the normalized scalar curvature of M^n. Then is from the Gauss equation, we have the following well-known relation

$$R=c[n(n-1)+2(n-1)|T|^2]+n^2H^2-S.$$

The Codazzi equation, Ricci equation and Ricci identity on M^n are given, respectively, by

$$h^{\alpha}_{ijk}-h^{\alpha}_{ikj}=\eta_{\alpha}(T_j\delta_{ik}-T_k\delta_{ij}) \tag{2}$$

$$R_{\alpha\beta ij}=\sum_k(h^{\alpha}_{ik}h^{\beta}_{kj}-h^{\beta}_{ik}h^{\alpha}_{kj}) \tag{3}$$

and

$$h^{\alpha}_{ijkl}-h^{\alpha}_{ijlk}=\sum_M h^{\alpha}_{mj}R_{mikl}+\sum_M h^{\alpha}_{mi}R_{mjkl}+\sum_{\beta}h^{\beta}_{ij}R_{\beta\alpha kl} \tag{4}$$

Lemma 1.1 Let M^n be the n-dimensional submanifold in product space form $M^m(c)\times\mathbb{R}$. Then M^n is a biharmonic submanifold if and only if M^n satisfies the following conditions.

$$\left.\begin{array}{l}\sum_{\alpha,i,k}(2h^{\alpha}_{iik}h^{\alpha}_{jk}+h^{\alpha}_{ii}h^{\alpha}_{kkj})=0,\ \forall j\\ \sum_{i,k}h^{\alpha}_{iikk}-\sum_{\beta,i,j,k}h^{\beta}_{ii}h^{\beta}_{jk}h^{\alpha}_{jk}+\\ \quad c(n-|T|^2)\sum_i h^{\alpha}_{ii}=0,\ \forall\alpha\end{array}\right\} \tag{5}$$

Lemma 1.2 Let $A_1, A_2, \cdots, A_M$ be $(n\times n)$-symmetric matrices $(m\geqslant 2)$, then

$$\sum_{\alpha,\beta}\mathrm{tr}(A_{\alpha}A_{\beta}-A_{\beta}A_{\alpha})^2-\sum_{\alpha,\beta}(\mathrm{tr}A_{\alpha}A_{\beta}))^2\geqslant-\frac{3}{2}\Big(\sum_{\alpha}\mathrm{tr}(A^2_{\alpha})\Big)^2.$$

Lemma 1.3 Let M^n be the n-dimensional submanifold in product space form $M^m(c)\times\mathbb{R}$. Then

$$2\sum_{\alpha,\beta}[\mathrm{tr}(A_{\alpha}A_{\beta})^2-\mathrm{tr}(A^2_{\beta}A^2_{\alpha})]-\sum_{\alpha,\beta}(\mathrm{tr}(A_{\alpha}A_{\beta}))^2\geqslant-\frac{3}{2}S^2 \tag{6}$$

$$\sum_{\alpha,\beta}\mathrm{tr}(A_{\alpha}A_{\beta})\,\mathrm{tr}(A_{\alpha})\mathrm{tr}(A_{\beta})\geqslant 0 \tag{7}$$

$$\sum_{\alpha,\beta}\mathrm{tr}(A^2_{\alpha}A_{\beta})\,\mathrm{tr}(A_{\beta})\geqslant-HS^{\frac{3}{2}} \tag{8}$$

Proof From Lemma 1.2, Eq.(1) is clear.

$$\sum_{\alpha,\beta}\mathrm{tr}(A_{\alpha}A_{\beta})\,\mathrm{tr}(A_{\alpha})\mathrm{tr}(A_{\beta})=\sum_{l,m}\Big(\sum_{\beta}\big(\sum_k h^{\beta}_{kk}\big)h^{\beta}_{lm}\Big)^2\geqslant 0.$$

For fixed α, we have $h^{\alpha}_{kl}=\lambda^{\alpha}_k\delta_{kl}$. According to Cauchy-Schwarz inequality, we have

$$\mathrm{tr}(A^2_{\alpha}A_{\beta})=\sum_k(\lambda^{\alpha}_k)^2h^{\beta}_{kj}\leqslant\sqrt{\sum_k(\lambda^{\alpha}_k)^2\sum_M(\lambda^{\alpha}_M)^2(h^{\alpha}_{mm})^2}\leqslant\sqrt{\sum_k(\lambda^{\alpha}_k)^2\sum_M(\lambda^{\alpha}_M)^2\sum_{\{k,m\}}(h_{\{km\}}{}^{\alpha})^2}=\mathrm{tr}(A^2_{\alpha})\sqrt{\mathrm{tr}(A^2_{\beta})}.$$

Then

$$\Big|\sum_{\alpha,\beta}\mathrm{tr}(A^2_{\alpha}A_{\beta})\,\mathrm{tr}(A_{\beta})\Big|\leqslant\sqrt{\sum_{\beta}\Big(\sum_{\alpha}\mathrm{tr}(A^2_{\alpha}A_{\beta})\Big)^2\sum_{\beta}\mathrm{tr}(A_{\beta})^2}\leqslant\sqrt{\sum_{\beta}\Big[\sum_{\alpha}\mathrm{tr}(A^2_{\alpha})\sqrt{\mathrm{tr}(A^2_{\beta})}\Big]^2\sum_{\beta}(\mathrm{tr}A_{\beta})^2}=HS^{\frac{3}{2}}.$$

Eqs.(6)～(8) Q.E.D.

2 Proofs of Theorems

Proof Let's prove the Theorem 0.1 first: Since the mean curvature vector of M^n is parallel, then

$$\sum_i h^{\alpha}_{iik}=0,\ \sum_i h^{\alpha}_{iikj}=0.$$

Multiplying $\sum_M h^{\alpha}_{mm}$ on the both sides of the second expression in Eq.(5) and summing up with respect on α, we obtain that

$$0=\sum_{\alpha,\beta,i,j,k,m}h^{\alpha}_{mm}h^{\beta}_{ii}h^{\alpha}_{jk}h^{\beta}_{jk}-(n-|T|^2)c\sum_{\alpha,i,m}h^{\alpha}_{ii}h^{\alpha}_{mm}=\sum_{j,k}\Big(\sum_{\alpha}\big(\sum_i h^{\alpha}_{ii}\big)h^{\alpha}_{jk}\Big)^2-(n-|T|^2)c\sum_{\alpha}\Big(\sum_i h^{\alpha}_{ii}\Big)^2\geqslant-(n-|T|^2)cH^2.$$

According to $c<0$, then $H^2=0$, then M^n is minimal.

Proof Proving Theorem 0.2: Similarly to the proof of Theorem 0.1, we get

$$0=\sum_{j,k}\Big(\sum_{\beta}\big(\sum_i h^{\beta}_{ii}\big)h^{\beta}_{jk}\Big)^2-(n-|T|^2)c\sum_{\alpha}\Big(\sum_i h^{\alpha}_{ii}\Big)^2=\sum_{j,k}\Big(\sum_{\beta}\big(\sum_i h^{\beta}_{ii}\big)^2\sum_{\alpha}(h^{\alpha}_{jk})^2\Big)-$$

$$(n-|T|^2)c\sum_\alpha\Big(\sum_i h_{ii}^\alpha\Big)^2=H^2[S-(n-|T|^2)c].$$

From $S<(n-|T|^2)c$, it follows that $H^2=0$.

Proof Now we prove Theorem 0.3: Suppose M^n is not a minimal submanifold, then $h\neq 0$. Choosing $e_{n+1}/\!/h$, we have

$$h=\frac{1}{n}\sum_i h_{ii}^{n+1};\ \sum_i h_{ii}^{n+1}=n\,|\,h\,|;$$
$$\sum_i h_{ii}^\alpha=0,\ \forall\alpha\neq n+1.$$

Since M^n is a parallel mean curvature submanifold, from Lemma 1.1 we have

$$\left.\begin{aligned}&\sum_{i,j,k}h_{ii}^{n+1}h_{jk}^{n+1}h_{jk}^\alpha=0,\ \alpha\neq n+1\\&\sum_{i,j,k}h_{ii}^{n+1}h_{jk}^{n+1}h_{jk}^{n+1}-(n-|T|^2)c\sum_i h_{ii}^{n+1}=0,\\&\qquad\forall\alpha\neq n+1\end{aligned}\right\}\quad(9)$$

From $h\neq0$ and Eq.(9) we get

$$\sum_{j,k}(h_{jk}^{n+1})^2=(n-|T|^2)c.$$

So $S_h=(n-|T|^2)c$, which results in a contradiction.

Proof In the proof of Theorem 0.4: Taking the covariant derivative with respect to j on the both sides of the first expression in Eq.(5), and summing up with respect to j, we have

$$\sum_{\alpha,i,j,k}(2h_{iikj}^\alpha h_{jk}^\alpha+2h_{iik}^\alpha h_{kjj}^\alpha+h_{iij}^\alpha h_{kkj}^\alpha+h_{kkij}^\alpha h_{ii}^\alpha)=0.$$

Then we get

$$\sum_{\alpha,i,j,k}h_{ij}^\alpha h_{kkij}^\alpha=-\frac{1}{2}\sum_{\alpha,i,j,k}(3h_{iik}^\alpha h_{jjk}^\alpha+h_{ii}^\alpha h_{jjkk}^\alpha)=$$
$$-\frac{3}{2}\sum_{\alpha,i,j,k}(h_{iik}^\alpha h_{jjk}^\alpha+h_{ii}^\alpha h_{jjkk}^\alpha)+\sum_{\alpha,i,j,k}h_{ii}^\alpha h_{jjkk}^\alpha\quad(10)$$

From

$$\frac{1}{2}\Delta(H^2)=\sum_{\alpha,i,j,k}(h_{iik}^\alpha h_{jjk}^\alpha+h_{ii}^\alpha h_{jjkk}^\alpha)\quad(11)$$

and from Eq.(5), we have

$$\sum_{\alpha,i,j,k}h_{ii}^\alpha h_{jjkk}^\alpha=\sum_{\alpha,\beta}\mathrm{tr}(A_\alpha)\mathrm{tr}(A_\beta)\mathrm{tr}(A_\alpha A_\beta)-(n-|T|^2)cH^2\quad(12)$$

Substituting Eqs.(8), (9) into Eq.(7), we have

$$\sum_{\alpha,i,j,k}h_{ij}^\alpha h_{kkij}^\alpha=-\frac{3}{4}\Delta(H^2)+\sum_{\alpha,\beta}\mathrm{tr}(A_\alpha)\mathrm{tr}(A_\beta)\mathrm{tr}(A_\alpha A_\beta)-(n-|T|^2)cH^2\quad(13)$$

From Eqs.(1)～(4), (5) and (10), we obtain

$$\begin{aligned}&\frac{1}{2}\Delta S+\frac{3}{4}\Delta(H^2)=\sum_{\alpha,i,j,k}(h_{ijk}^\alpha)^2+\sum_{\alpha,\beta}\mathrm{tr}(A_\alpha)\mathrm{tr}(A_\beta)\mathrm{tr}(A_\alpha A_\beta)-(n-|T|^2)cH^2+\\&2\sum_{\alpha,\beta}[\mathrm{tr}(A_\alpha A_\beta)^2-\mathrm{tr}(A_\beta^2A_\alpha^2)]-\sum_{\alpha,\beta}(\mathrm{tr}(A_\alpha A_\beta))^2+3nc\sum_{\alpha,i,j}h_{ij}^\alpha H^\alpha T_iT_j-\\&n^2c\Big(\sum_\alpha H^\alpha\eta_\alpha\Big)^2-2nc\sum_{\alpha,i,j,k}h_{ij}^\alpha h_{ik}^\alpha T_kT_j+\\&nc\sum_{\alpha,\beta,i,j}h_{ij}^\alpha h_{ij}^\beta\eta_\alpha\eta_\beta+(n-|T|^2)cS-cn^2H^2+\sum_{\alpha,\beta}\mathrm{tr}(A_\alpha^2A_\beta)\,\mathrm{tr}(A_\beta)\end{aligned}\quad(14)$$

From Lemma 1.3 and Eq.(14) we have

$$\begin{aligned}&\frac{1}{2}\Delta S+\frac{3}{4}\Delta(H^2)\geqslant-(n-|T|^2)cH^2-\frac{3}{2}S^2+(n-|T|^2)S-HS^{\frac{3}{2}}-2cn^2H^2-\\&2ncS\,|\,T\,|^2+ncS=cH^2[-(n-|T|^2)-2n^2]+S\Big[-HS^{\frac{1}{2}}+2nc-(1+2n)c\,|\,T\,|^2-\frac{3}{2}S\Big].\end{aligned}$$

Since M^n is compact, we get

$$\int_{M^n}\Big\{(2n^2+n-|T|^2)H^2c+S\Big[\frac{3}{2}S+HS^{\frac{1}{2}}-2nc+(1+2n)c\,|\,T\,|^2\Big]\Big\}\mathrm{d}V_{M^n}\geqslant0.$$

Proof The last we prove Theorem 0.5. From Theorem 0.1, we know M^n is minimal. Since M^n is compact, from Theorem 0.4, we have

$$\int_{M^n}S\Big[\frac{3}{2}S-2nc+(1+2n)c\,|\,T\,|^2\Big]\mathrm{d}V_{M^n}\geqslant0.$$

Noting

$$S\leqslant\frac{2}{3}[2nc-(1+2n)c\,|\,T\,|^2],$$

we know M^n is totally geodesic, or $S=\frac{2}{3}[2nc-(1+2n)c|T|^2]$.

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