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共振怪波的预测

孟 勇

(宁波大学物理科学与技术学院,浙江宁波 315211)

摘要:怪波是一种分布非常陡峭、存在时间极短、波峰峰值远高于周围的波浪的局域波.以(2+1)维SK方程为例,运用Hirota双线性方法探究一种新型的怪波(共振怪波),该怪波的形成与lump型孤子密切相关.当lump型孤子在双条纹孤子的影响下,只在一瞬间出现,然后立即消失,于是lump型孤子就变成了怪波.并且通过理论计算和数形结合的方法求得怪波的运动轨迹、存在时间、面积、体积等特征量.

关键词:共振怪波;(2+1)维SK方程;运动轨迹;存在时间

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Resonance rogue wave prediction

MENG Yong

(*School of Physical Science and Technology, Ningbo University, Ningbo* 315211, *China*)

Abstract: Rogue wave is a kind of local wave with very steep distribution, very short existence time, and whose peak value is much higher than the surrounding waves. Therefore, taking the (2+1)-dimensional SK equation as an example, the Hirota bilinear method was used to explore a new type of rogue wave (resonance rogue wave), whose formation is closely related to the lump-type soliton. When the lump-type soliton is under the influence of a double-striped soliton, it will appear only momentarily and then disappear immediately, so it becomes a rogue wave. And the characteristic quantities such as the movement track, the existence time, the area and the volume of the strange wave were obtained by the method of theoretical calculation and the combination of number and shape.

Key words: resonance rogue wave; (2+1)-dimensional SK equation; track of motion; existence time

0 引言

怪波,也称之为畸形波、凶波、杀人波、巨波,最早发现于海洋之中,它是一种分布非常陡峭、存在时间极短、峰值远高于周围波浪的局域波.也正因为它“来无影、去无踪”和峰值高的特性,所以在突发时具有强破坏性,从1826年最早记录的怪波事件起,时有发生关于怪波引起的灾难报导[1-8].除此之外,怪波不仅出现在海洋之中,而且也出现在非线性光学、超流体、等离子体、冷原子凝聚等研究领域以及重要

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作者简介:孟勇,男,1992年生,硕士.研究方向:非线性物理. E-mail: wanguozhifu@yeah.net

的经济系统中[9-17].

由于孤子理论和实验的不断发展,推动了人们通过非线性波理论建立的非线性模型对怪波的产生和传播进行研究.类比孤立波,可以用孤子碰撞、呼吸子碰撞产生的高振幅波解释怪波的形成. lump 型孤子作为一种在局限在全空间上的有理函数解,已经越来越引起研究者的关注[18-21].文献[22]首次发现一种新型怪波,当全部时间内可见的 lump 型孤子在双条纹孤子的作用下突然出现又忽然消失,于是 lump 型孤子就变成了怪波;该类型怪波被命名为共振怪波.针对这种新型怪波,本文以(2+1)维 SK 方程为例,利用 Hirota 双线性方法得到了 lump 孤子以及 lump 型孤子与双条纹孤子的作用解;并且研究了共振怪波的若干动力学性质,以便更深入地理解它.

1　(2+1)维 SK 方程的怪波解

(2+1)维 SK 方程为

$$u_t=(u_{xxxx}+5uu_{xx}+\frac{5}{3}u^3+5u_{xy})_x-5\int u_{yy}\mathrm{d}x+5uu_y+5u_x\int u_y\mathrm{d}x \tag{1}$$

该方程被广泛地应用于物理学中的各个分支领域,比如二维量子重力测量仪领域、共形场理论和非线性科学 Liouville 流守恒方程.

为了方便求解式(1),首先对其进行变换

$$u=6(\ln f)_{xx} \tag{2}$$

得到(2+1)维 SK 方程的双线性形式[23]:

$$(D_xD_t+5D_y^2-D_x^6-5D_x^3D_y)f\cdot f=0 \tag{3}$$

式中,D 是双线性算子.然后为了产生共振怪波,将代表 lump 型孤子的代数解和代表双条纹孤子的双曲函数解进行组合叠加,所以设 f 的表达式为

$$f=\xi^2+f_0+k_0\cosh(\theta) \tag{4}$$

$$\theta=k_1x+k_2y+k_3t+k_4 \tag{5}$$

式中,f_0,k_0 为待定常数,$\xi^2=\xi\cdot\xi$,ξ 为一个关于常矢量 κ,p,ω,α 的四维矢量,

$$\xi=x\kappa+yp+t\omega+\alpha \tag{6}$$

并标注

$$\left.\begin{aligned}&k_{00}=\alpha\cdot\alpha,\ k_{10}=\kappa\cdot\alpha,\\&k_{11}=\kappa\cdot\kappa\ ,k_{12}=\kappa\cdot p,\\&k_{13}=\kappa\cdot\omega\ ,k_{22}=p\cdot p,\\&k_{23}=p\cdot\omega\ ,k_{33}=\omega\cdot\omega,\\&\cdots\cdots\end{aligned}\right\} \tag{7}$$

再将式(4)～(7)代入式(3),整理化简为含有 $x,y,t,\sinh(\theta),\cosh(\theta)$ 的幂次多项式.并且令 x,$y,t,\sinh(\theta),\cosh(\theta)$ 各次幂前的系数为 0,得到一组代数方程组,从而求解出以下参数关系式:

$$\left.\begin{aligned}&k_{13}=\frac{5(k_{11}k_{22}-2k_{12}^2)}{k_{11}},\\&k_{23}=-\frac{5k_{12}k_{22}}{k_{11}},k_{30}=\frac{5(k_{13}k_{22}-2k_{12}k_{20})}{k_{11}},k_{33}=\frac{25k_{22}^2}{k_{11}},\\&f_0=-k_{00}+\frac{3k_0^2k_1^6k_{11}+3k_0^2k_1^3k_{11}k_2+4k_{10}^2k_{22}-8k_{10}k_{12}k_{20}+12k_{11}^2k_{12}+4k_{11}k_{20}^2}{4(k_{11}k_{22}-k_{12}^2)},\\&k_1^2=\frac{2(-k_{12}\pm\sqrt{k_{11}k_{22}})}{3\ k_{11}},k_2=\frac{k_1(k_1^2k_{11}+2k_{12})}{2k_{11}},k_3=\frac{5k_2^2-5k_1^3k_2-k_1^6}{k_1}\end{aligned}\right\} \tag{8}$$

其中需要保证两个限制条件:

$$\left.\begin{aligned}&k_{11}\neq 0,\\&k_{11}k_{22}-k_{12}^2\neq 0\end{aligned}\right\} \tag{9}$$

最后将式(8)代入式(4)～(5),并利用式(2),就得到了(2+1)维 SK 方程的怪波解.

此外,为了通过图像来观察该类型怪波的运动情况,取式(8)中 $k_1^2=\frac{2(-k_{12}+\sqrt{k_{11}k_{22}})}{3k_{11}}$,然后选择如下参数:

$$\left.\begin{aligned}&k_{00}=1,k_{10}=2,k_{11}=2,k_{12}=1,\\&k_{20}=1,k_{22}=2,k_0=5,k_4=1\end{aligned}\right\} \tag{10}$$

得到

$$f=2x^2+2y^2+50t^2+2xy+10xt-10yt+4x+2y+10t+\frac{133}{18}+5\cosh\left(\frac{\sqrt{3}}{3}x+\frac{2\sqrt{3}}{9}y-\frac{\sqrt{3}}{3}t+1\right) \tag{11}$$

再运用式(2),画出共振怪波其不同时刻的 3D 图(图 1)和密度图(图 2).

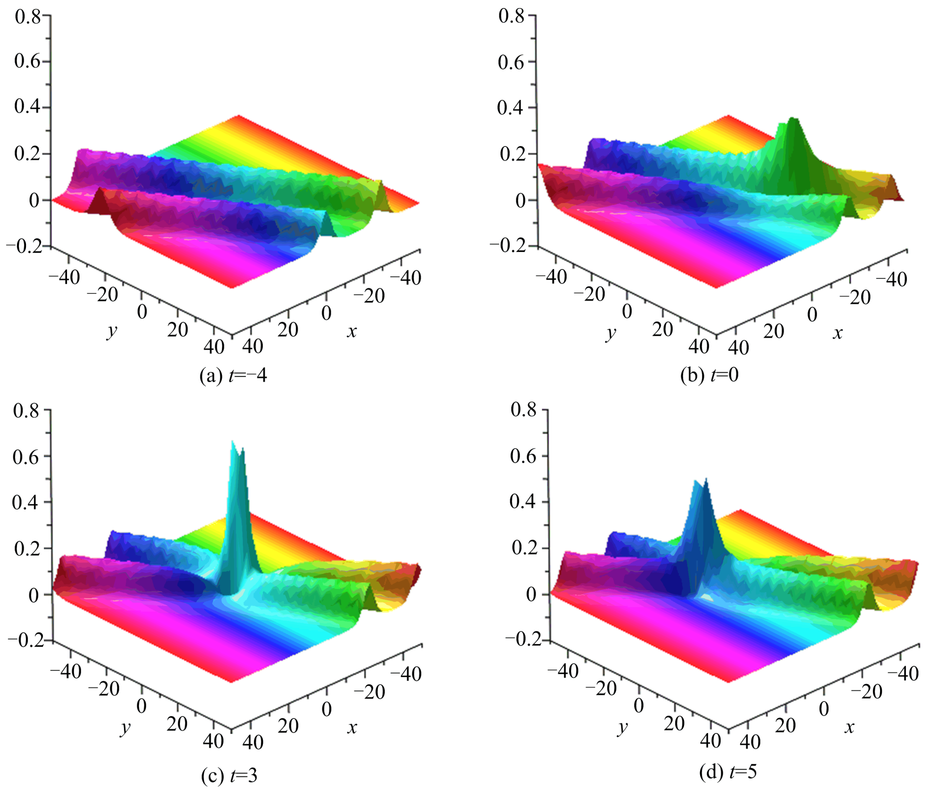


图 1 共振怪波的 3D 图

Fig. 1 Three-dimensional view of the resonance rogue wave

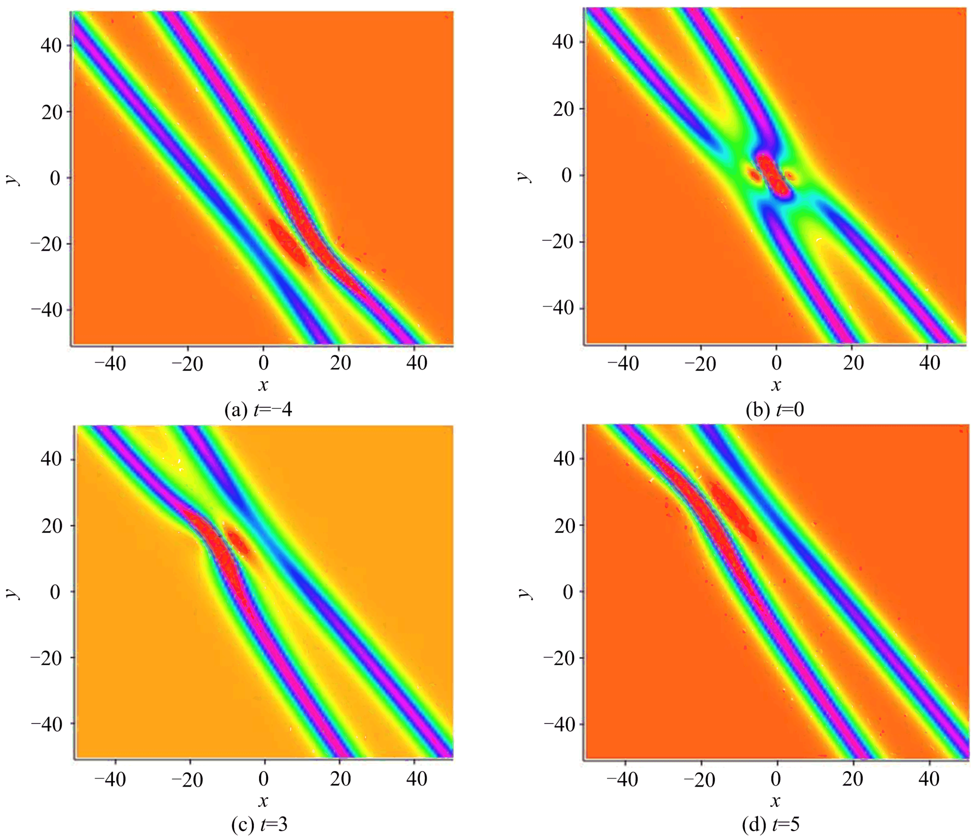


图 2 共振怪波的密度图

Fig. 2 Density map of the resonance rogue wave

从图 1 和图 2 可以看出，lump 型孤子在双条纹孤子背景下忽然以极大的峰值出现，又很快消失．这符合怪波的特征．

针对 lump 型孤子在双条纹孤子作用下表现出来的瞬态性特征可以作如下的解释．在式（4）中将 $\cosh(\theta)$ 写成 $\frac{e^{\theta}+e^{-\theta}}{2}$ 形式，有

$$f=\xi^2+f_0+\frac{k_0}{2}(e^{\theta}+e^{-\theta}) \tag{12}$$

式中，ξ^2 作为幂函数在 $\theta\to\pm\infty$ 没有作为指数函数 $e^{\pm\theta}$ 增长快，f 整体表现为指数函数．于是当 $\theta\to\pm\infty$ 时，u 表现为孤子解．而在 $\theta\to 0$ 时，$e^{\pm\theta}\to 1$，所以 f 整体就表现为 ξ^2+f_0 所代表的 lump 解，于是 u 表现为 lump 型孤子．但是当 $\theta\nrightarrow 0$ 时，lump 型孤子就会立刻消失，所以才有上述怪波现象的发生．

2　怪波的运动轨迹和存在时间

为了方便研究共振怪波的运动轨迹，在解的表达式中将代表双条纹孤子的双曲函数部分排除，只单独保留代表怪波的有理函数解的部分，即在式（4）和式（8）中取 $k_0=0$，得

$$\begin{aligned}f=&k_{11}x^2+k_{22}y^2+\frac{25k_{22}^2}{k_{11}}t^2+2k_{12}xy-\frac{10\,k_{12}k_{22}}{k_{11}}yt+\\&\frac{10\,(k_{11}k_{22}-2k_{12}^2)}{k_{11}}xt+2k_{10}x+2k_{20}y+\frac{10\,(k_{13}k_{22}-2k_{12}k_{20})}{k_{11}}t+\\&\frac{k_{10}^2k_{22}-2k_{10}k_{12}k_{20}+3k_{11}^2k_{12}+k_{11}k_{20}^2}{k_{11}k_{22}-k_{12}^2}\end{aligned} \tag{13}$$

再利用式（2）得到（2＋1）维 SK 方程的 lump 解：

$$u=\frac{12k_{11}}{f}-\frac{24\,(k_{11}x+k_{12}y+k_{13}t+k_{10})^2}{f^2} \tag{14}$$

在式（14）中分别对 x，y 求偏导数并且让偏导数为 0，得到 3 个驻点的坐标：

$$x_1=-\frac{5k_{22}t}{k_{11}}+\frac{k_{12}k_{20}-k_{10}k_{22}}{k_{11}k_{22}-k_{12}^2} \tag{15}$$

$$y_1=\frac{10k_{12}t}{k_{11}}+\frac{k_{10}k_{12}-k_{11}k_{20}}{k_{11}k_{22}-k_{12}^2} \tag{16}$$

$$x_2=-\frac{5k_{22}t}{k_{11}}+\frac{k_{12}k_{20}-k_{10}k_{22}}{k_{11}k_{22}-k_{12}^2}+3\sqrt{\frac{k_{11}k_{13}}{k_{11}k_{22}-k_{12}^2}} \tag{17}$$

$$y_2=\frac{10k_{12}t}{k_{11}}+\frac{k_{10}k_{12}-k_{11}k_{20}}{k_{11}k_{22}-k_{12}^2} \tag{18}$$

$$x_3=-\frac{5k_{22}t}{k_{11}}+\frac{k_{12}k_{20}-k_{10}k_{22}}{k_{11}k_{22}-k_{12}^2}-3\sqrt{\frac{k_{11}k_{13}}{k_{11}k_{22}-k_{12}^2}} \tag{19}$$

$$y_3=\frac{10k_{12}t}{k_{11}}+\frac{k_{10}k_{12}-k_{11}k_{20}}{k_{11}k_{22}-k_{12}^2} \tag{20}$$

通过计算可得，在驻点 (x_1, y_1) 处的 Hessian 矩阵和 u_{xx} 分别为

$$\Delta=\left|\begin{matrix}\frac{\partial^2u(x,y)}{\partial x^2} & \frac{\partial^2u(x,y)}{\partial x\partial y}\\ \frac{\partial^2u(x,y)}{\partial x\partial y} & \frac{\partial^2u(x,y)}{\partial y^2}\end{matrix}\right|_{(x_1,y_1)}=\frac{64(k_{11}k_{22}-k_{12}^2)^5}{3k_{12}^4k_{11}^6}>0 \tag{21}$$

$$u_{xx}(x_1,y_1)=-\frac{8(k_{11}k_{22}-k_{12}^2)^2}{k_{11}^2k_{12}^2}<0 \tag{22}$$

而在驻点 $(x_2 y_2)$，(x_3, y_3) 处 Hessian 矩阵和 u_{xx} 分别为

$$\Delta=\left|\begin{matrix}\frac{\partial^2u(x,y)}{\partial x^2} & \frac{\partial^2u(x,y)}{\partial x\partial y}\\ \frac{\partial^2u(x,y)}{\partial x\partial y} & \frac{\partial^2u(x,y)}{\partial y^2}\end{matrix}\right|_{(x_i,y_i)}=\frac{(k_{11}k_{22}-k_{12}^2)^5}{12k_{12}^4k_{11}^6}>0,\ i=2,3 \tag{23}$$

$$u_{xx}(x_i,y_i)=\frac{(k_{11}k_{22}-k_{12}^2)^2}{4k_{11}^2k_{12}^2}>0,\ i=2,3 \tag{24}$$

所以 u 在 (x_1, y_1) 位置处有最大值，即波峰高度为

$$H=u_{\max}=\frac{4(k_{11}k_{22}-k_{12}^2)}{k_{11}k_{12}} \tag{25}$$

然后在式（15）～（16）中，对时间 t 求导以及把坐标表达式联立消去时间 t 得到波峰运动速度与波峰的运动轨迹方程分别为

$$\left.\begin{aligned}&v_x=-\frac{5k_{22}}{k_{11}},\ v_y=\frac{10k_{12}}{k_{11}},\\&v=\sqrt{{v_x}^2+v_y^2}=5\,\frac{\sqrt{4k_{12}^2+k_{22}^2}}{k_{11}}\end{aligned}\right\} \tag{26}$$

$$y=-\frac{2k_{12}x}{k_{22}}-\frac{k_{10}k_{12}k_{22}+k_{11}k_{20}k_{22}-2k_{12}^2k_{20}}{k_{22}(k_{11}k_{22}-k_{12}^2)} \tag{27}$$

选择参数与式（10）相同，代入式（14）和（25）～（27）得到

$$u_1=6\frac{y^2-2x^2+25t^2-2xy-10xt-20yt-4x-2y-10t+2}{(x^2+y^2+25t^2+xy+5xt-5yt+5t+2x+y+3)^2} \tag{28}$$

$$H=u_{\max}=6 \tag{29}$$

$$v_x=5,v_y=-5,v=5\sqrt{2} \tag{30}$$

$$y=-x-1 \tag{31}$$

并画出 lump 型孤子(怪波)运动的轨迹图,如图 3 所示.

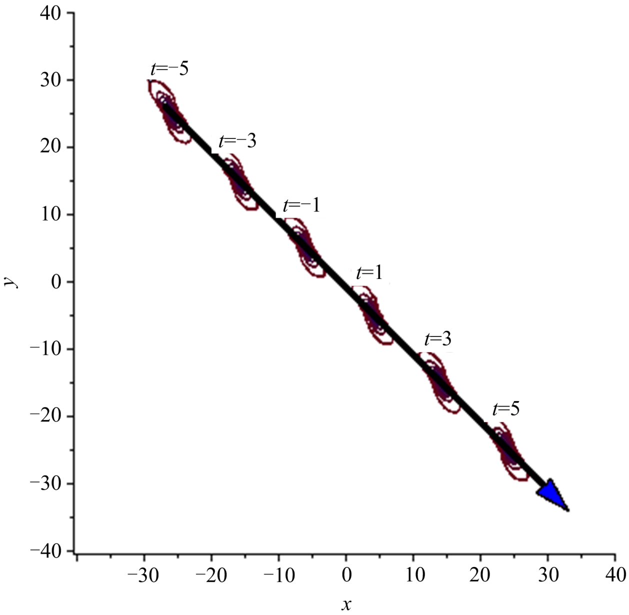


图 3 lump 型孤子(怪波)的不同时刻位置与运动轨迹图

Fig. 3 A map of locations and motion tracks of a lump soliton (a rogue wave) at different moments

将怪波的轨迹方程式(31)代入式(11)中,并运用式(2)得到

$$u_2=\frac{6\left(4+\frac{5}{3}\cosh\left(\frac{\sqrt{3}}{9}x-\frac{\sqrt{3}}{3}t-\frac{2\sqrt{3}}{9}+1\right)\right)}{2x^2+50t^2+20xt+4x+20t+\frac{133}{18}+5\cosh\left(\frac{\sqrt{3}}{9}x-\frac{\sqrt{3}}{3}t-\frac{2\sqrt{3}}{9}+1\right)}-$$
$$\frac{6\left(2x+10t+2+\frac{5}{\sqrt{3}}\sinh\left(\frac{\sqrt{3}}{9}x-\frac{\sqrt{3}}{3}t-\frac{2\sqrt{3}}{9}+1\right)\right)^2}{\left(2x^2+50t^2+20tx+4x+20t+\frac{133}{18}+5\cosh\left(\frac{\sqrt{3}}{9}x-\frac{\sqrt{3}}{3}t-\frac{2\sqrt{3}}{9}+1\right)\right)^2} \tag{32}$$

然后画出不同时刻的图像(图 4).从图 4 可以看出当怪波从左条纹孤子运动到右条纹孤子后,最高峰也就从左条纹孤子转移到右条纹孤子.于是将 $t=-15$ 代入式(32),并通过 $u_x=0$ 得到条纹孤子的最大高度 ,$h=0.522\ 917\ 341\ 6$.因此在式(32)中令 ,$u_2=0.522\ 917\ 341\ 6$,画出 x-t 图像,如图 5 所示.

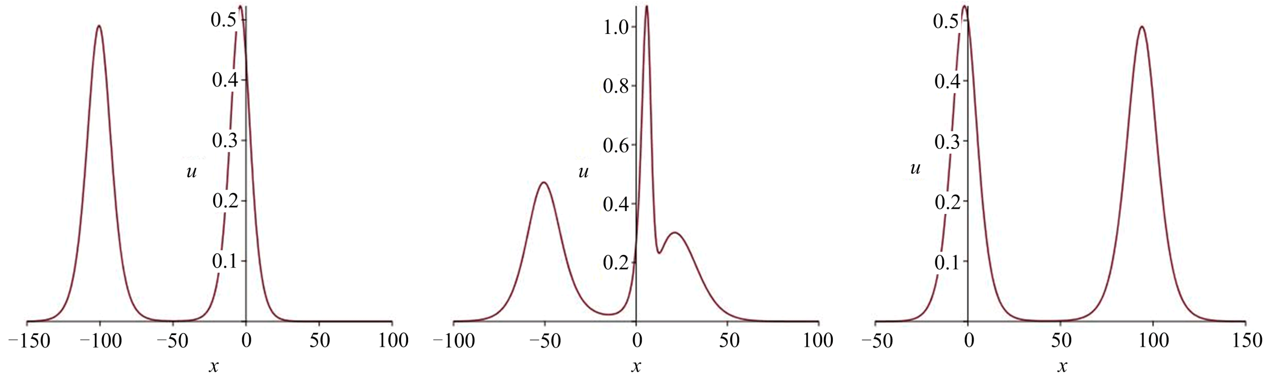


图 4　共振怪波不同时刻的 x-u 图像

Fig. 4　x-u images of resonance rogue waves at different times

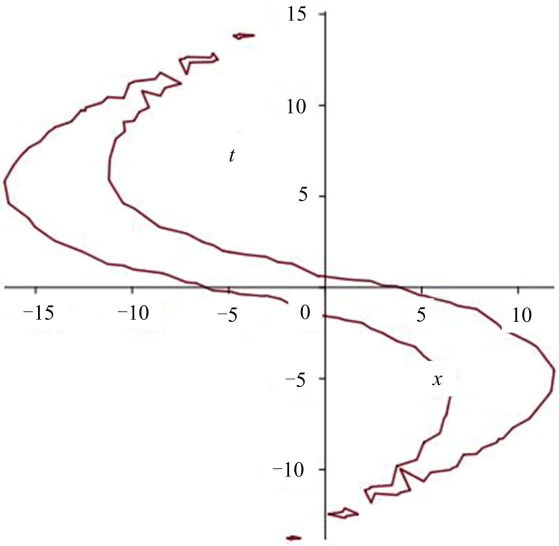
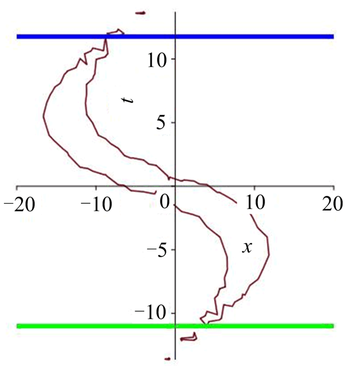


图 5　$u_2=0.5229173416$ 时 x-t 图像

Fig. 5　The graph of x-t when $u_2=0.5229173416$

从图 5 中可以看出共振怪波大致出现在 $t=-11$，而消失在 $t=11$ 左右，然后通过作图法(图 6)更精确地得到怪波存在的时间为

$$t=11.7568310-(-11.0015028)=22.7583338 \tag{33}$$



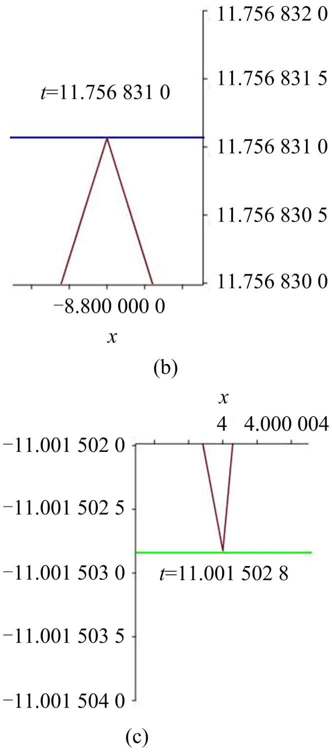


图 6　通过不断试探画出时间图像与图 5 交汇，得到怪波出现与消失的时刻

Fig. 6　The time when the wave appeared and disappeared was obtained by drawing the intersection of the time image and Fig. 5

3　怪波的有效面积和有效体积

孤立子作为非线性系统中最重要的基本激发，在 FPU 实验[24]中可以看成分布在有限范围内携带能量的振动模式. 于是类比于振子的共振曲线的半宽度概念[25]，定义这种新型怪波的有效面积为其峰值高度一半所对应的面积.

将 $t=0$ 代入式(28)中，得到

$$u_3=6\frac{y^2-2x^2-2xy-4x-2y+2}{(x^2+y^2+xy+2x+y+3)^2} \quad (34)$$

因为峰值为 6，所以将 $u_3=3$ 代入式(34)中，得到

$$6\frac{y^2-2x^2-2xy-4x-2y+2}{(x^2+y^2+xy+2x+y+3)^2}=3 \quad (35)$$

通过有效面积方程式(35)计算出有效面积以及对应的有效体积分别为

$$S_{\text{eff}}=3.723219159 \quad (36)$$

$$V_{\text{eff}}=15.66988572 \quad (37)$$

然后可以画出有效面积和有效体积的截面图像，如图 7 所示. 从图 7(a)中可以看出深蓝色的椭圆形区域就是代表着共振怪波的有效面积，而在图 7(b)中从高耸的怪波沿 y 轴的分布可以观察到整个波峰截面几乎被上文中所定义的粉红色的有效面积截面所占据. 这说明本文通过半宽度所定义的怪波的有效面积和有效体积的正确性.

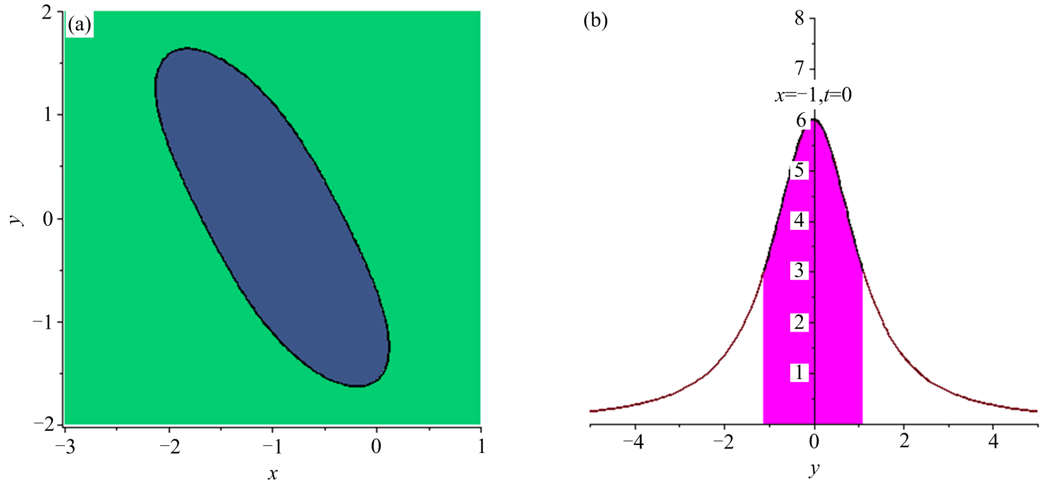


图 7 (a)在 x-y 平面中的有效面积，(b)有效体积在 y 轴方向上的截面

Fig. 7 The effective area in the x-y plane (a) and the cross section of the effective volume in the y-axis direction (b)

4 结论

本文通过 Hirota 双线性方法探究共振怪波，该怪波的形成源于 lump 型孤子在双条纹孤子的作用下，只能在短时间内可见. 针对这种现象，可以解释为，由于时间的变化导致式(12) 中 θ 的不同，造成 f 中主要部分的变化. 并且求出了怪波的若干特征量，尤其是存在时间、有效面积、有效体积，这是以往文献[22,26-29]没有讨论过的，这对今后研究怪波出现时间、持续时长以及所含能量大小的计算都有指导性的意义. 需要指出的是，本文没有对怪波高度随时间的变化规律做具体细致的研究，希望在以后的研究中进行这方面的探讨.

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