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On 2-extra edge connectivity of folded crossed cube

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Abstract: The g-extra edge connectivity is an important parameter in measuring the reliability and fault tolerance of large interconnection networks. Let G be a connected graph and an integer $g\geqslant 0$, the g-extra edge connectivity of G, denoted by $\lambda_g(G)$, is the minimum cardinality of a set of edges of G, if it exists, whose deletion disconnects G and leaves each remaining component to have at least $g+1$ vertices. Note that $\lambda_0(G)=\lambda(G)$ and $\lambda_1(G)$ is the super edge connectivity of G. The n-dimensional folded crossed cube FCQ_n is obtained from the crossed cube CQ_n by adding extra 2^{n-1} edges. Here it was proved that $\lambda_2(FCQ_n)=3n-1$ for $n\geqslant 5$.

Key words: crossed cube; folded crossed cube; g-extra edge connectivity; interconnection network

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折叠交叉立方体的2-外边连通度

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摘要:g-外边连通度是衡量大型互连网络可靠性和容错性的一个重要参数. 设G是连通图且g是非负整数,如果G中存在某种边子集使得G删除这种边子集后得到的图不连通并且每个分支至少有$g+1$个点,则所有这种边子集中基数最小的边子集的基数称为图G的g-外边连通度,记作$\lambda_g(G)$. 由定义可知$\lambda_0(G)=\lambda(G)$并且$\lambda_1(G)$是图G的超边连通度. n维折叠交叉立方体FCQ_n是由交叉立方体CQ_n增加2^{n-1}条边后所得. 证明了$\lambda_2(FCQ_n)=3n-1$,$n\geqslant 5$.

关键词:交叉立方体;折叠交叉立方体;g-外边连通度;互连网络

0 Introduction

We consider only finite, undirected, simple and connected graphs. For the graph definitions and notation we follow Ref.[1].

Let $G=(V,E)$ be a graph and $K\subset V(G)$, the neighborhood(neighbors) of K in G, denoted by $N_G(K)$, is the set of vertices in G adjacent to

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some vertex in K. The edge neighborhood (neighboring edges) of K in G, denoted by $NE_G(K)$, is the set of edges in G incident to some vertex in K. In particular, if K consists of one vertex v, we omit the brackets, and let $d_G(v)=|N_G(v)|$ be the degree of v in G and $NE_G[v]=NE_G(v)\cup\{v\}$. If no ambiguity can arise, we omit the indices and just write $d(v)$, $N(v)$ and $N[v]$, etc.

The underlying topology of an interconnection network is usually presented by a connected graph, where the vertices represent the processors and the edges represent the communication links in the network[2]. The classical connectivity $\kappa(G)$ and edge connectivity $\lambda(G)$ are the two significant factors for measuring the reliability and fault tolerance of the network[3]. However, these two parameters have an obvious deficiency, that is, they tacitly assume that either all vertices adjacent to or all edges incident with the same vertex of G can potentially fail simultaneously, which is almost unlikely to happen in the practical applications of networks. Consequently, these two measurements are inaccurate for large-scale processing systems in which all processors adjacent to or all links incident with the same processor cannot fail at the same time. To address this deficiency, it would seem natural to generalize the notion of the classical connectivity (resp. edge connectivity) by imposing some conditions or restrictions on the components of G-S. Harary[4] first considered this problem by introducing the concept of the conditional connectivity (resp. edge connectivity).

Let G be a graph, and $\mathscr{P}$ a graph-theoretic property. Harary[4] defined the conditional connectivity $\kappa(G:\mathscr{P})$ (resp. edge connectivity $\lambda(G:\mathscr{P})$) as the minimum cardinality of a set of vertices (resp. edges), if any, whose deletion disconnects G and every remaining component has property $\mathscr{P}$. Subsequently, Fábrega and Fiol[5-6] investigated the following kind of conditional connectivity (resp. edge connectivity).

A vertex cut S (resp. edge cut) of G is a vertex (resp. edge) set of G such that G-S is disconnected or trivial. A vertex cut S (resp. edge cut) is called an R_g-vertex cut (resp. R_g-edge cut), where g is a non-negative integer, if every component of G-S has at least $g+1$ vertices. If G has at least one R_g-vertex cut (resp. R_g-edge cut), the g-extra connectivity (resp. g-extra edge connectivity) of G, denoted $\kappa_g(G)$ (resp. $\lambda_g(G)$), is then defined as the minimum cardinality over all R_g-vertex cuts (resp. R_g-edge cut) of G. Clearly, $\kappa_0(G)=\kappa(G)$ and $\lambda_0(G)=\lambda(G)$ for any graph G that is not a complete graph. Thus, the g-extra connectivity (resp. g-extra edge connectivity) can be regarded as a generalization form of the classical connectivity and can provide more accurate measures for the reliability and the tolerance of a large-scale parallel processing system. During the last two decades, among a large number of results on a variety kind of conditional connectivity measures of networks (graphs), many research works on the g-extra connectivity (resp. g-extra edge connectivity) of networks (graphs) have been reported, please refer to Refs. [7-27] for more details.

The n-dimensional crossed cube CQ_n[28-30] and n-dimensional folded hypercube FQ_n[31] are regarded as two of the most versatile and efficient interconnection networks for parallel computer systems. Based on the n-dimensional crossed cube and folded hypercube, an interconnection network called folded crossed cube, denoted by FCQ_n, was introduced in Refs. [32-33]. Many properties of folded crossed cube have been studied. It turns out that FCQ_n is a high-performance-low-cost architecture with some appealing properties such as short diameter, short mean internode distance and very low message traffic density. Very recently, Pai et al. studied the vertex-transitivity of FCQ_n[34]. For the details of the researches on FCQ_n see Refs. [7,32-34].

Previous results have shown that $\kappa(FCQ_n)=\kappa_0(FCQ_n)=\lambda(FCQ_n)=\lambda_0(FCQ_n)=n+1$ for $n\geqslant$

2 in Ref. [32]. In Ref. [7], we determined $\kappa_1(FCQ_n)=\lambda_1(FCQ_n)=2n$ for $n\geqslant 4$. In this paper, we will show that $\lambda_2(FCQ_n)=3n-1$ for $n\geqslant 5$.

1 Definitions and lemmas

Definition 1.1 Two binary strings $x=x_1x_0$ and $y=y_1y_0$ are pair related, denoted by $x\sim y$, if and only if $(x,y)\in\{(00,00),(10,10),(01,11),(11,01)\}$. Otherwise, they are not pair related, denoted by $x\nsim y$.

The vertex set of an n-dimensional crossed cube CQ_n is $\{b_{n-1}b_{n-2}\cdots b_0 \mid b_i\in\{0,1\},0\leqslant i\leqslant n-1\}$, and CQ_n is defined inductively as follows.

Definition 1.2[28] CQ_1 is the complete graph on two vertices with labels 0 and 1. For $n\geqslant 2$, CQ_n consists of two subcubes CQ_{n-1}^0 and CQ_{n-1}^1 such that the $(n-1)$th bit of every vertex in CQ_{n-1}^0 and CQ_{n-1}^1 is 0 and 1, respectively. Two vertices $u=0u_{n-2}\cdots u_0\in V(CQ_{n-1}^0)$ and $v=1v_{n-2}\cdots v_0\in V(CQ_{n-1}^1)$ are adjacent in CQ_n if and only if ① $u_{n-2}=v_{n-2}$ if n is even, and ② for $0\leqslant i<\lfloor\frac{n-1}{2}\rfloor$, $u_{2i+1}u_{2i}\sim v_{2i+1}v_{2i}$.

From the above definition, it is immediate to get the following observation.

Observation 1.1[28] Two vertices $u=u_{n-1}u_{n-2}\cdots u_0$ and $v=v_{n-1}v_{n-2}\cdots v_0$ of CQ_n are adjacent if and only if there exists an integer l, with $1\leqslant l\leqslant n$, such that the following four conditions are satisfied: ①$u_{n-1}\cdots u_l=v_{n-1}\cdots v_l$; ② $u_{l-1}\neq v_{l-1}$; ③$u_{l-2}=v_{l-2}$ if l is even; ④$u_{2i+1}u_{2i}\sim v_{2i+1}v_{2i}$ for $0\leqslant i<\lfloor\frac{l-1}{2}\rfloor$.

By definition, CQ_n is an n-regular graph with 2^n vertices and $n2^{n-1}$ edges. In addition, CQ_n $(n\geqslant 2)$ can be partitioned into two sub-crossed cubes CQ_{n-1}^0 and CQ_{n-1}^1, where CQ_{n-1}^i is the subgraph of CQ_n induced by $\{iu_{n-2}\cdots u_0\}$ for $i\in\{0,1\}$. Note that CQ_{n-1}^0 and CQ_{n-1}^1 are connected by a perfect matching.

Definition 1.3 The n-dimensional folded crossed cube, denoted by FCQ_n, is the graph obtained from the crossed cube CQ_n by adding an edge between any two complementary vertices $x=x_{n-1}x_{n-2}\cdots x_0$ and $\overline{x}=\overline{x}_{n-1}\overline{x}_{n-2}\cdots\overline{x}_0$, where $\overline{x}_i=1-x_i$. These edges, denoted by $\overline{M}$, are called complementary edges of FCQ_n, and the edges in $FCQ_n-\overline{M}$ are called cross edges.

Fig. 1 shows CQ_3 and FCQ_3. By definition, FCQ_n is $(n+1)$-regular with 2^n vertices and $(n+1)2^{n-1}$ edges.

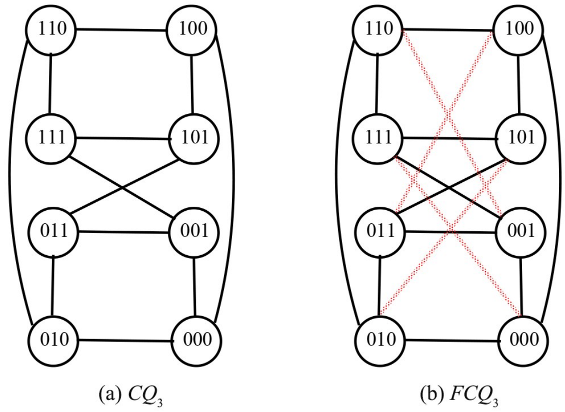

Fig. 1 CQ_3 **and** FCQ_3

If two vertices u and v in FCQ_{n+1} are adjacent by cross edges and their leftmost differing bit position is i, $0\leqslant i\leqslant n$, we say that they are adjacent along dimension i. We also say that v is the i-neighbor of u, denoted by $v=u_i$. Similarly, we use u_{ij} to denoted the j-neighbor of u_i. Clearly, $u_{ii}=u$. The edge $(u,u_i)\in E(FCQ_{n+1}-\overline{M})$, denoted by $e_i(u)$, is referred to as a i-edge in $FCQ_{n+1}-\overline{M}$ incident with vertex u. We use $\overline{e}(u)\in\overline{M}$ to denoted the edge in $\overline{M}$ incident with u.

Note that in an $(n+1)$-dimensional folded crossed cube FCQ_{n+1} the two isomorphic n-dimensional crossed cubes $L=CQ_n^0$ and $R=CQ_n^1$ with respectively the prefix bits 0 and 1 in each vertex, are connected by two perfect matchings $\overline{M}$ and M_n, where $M_n=\{(u,u_n)\in E(FCQ_{n+1})\mid u\in V(L), u_n\in V(R)\}$. We write $FCQ_{n+1}=L\oplus R$.

Lemma 1.1[30] $\kappa_1(CQ_n)=\lambda_1(CQ_n)=2n-2$ for $n\geqslant 3$.

Lemma 1.2[3] CQ_n contains no triangle for $n\geqslant 3$.

Ref. [7] proved the following lemma.

Lemma 1.3[7] FCQ_n contains no triangle for $n\geqslant 4$.

For the sake of convenience, in the following discussion, we consider FCQ_{n+1} rather than FCQ_n.

2 Main results

Lemma 2.1 Let K be a set of edges in $FCQ_{n+1}=L\oplus R$, where $L=CQ_n^0$ and $R=CQ_n^1$, and let $K_L=K\cap E(L)$, $K_R=K\cap E(R)$, $K_{M_n}=K\cap M_n$ and $K_{\overline{M}}=K\cap\overline{M}$. If $|K|\leqslant 3n+1$ and there are neither isolated vertices nor isolated edges in $FCQ_{n+1}-K$, then every vertex in $L-K_L$ (resp. $R-K_R$) is connected to a vertex in $R-K_R$ (resp. $L-K_L$).

Proof Let $u\in V(L-K_L)$. If $e_n(u)\notin K_{M_n}$ or $\bar{e}(u)\notin K_{\overline{M}}$, then we are done. Hence, suppose that $e_n(u)\in K_{M_n}$ and $\bar{e}(u)\in K_{\overline{M}}$. Let $I_n=\{0,1,2,\cdots,n-1\}$. Set $A=\{e_i(u),\ e_n(u_i)\mid i\in I_n\}\cap K$. If $|A|<n$, then there is some i such that $e_i(u)\notin K$ and $e_n(u_i)\notin K$, then we are done. Hence, suppose that $|A|\geqslant n$. Because $FCQ_{n+1}-K$ contains no isolated vertices, there is some $p\in I_n$ such that $e_p(u)\notin K$. If $e_n(u_p)\notin K$ or $\bar{e}(u_p)\notin K$, we are done. Hence, suppose that $e_n(u_p)\in K$ and $\bar{e}(u_p)\in K$. Let $B=\{e_j(u_p),\ e_n(u_{pj})\mid j\in I_n-\{p\}\}\cap K$. If $|B|<n-1$, then there is some $j\in I_n-\{p\}$ such that $e_j(u_p)\notin K$ and $e_n(u_{pj})\notin K$, we are done. Hence, suppose that $|B|\geqslant n-1$. Because $FCQ_{n+1}-K$ contains no isolated edges, there is some q such that $e_q(u_p)\notin K$. If $e_n(u_{pq})\notin K$ or $\bar{e}(u_{pq})\notin K$, then we are done. Hence, suppose that $e_n(u_{pq})\in K$ and $\bar{e}(u_{pq})\in K$. For convenience, let $K'=\{e_n(u),\bar{e}(u),e_n(u_p),\bar{e}(u_p),e_n(u_{pq}),\bar{e}(u_{pq})\}$. Now let $C=\{e_r(u_{pq}),\ e_n(u_{pqr})\mid r\in I_n-\{p,q\}\}$. Then

$$|C\cap K|\leqslant|K-(A\cup B\cup K')|\leqslant(3n+1)-n-(n-1)-6=n-4.$$

Because there are $n-2$ pairs of edges in C, so there exists a pair of edges $e_r(u_{pq})$, $e_n(u_{pqr})$ $(r\in I_n-\{p,q\})$ which is not in K. Hence, this implies u is connected to some vertex in $R-K_R$.

In the same way, we can prove that every vertex in $R-K_R$ is connected to a vertex in $L-K_L$.

Theorem 2.1 $\lambda_2(FCQ_{n+1})=3n+2$ for $n\geqslant 4$.

proof Suppose $n\geqslant 4$. Let P_3 be a path of length two in FCQ_{n+1}. By the definition of FCQ_{n+1}, we may know that $|NE_{FCQ_{n+1}}(P_3)|=3n+2$. Let $Y=FCQ_{n+1}-V(P_3)$. Because $|V(P_3)|=3$ and $\kappa(FCQ_{n+1})=n+2\geqslant 6$, Y is connected. Furthermore, $|V(Y)|=2^{n+1}-3\geqslant 3$. Therefore, $NE_{FCQ_{n+1}}(P_3)$ is a R_2-edge cut of FCQ_{n+1} implying $\lambda_2(FCQ_{n+1})\leqslant 3n+2$ for $n\geqslant 4$.

Next, we will show that $\lambda_2(FCQ_{n+1})\geqslant 3n+2$ for $n\geqslant 4$. Let K be an arbitrary set of edges in FCQ_{n+1} such that $|K|=3n+1$ and there are neither isolated vertices nor isolated edges in $FCQ_{n+1}-K$. To prove $\lambda_2(FCQ_{n+1})\geqslant 3n+2$, it suffices to show that $FCQ_{n+1}-K$ is connected. Let $FCQ_{n+1}=L\oplus R$.

Let M_i for $i=0,1,\cdots,n-1$ be set of edges in $E(FCQ_{n+1}-(M_n\cup\overline{M}))$ in which the two end-vertices differ in the leftmost i-bit position. Note that $M_0,M_1,\cdots,M_n$ and $\overline{M}$ form a partition of $E(FCQ_{n+1})$. If $|M_i\cap K|\leqslant 2$ for every $i\in\{0,1,\cdots,n\}$ and $|\overline{M}\cap K|\leqslant 2$, then $2(n+2)<3n+1=|K|$ for $n\geqslant 4$. Hence, $|M_i\cap K|\geqslant 3$ for some $i\in\{0,1,\cdots,n\}$ or $|\overline{M}\cap K|\geqslant 3$. In addition, the vertices of FCQ_{n+1} can be labeled such that

$$|K\cap(M_n\cup\overline{M})|\geqslant 3.$$

For convenience, let $K_L=K\cap E(L)$, $K_R=K\cap E(R)$, $K_{M_n}=K\cap M_n$ and $K_{\overline{M}}=K\cap\overline{M}$. Therefore, $|K_L|+|K_R|\leqslant 3n+1-3=3n-2$.

Since $3n-2<4n-4$ for $n\geqslant 3$, $|K_L|<2n-2$ or $|K_R|<2n-2$. Without loss of generality, we suppose that $|K_L|\leqslant|K_R|$. Then $|K_L|<2n-2$. We distinguish the following cases.

Case 1 There are no isolated vertices in $L-K_L$. Then $L-K_L$ contains no isolated edges since $|K_L|<2n-2=\lambda_1(L)=\lambda_1(CQ_n)$. Hence, $L-K_L$ is connected. By using Lemma 2.1, every vertex in $R-K_R$ is connected to a vertex in $L-K_L$. Thus $FCQ_{n+1}-K$ is connected.

Case 2 There is an isolated vertex u in $L-$

K_L. Since $\lambda(L-\{u\})\geqslant\kappa(L-\{u\})\geqslant n-1$ and $|E(L-\{u\})\cap K_L|\leqslant|K_L|-|NE_L(u)|<n-2$, $L-K_L-\{u\}$ is connected. Next, we only need to prove that u is connected to a vertex in $L-K_L-\{u\}$ under $FCQ_{n+1}-K$. Because $FCQ_{n+1}-K$ has no isolated vertices, $e_n(u)\notin K_{M_n}$ or $\bar{e}(u)\notin K_{\bar{M}}$.

Suppose that $e_n(u)\notin K_{M_n}$ and $\bar{e}(u)\notin K_{\bar{M}}$. If $\bar{e}(u_n)\notin K_{\bar{M}}$ or $e_n(\bar{u})\notin K_{M_n}$, we are done. So, suppose that $\bar{e}(u_n)\in K_{\bar{M}}$ and $e_n(\bar{u})\in K_{M_n}$. Let $N_R(u)=\{u_n,\bar{u}\}$ and $S=\{\bar{e}(u_n), e_n(\bar{u})\}$. By definition of FCQ_{n+1} and Lemma 1.3, there are in total $|NE_R(\{u_n,\bar{u}\})|=|NE_R(u_n)|+|NE_R(\bar{u})|=n+n=2n$ pairwise edge-disjoint paths in which each connects $\{u_n,\bar{u}\}$ to $L-K_L-\{u\}$ under $FCQ_{n+1}-S$. However, these paths contain at most $|K|-|K_L|-|S|\leqslant(3n+1)-n-2=2n-1$ edges in K. Hence, u can be connected to a vertex in $L-K_L-\{u\}$ under $FCQ_{n+1}-K$.

Next we suppose that ($e_n(u)\notin K_{M_n}$ and $\bar{e}(u)\in K_{\bar{M}}$) or ($e_n(u)\in K_{M_n}$ and $\bar{e}(u)\notin K_{\bar{M}}$). Without loss of generality, we say the former. If $\bar{e}(u_n)\notin K_{\bar{M}}$, then we are done. Therefore, we suppose that $\bar{e}(u_n)\in K_{\bar{M}}$. Because FCQ_{n+1} contains no isolated edges, u_n has a neighbor v in $R-K_R$. If $e_n(v)=(v,v_n)\notin K_{M_n}$ or $\bar{e}(v)=(v,\bar{v})\notin K_{\bar{M}}$, then we are done. Hence, suppose that $e_n(v)=(v,v_n)\in K_{M_n}$ and $\bar{e}(v)=(v,\bar{v})\in K_{\bar{M}}$. For convenience, $K'=\{\bar{e}(u), \bar{e}(u_n), e_n(v), \bar{e}(v)\}$ and $A=\{u_n, v\}$. By Lemma 1.2 and Definition 1.3, there are in total

$$|NE_R(A)|=|NE_{R-v}(u_n)|+|NE_{R-u_n}(v)|=(n-1)+(n-1)=2n-2$$

pairwise edge-disjoint paths in which each connects A to $L-K_L-\{u\}$ under $FCQ_{n+1}-(K'\cup\{e_n(u)\})$. However, these paths contain at most $|K|-|K_L|-|K'|\leqslant(3n+1)-n-4=2n-3$ edges in K. Hence, u can be connected to a vertex in $L-K_L-\{u\}$ under $FCQ_{n+1}-K$.

3 Conclusion

In this paper, we concentrate on a novel measurement parameter for the reliability and the tolerance of networks: 2-extra edge connectivity $\lambda_2(G)$ of a graph G, which not only compensates for some shortcomings but also generalizes the classical edge connectivity $\lambda(G)$, and so can provide more accurate measurement for the reliability and the tolerance of a large-scale parallel processing system. For the folded crossed cube FCQ_n, a significant variant of the crossed cube CQ_n, we show that $\lambda_2(FCQ_n)=3n-1$ for $n\geqslant5$. In other words, The result determines that at least $3n-1(n\geqslant5)$ edges must be removed to disconnect the n-dimensional folded crossed cube, provided that the removal of these edges does not isolate either a vertex or an edge. Future work will consider the g-extra edge connectivity ($g\geqslant3$) of the folded crossed cube FCQ_n.

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