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Measuring systemic risk contribution with CoGVaR approach

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Abstract: We propose a new conditional risk measure, conditional generalized value-at-risk (CoGVaR), from the perspective of measuring systemic risk. The new class of risk measures is a natural generalization of the conditional quantiles including the classic CoVaR. Compared with the classic conditional value-at-risk (CoVaR) and conditional expectile (CoExpectile), it has more potential application in reality as it takes the risk attitude of the decision maker into consideration, which has not been the focus of much study to date. Using generalized quantile regression approach with state variables added, some calculation results are presented in the Dow Jones U. S. Financials Index case, and it is shown that it provides a new perspective on systemic risk contribution. In addition, the result shows that our risk measure can capture the tail risk by using more convex disutility function.

Keywords: generalized quantile; conditional risk measure; systemic risk contribution

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1 Introduction

The systemic risk is a risk that could trigger severe instability or collapse of an entire industry or economy. It has attracted a lot of interests recently. Usually there exist financial links between the institutions in the financial systems. The caused failure of one institution may spread to other institutions. The contribution of a financial institution to the systemic crisis thus plays an important role in systemic risk measurement and management of the whole financial system. Measuring the contribution of each institution to overall systemic risk can help regulators identify institutions that make significant contributions to systemic risk. With strict constraints on these institutions, the tendency to generate systemic risk can be restrained. Since the constructive work of Ref. [1] with a corisk, conditional value-at-risk (CoVaR), for systemic risk measurement was proposed, many corisks has been discussed for measuring systemic risk.

CoVaR in Ref. [1] described the VaR of the financial system conditional on an institution being in financial distress (the loss of an institution being exactly its VaR). Ref. [1] defined the systemic contribution of a financial institution as the difference between CoVaR conditional on the institution being under distress and CoVaR in the median state. Girardi and Ergün[2] modified the definition of distress in Ref. [1] to no less than its VaR to consider more severe distress events. Huang and Uryasev[3] changed the systemic risk in Ref. [1] from VaR to CVaR to propose a new corisk, CoCVaR. Brownlees and Engle[4] introduced SRISK, a function of the firm's size, leverage and risk, to measure the systemic risk contribution of a financial firm. Acharya et al.[5] proposed the systemic expected shortfall to measure financial institutions' contribution to the systemic risk with an economic model of systemic risk presented in their paper.

Generalized quantile, introduced by Bellini et al.[6], includes VaR and expectiles[7] as special cases. It is a more general class of risk measures with elicitability property satisfied. At the same time, it shares several good properties with the usual quantiles, such as monotonicity and translation invariance. According to past works, we know that such risk measure is closely related to shortfall risk measures[8]. In this paper, we adopt the definition of generalized quantile of a risk X in Ref. [6] as the minimizer of the minimization problem $\min_{x\in\mathbb{R}} \pi_\alpha(X,x)$, where

$$\pi_\alpha(X,x) = \alpha\mathbb{E}[u_1((X-x)^+)] + (1-\alpha)\mathbb{E}[u_2((X-x)^-)].$$

Here, X represents the loss a financial institution face and x is the required capital. $a^+ = \max\{a,0\}$, $a^- = \max\{-a,0\}$, $\alpha \in (0,1)$ is the confidence level to

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balance the shortfall risk $(X-x)^+$ and over-required capital risk $(X-x)^-$. u_1 and u_2, two strictly increasing, convex functions on $\mathbb{R}^+$, are disutility (loss) functions in the expected utility model, they are used to transform the two risks $(X-x)^+$ and $(X-x)^-$ respectively. If $u_1(x)=u_2(x)=x$, then the generalized quantile reduces to the classic quantile (VaR), and if $u_1(x)=u_2(x)=x^2$, the generalized quantile reduces to the expectile. While VaR and expectile are symmetrically viewing the two risks with same disutility functions $u_1=u_2$, the generalized quantile can asymmetrically view them by using different disutility functions.

There is a generally believed point of view in risk management that underestimating is much more disastrous than overestimating. Thus it is natural to use more convex disutility function u_1 than u_2 to transform shortfall risk $(X-x)^+$. For example, the generalized quantile will work well on such an occasion when the decision maker (regulator) is risk averse, i. e., is more concerned about upper-tail realizations of a loss random variable[9]. That is the main occasion that our paper focuses on the systemic risk measurement, to the best of our knowledge, which has not been discussed by other people.

Motivated by past work on the systemic risk, our paper propose a systemic risk measure, CoGVaR, similar to the definition of CoVaR in Ref. [1]. CoGVaR is defined as the GVaR of financial system conditional on some financial institution being under distress. We define systemic risk contribution of a financial institution as ΔCoGVaR, which describes the change from its CoGVaR in its median state to its CoGVaR under distress. We consider disutility functions u_1 and u_2 with the form of

$$u_1(x)=x^a, u_2(x)=x^b, a,b>1.$$

The rest of paper is organized as follows. In Section 2, we first recall the definition and some properties of generalized quantiles that will be used in the context. Next we give the definition of GVaR and CoGVaR and explore the stronger emphasis a GVaR with relatively large value of a may apply on upper tail of a loss. Section 3 illustrates the approach to do estimations. Section 4 presents the Dow Jones U. S. Financials Index case study and Section 5 concludes.

2 GVaR and CoGVaR

Recall that the generalized quantile[6] of a risk X is defined as follows.

$$\underset{x\in\mathbb{R}}{\operatorname{argmin}}\, \pi_\alpha(X,x),$$

where

$$\pi_\alpha(X,x)= \alpha\mathbb{E}[u_1((X-x)^+)] + (1-\alpha)\mathbb{E}[u_2((X-x)^-)] \tag{1}$$

$\alpha\in(0,1)$ and u_1, u_2 are two strictly increasing, convex functions on $\mathbb{R}_+$ with $u_i(0)=0, u_i(1)=1, i=1,2$.

Proposition 2.1 When u_1, u_2 are strictly convex functions, the minimization problem $\min_{x\in\mathbb{R}} \pi_\alpha(X,x)$ has a unique minimizer.

Proof The proof can be seen in Proposition 1 of Ref. [6].

Throughout the paper, we only focus on disutility functions u_1 and u_2 with the form of $u_1(x)=x^a, u_2(x)=x^b, a,b>1$. Let X^i be a random variable representing the loss of a financial institution, $X=(X^1,X^2,\cdots,X^n)$ be a vector of random variables, and X^{sys} be the financial system loss.

Definition 2.1 The generalized quantile with a confidence level α of a financial institution i denoted as $\mathrm{GVaR}_\alpha(X^i)$,

$$\mathrm{GVaR}_\alpha(X^i)= \underset{x\in\mathbb{R}}{\operatorname{argmin}}\ \mathbb{E}[\alpha((X^i-x)^+)^a+(1-\alpha)((X^i-x)^-)^b].$$

Proposition 2.2 GVaR is a convex/coherent risk measure if and only if $a=b=2$ and $\alpha\geqslant 1/2$.

Proof When $a=b=2$, it refers to expectile in Ref. [7], and by Proposition 6 (a) and (b) of Ref. [6], we can conclude that expectile is the only class of GVaRs that are convex/coherent risk measures.

Remark 2.1 Disutility functions u_1 and u_2 with the form of $u_1(x)=x^a, u_2(x)=x^b$, $a,b>1$, become more convex functions $(x\geqslant 1)$ as a and b increase. Here, we want to discuss the stronger emphasis on tail risks through more convex disutility functions u_1, u_2. The GVaR with confidence level α of a risk X' is shown as follows.

$$\mathrm{GVaR}_\alpha(X')= \underset{x\in\mathbb{R}}{\operatorname{argmin}}\{\alpha\mathbb{E}((X'-x)^+)^a + (1-\alpha)\mathbb{E}((X'-x)^-)^b\}.$$

As is often the case, X' obeys heavy-tailed distribution in finance with much probability weight on tail. From the view of a regulator, underestimating is much more disastrous than overestimating, thus $(X'-x)^+$ is much more focused on with large value of α and relatively large value of a, and for fixed b, $\mathrm{GVaR}_\alpha(X')$ increases in general as a increases. Supposing $X'\sim$ Pareto $(3,1)$ with distribution function $F(x)=1-x^{-3}$, $x\geqslant 1$, we calculate the corresponding results with $\alpha=0.9$ in Table 1. It is clear that the larger value of a brings more conservative outcome.

Definition 2.2 The CoGVaR of the financial system is defined as GVaR with confidence level α of X^{sys}, conditional on the event that X is in a measurable set C,

$$\mathrm{CoGVaR}_\alpha^{sys} = \mathrm{GVaR}_\alpha(X^{sys}\mid X\in C)= \underset{x\in\mathbb{R}}{\operatorname{argmin}}\ \mathbb{E}[\alpha((X^{sys}-x)^+)^a + (1-\alpha)((X^{sys}-x)^-)^b \mid X\in C].$$

Table 1. Results of generalized quantiles with $\alpha = 0.9$.

a	Results	
	$b=2.0$	$b=1.5$
3	4.49	7.41
2.7	3.26	4.33
2.5	2.83	3.45
2.1	2.51	2.60
2.0	2.27	2.48
1.7	2.11	2.23
1.5	2.05	2.13
1.1	1.97	2.03

Definition 2.1 and Definition 2.2 hold because u_1 and u_2 are strictly convex functions and by Proposition 2.1, the minimizer is unique.

Theorem 2.1 CoGVaR has the following form:

$$\mathrm{CoGVaR}_\alpha^{\mathrm{sys}} = \inf\{x \in \mathbb{R}: \mathbb{E}\, v[(X^{\mathrm{sys}} - x) \mid X \in C] \leqslant 0\},$$

where

$$v(x) = \begin{cases} \alpha a x^{a-1}, & x > 0; \\ -(1-\alpha) b (-x)^{b-1}, & x \leqslant 0. \end{cases}$$

Proof It follows directly from Ref. [6, Proposition 1 (c)] about the proposition of minimizers of $\min\limits_{x \in \mathbb{R}} \pi_\alpha(X, x)$.

Similar to the method in Ref. [1], the contribution to systemic risk of financial institution i can be measured as the difference between the GVaR of X^{sys} conditional on the distress of a particular financial institution i (X^i is at its GVaR_α) and the GVaR of X^{sys} conditional on the median state of the institution i (X^i is at its median $\mathrm{GVaR}_{0.5}$). We define financial institution i's contribution to the financial system, $\Delta\mathrm{CoGVaR}_\alpha^{\mathrm{sys}|i}$, by

$$\begin{aligned} &\Delta\mathrm{CoGVaR}_\alpha^{\mathrm{sys}|i} = \\ &\mathrm{CoGVaR}_\alpha^{\mathrm{sys}|X^i = \mathrm{GVaR}_\alpha^i} - \mathrm{CoGVaR}_\alpha^{\mathrm{sys}|X^i = \mathrm{GVaR}_{0.5}^i} = \\ &\mathrm{GVaR}_\alpha(X^{\mathrm{sys}} \mid X^i = \mathrm{GVaR}_\alpha^i) - \\ &\mathrm{GVaR}_\alpha(X^{\mathrm{sys}} \mid X^i = \mathrm{GVaR}_{0.5}^i). \end{aligned}$$

3 Methodology for estimation

3.1 Generalized quantile regression in static setting

Consider the following regression models:

$$X^i = \beta_1^i + \varepsilon^i,$$
$$X^{\mathrm{sys}} = \beta_1^{\mathrm{sys}} + \beta_2^{\mathrm{sys}} X^i + \varepsilon^{\mathrm{sys}}.$$

The coefficients are estimated based on minimizing a loss function of the form

$$l_\alpha(x) = \alpha((x)^+)^a + (1-\alpha)((x)^-)^b,$$

that is to say we need to solve the following minimization problems

$$\min_\beta \sum_{m=1}^n l_\alpha(\varepsilon_m^i) \text{ and } \min_\beta \sum_{m=1}^n l_\alpha(\varepsilon_m^{\mathrm{sys}}),$$

where $\varepsilon_m^i = X_m^i - \beta_1^i$, $\varepsilon_m^{\mathrm{sys}} = X_m^{\mathrm{sys}} - \beta_1^{\mathrm{sys}} - \beta_2^{\mathrm{sys}} X_m^i$, $\{X_m^i\}_{m=1}^n$ are n observations of X^i and $\{X_m^{\mathrm{sys}}\}_{m=1}^n$ are n observations of X^{sys}.

According to the quadrangle theory in Ref. [10], in a quadrangle including "error", "regret", "deviation", "risk" and a statistics, the statistics can be estimated by the regression with the "error". Let $\mathscr{L} = X^{\mathrm{sys}} - \beta_1^{\mathrm{sys}} - \beta_2^{\mathrm{sys}} X^i$, and the GVaR error function be

$$\mathscr{E}(\mathscr{L}) = \mathbb{E}[l_\alpha(\mathscr{L})],$$

the GVaR statistics be

$$S(\mathscr{L}) = \mathrm{GVaR}(\mathscr{L}).$$

Thus we can use the estimated regression vector $\widehat{\beta}$ to obtain

$$\mathrm{GVaR}_\alpha^i = \widehat{\beta}_1^i,$$

$$\begin{aligned} &\mathrm{CoGVaR}_\alpha^{\mathrm{sys}|i} = \\ &\mathrm{GVaR}_\alpha(X^{\mathrm{sys}} \mid X^i = \mathrm{GVaR}_\alpha^i) = \widehat{\beta}_1^{\mathrm{sys}} + \widehat{\beta}_2^{\mathrm{sys}}\, \mathrm{GVaR}_\alpha^i. \end{aligned}$$

Financial institution i's contribution to the financial system, $\Delta\mathrm{CoGVaR}_\alpha^{\mathrm{sys}|i}$, is

$$\begin{aligned} &\Delta\mathrm{CoGVaR}_\alpha^{\mathrm{sys}|i} = \\ &\mathrm{CoGVaR}_\alpha^{\mathrm{sys}|X^i = \mathrm{GVaR}_\alpha^i} - \mathrm{CoGVaR}_\alpha^{\mathrm{sys}|X^i = \mathrm{GVaR}_{0.5}^i} = \\ &\mathrm{GVaR}_\alpha(X^{\mathrm{sys}} \mid X^i = \mathrm{GVaR}_\alpha^i) - \\ &\mathrm{GVaR}_\alpha(X^{\mathrm{sys}} \mid X^i = \mathrm{GVaR}_{0.5}^i) = \\ &\widehat{\beta}_2^{\mathrm{sys}}(\mathrm{GVaR}_\alpha^i - \mathrm{GVaR}_{0.5}^i). \end{aligned}$$

3.2 Estimation in dynamic setting

To capture time variation, we estimate X^i and X^{sys} as functions of state variables, which allows us to model the evolution of the joint distributions over time. We indicate time-varying $\mathrm{CoGVaR}_{\alpha,t}^i$ and $\mathrm{GVaR}_{\alpha,t}^i$ with a subscript t and estimate the time variation conditional on a vector of lagged state variables M_{t-1}. We consider the following models:

$$X_t^i = \beta_1^i + \beta_2^i M_{t-1} + \varepsilon_t^i,$$
$$X_t^{\mathrm{sys}} = \beta_1^{\mathrm{sys}} + \beta_2^{\mathrm{sys}} X_t^i + \beta_3^{\mathrm{sys}} M_{t-1} + \varepsilon_t^{\mathrm{sys}} \tag{2}$$

The coefficients are estimated based on

$$\min_\beta \sum_{t=1}^T l_\alpha(\varepsilon_t^i) \text{ and } \min_\beta \sum_{t=1}^T l_\alpha(\varepsilon_t^{\mathrm{sys}}),$$

where $\varepsilon_t^i = X_t^i - \beta_1^i - \beta_2^i M_{t-1}$, $\varepsilon_t^{\mathrm{sys}} = X_t^{\mathrm{sys}} - \beta_1^{\mathrm{sys}} - \beta_2^{\mathrm{sys}} X_t^i - \beta_3^{\mathrm{sys}} M_{t-1}$, $\{X_t^i\}_{t=1}^{\mathrm{T}}$ are observations of X^i and $\{X_t^{\mathrm{sys}}\}_{t=1}^{\mathrm{T}}$ are observations of X^{sys}. We then use the estimated regression vectors $\widehat{\beta}$ to obtain

$$\mathrm{GVaR}_{\alpha,t}^i = \widehat{\beta}_1^i + \widehat{\beta}_2^i M_{t-1},$$

$$\begin{aligned} \mathrm{CoGVaR}_{\alpha,t}^{\mathrm{sys}|i} &= \mathrm{GVaR}_{\alpha,t}(X^{\mathrm{sys}} \mid X^i = \mathrm{GVaR}_{\alpha,t}^i) = \\ &\widehat{\beta}_1^{\mathrm{sys}} + \widehat{\beta}_2^{\mathrm{sys}}\, \mathrm{GVaR}_{\alpha,t}^i + \widehat{\beta}_3^{\mathrm{sys}} M_{t-1}. \end{aligned}$$

Finally, $\Delta\mathrm{CoGVaR}_{\alpha,t}^{\mathrm{sys}|i}$ is computed as

$$\Delta\mathrm{CoGVaR}_{\alpha,t}^{\mathrm{sys}|i} = \widehat{\beta}_2^{\mathrm{sys}}(\mathrm{GVaR}_{\alpha,t}^i - \mathrm{GVaR}_{0.5,t}^i).$$

3.3 Validation analysis

General OLS (ordinary least squares) regression models are often assessed by coefficient determination

$$R^2 = 1 - \frac{\mathrm{SSE}}{\mathrm{SST}} = 1 - \frac{\mathbb{E}[(y - \widehat{y})^2]}{\mathbb{E}[(y - \bar{y})^2]},$$

where SSE denotes the residual sum of squares and SST denotes the total sum of squares. y is the response we focus on, $\widehat{y}$ is the corresponding estimated value via the OLS regression model, and $\bar{y}$ is the mean of the response. Analogous to the definition of R^2, the determination for our generalized quantile regression is given as

$$\bar{R}_\alpha{}^2 = 1 - \frac{\mathscr{E}_\alpha}{\mathscr{D}_\alpha},$$

where $\mathscr{E}_\alpha = \min_\beta \mathbb{E}\left[l_\alpha(\varepsilon^{\mathrm{sys}})\right]$ and

$$\mathscr{D}_\alpha = \min_{C \in \mathbb{R}} \mathbb{E}\left[l_\alpha(X^{\mathrm{sys}} - C)\right].$$

$\mathscr{E}_\alpha$ represents the error, while $\mathscr{D}_\alpha$ represents the deviation. The deviation measure is further discussed in Ref. [10]. As in general OLS regression, higher values of $\bar{R}^2$ are better in most cases.

4 Case study

In this section, we compare CoGVaR with CoVaR and CoExpectile in a case study after the estimation approach has been given before.

4.1 Data description

We use the data from the Dow Jones U.S. Financials Index (DJUSFN) and 10 institutions from the constitutions of the index. The DJUSFN represents the financial industry as defined by the industry classification benchmark (ICB). It measures the performance of the financial sector of the U.S. equity market. We denote by X_t^{sys} a scaled weekly log-loss of this index

$$X_t^{\mathrm{sys}} = -100 \ln \frac{I_t}{I_{t-1}},$$

where I_t is the index value at the end of day t. Similarly, the ith financial institution's scaled weekly log-loss X_t^i equals

$$X_t^i = -100 \ln \frac{P_t^i}{P_{t-1}^i},$$

where P_t^i is the closing price at the end of week t. We denote the weekly log-loss for the ith institution by X^i, the tth observation of this vector X^i equals X_t^i.

The data are considered in the period from January 1, 2002 to January 1, 2015 (679 weeks), which covers a recession (2007–2009) and a financial crisis (2008). We download the weekly Dow Jones U.S. Financials Index and financial institutions' closing prices from Yahoo! Finance①.

Table 2 lists ten publicly traded banks in the United States ranked by total assets as of December 31, 2014.

To estimate the time-varying $\mathrm{GVaR}_{\alpha,t}$ and $\mathrm{CoGVaR}_{\alpha,t}$, we choose the following state variables.

(Ⅰ) The change in the three-month yield (TC): Ref. [1] found that the change in the three-month Treasury bill rate is most significant in explaining the tails in asset returns of financial institutions.

Table 2. The ten publicly traded banks in the U.S..

No.	Banks
1	JP Morgan Chase & Company (JPM)
2	Bank of America (BAC)
3	Citigroup Inc (C)
4	Wells Fargo & Company (WFC)
5	The Bank of New York Mellon Corporation (BK)
6	US Bancorp (USB)
7	Capital One Financial Corporation (COF)
8	PNC Financial Services Group Inc (PNC)
9	State Street Corporation (STT)
10	The BB & T Corporation (TFC)

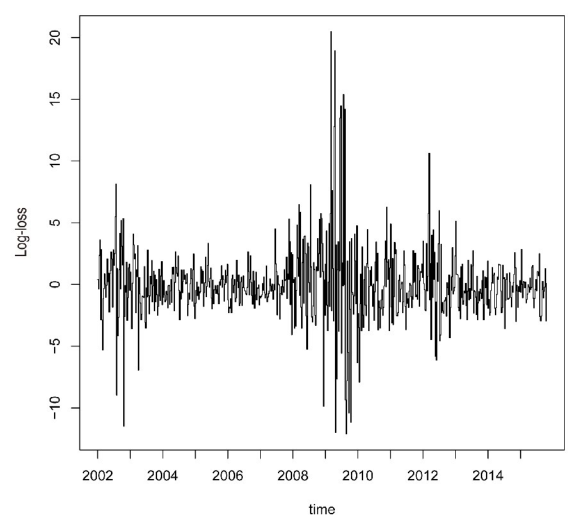


Figure 1. Log-loss of DJUSFN.

(Ⅱ) The change in the slope of the yield curve (TSC): measured by the yield spread between the ten-year Treasury rate and the three-month bill rate obtained from Federal Reserve Bank's H.15 report②.

(Ⅲ) The credit spread change (CSC): measured by the change between Baa-rated bonds and the Treasury rate (with the same ten-year maturity) from the Federal Reserve Bank's H.15 report.

(Ⅳ) Equity volatility (VIX): The Chicago Board Options Exchange (CBOE) Volatility Index (VIX), which captures the implied volatility in the stock market reported by the CBOE.

(Ⅴ) The weekly equity market return (MER): we use the Standard & Poor's 500 Index to calculate the

① Data are available at https://au.finance.yahoo.com/.

② www.federalreserve.gov/releases/h15/.

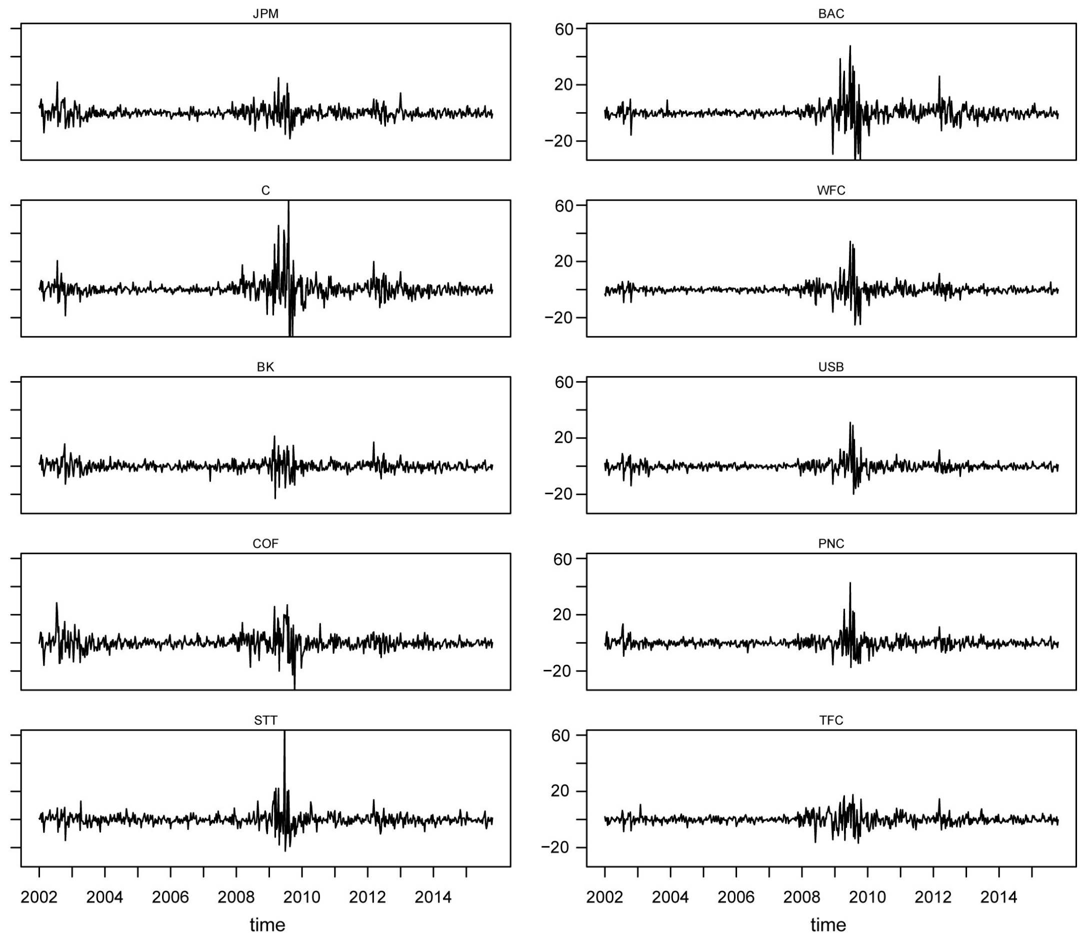


Figure 2. Log-loss of institutions.

equity market return.

(Ⅵ) A short-term "liquidity spread" (LS): defined as the difference between the three-month LIBOR rate and the three-month Treasury bill rate. This liquidity spread measures short-term liquidity risk. We obtain the three-month Treasury bill rate from the Federal Reserve Bank's H. 15 report. We use the three-month LIBOR rate from Wind.

Figure 1 shows the log-loss of the DJUSFN, and Figure 2 includes the log-loss of the related 10 institutions. They factually share a similar tendency in general, which coincides with the DJUSFN.

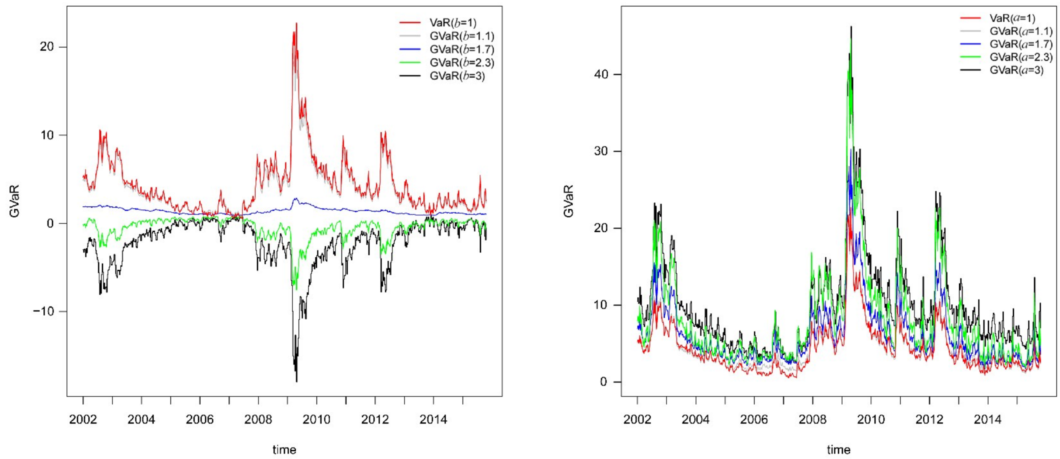
4.2 Caculation results

According to the definition of GVaR, there is a tendency that with a fixed index $a=1$[①] and $\alpha=0.9$, GVaR will decrease as index b increases in most cases. We only exhibit the results focusing on JPM to verify these changes here, they certainly also hold for other institutions. The plots on the left side of Figure 3 reveal the changes of a varying b, and it is clear that when b is relatively large enough, there is no practical significance (above 2.3 in the plots). Moreover, we discuss index a's variation in the plots on the right side of Figure 3. GVaR increases as a increases in most cases. We also linearise the risk measure VaR here, as it is factually a GVaR with $a=b=1$. The left and right parts of Figure 4 show CoGVaR's change in varying b and varying a, respectively, and they share similar fluctuations with GVaR in Figure 3.

In the following, we calculate CoGVaR with $a=3$ and $b=1.1$, which are chosen according to our calculation results for a relatively conservative corisk (compare to VaR and expectile). In the meantime, the choice reflects the regulator's risk aversion, that is he/she concerns much more about upper tail of risks.

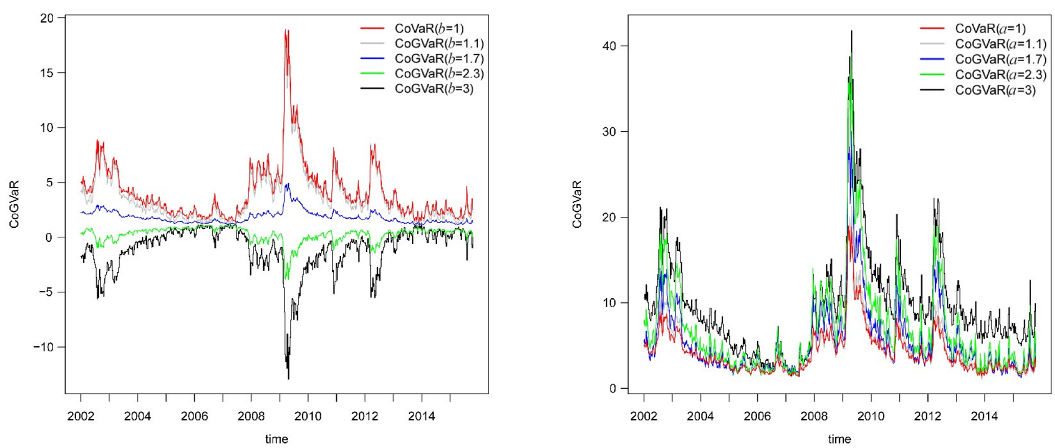
Table 3 presents the coefficients from model (2) for 10 institutions, which conveys the message that the state variables have different sensitivities for most institutions and some even have the opposite sign. For example, the change in the three-month yield (TC) has a positive effect on most banks, while it has a negative effect on JPM and PNC.

① When $a=1$ or $b=1$, the value of GVaR is not unique according to its definition and we choose the smallest one. We take the value of a or b as 1 just for the demonstration of monotonicity tendency and comparison to VaR.



(a) (b)

Figure 3. (a) depicts the GVaR of JPM with $a=1$ and varying b, and (b) depicts its GVaR of with $b=1$ and varying a.



(a) (b)

Figure 4. (a) is the $\mathrm{CoGVaR}_{0.9,t}$ of JPM with varying b and fixed $a=1$, and (b) is its $\mathrm{CoGVaR}_{0.9,t}$ with varying a and fixed $b=1$.

Table 3. Variable coefficients from regression.

No.	Banks	Intercept	GVaR	TC	TSC	CSC	VIX	MER	LS	$\bar{R}^2_{0.9}$
1	JPM	−0.201	0.506	−0.012	−0.339	−0.729	0.288	0.426	1.066	0.822
2	BAC	0.094	0.421	0.2	0.085	−0.332	0.164	0.038	−0.001	0.843
3	C	−0.198	0.356	0.397	0.391	−0.389	0.173	0.242	0.317	0.828
4	WFC	−0.295	0.437	0.2	−0.092	−0.872	0.337	0.993	0.823	0.782
5	BK	−0.291	0.54	0.306	−0.093	−0.34	0.235	0.409	−0.132	0.788
6	USB	0.541	0.458	0.061	−0.358	−1.302	0.437	−0.215	0.419	0.755
7	COF	0.079	0.35	0.06	−0.262	−0.757	0.311	0.509	0.801	0.768
8	PNC	0.37	0.488	−0.313	−0.219	−0.645	0.346	0.997	0.799	0.716
9	STT	0.304	0.431	0.416	−0.355	−0.899	0.329	−0.515	0.336	0.771
10	TFC	−0.557	0.343	0.16	0.369	−0.526	0.354	−0.305	0.195	0.685

In Figure 5, we calculate the corresponding $\mathrm{CoGVaR}_{0.9,t}$, $\mathrm{CoGVaR}_{0.95,t}$ and $\mathrm{CoGVaR}_{0.8,t}$ to explore the change generated by varying α. $\mathrm{CoGVaR}_{0.95,t}$ seems to be above the others, which is natural for GVaR's monotonicity in confidence level α. And in reality, large enough value of α brings large enough capital

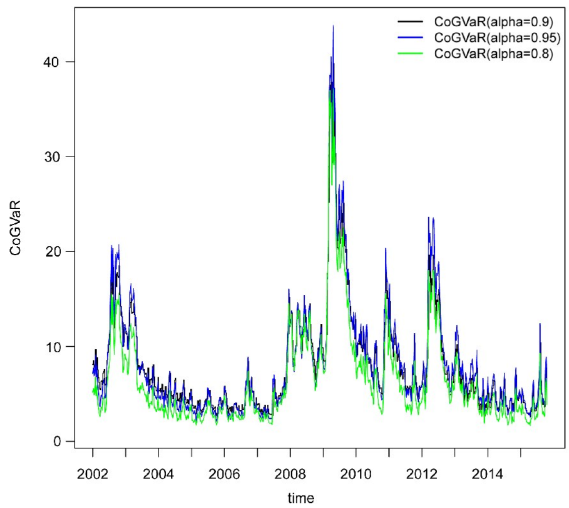


Figure 5. The $\mathrm{CoGVaR}_{0.9,t}$, $\mathrm{CoGVaR}_{0.95,t}$ and $\mathrm{CoGVaR}_{0.8,t}$ of JPM with $a=3$, $b=1.1$.

reservation to protect the system.

4.3 CoGVaR vs. CoVaR, CoExpectile

In this section, we perform a simple comparison between CoVaR, CoExpectile and CoGVaR ($a=3$, $b=1.1$) at different confidence levels α. Note that CoExpectile is the special case of CoGVaR with $a=b=2$. Figure 6 shows the concrete time series of the change between the three kinds of risk measures with $\alpha=0.9$. The time series plots show that for each institution, CoGVaR ($a=3$, $b=1.1$) is more conservative than CoVaR and CoExpectile, especially during the financial crisis period. As a result of the finding that the state variables have similar effects on most institutions according to Table 3, the CoRisks' values of institutions share similar trends.

Table 4 summarizes the average results of CoVaR, CoExpectile and CoGVaR ($a=3$, $b=1.1$) with $\alpha=0.9$. The Rk column is given according to the institution's contribution to the systemic risk. We find that a high rank may not always coincide with a high value of VaR, expectile or GVaR, and obviously CoVaR, CoExpectile and CoGVaR ($a=3$, $b=1.1$) provide rather different ranks for systemic risk contributions. Tables 5 and 6 summarize the average results with $\alpha=0.95$ and $\alpha=0.8$. Clearly, the case with $\alpha=0.95$ yields larger values of VaR, expectile and GVaR,

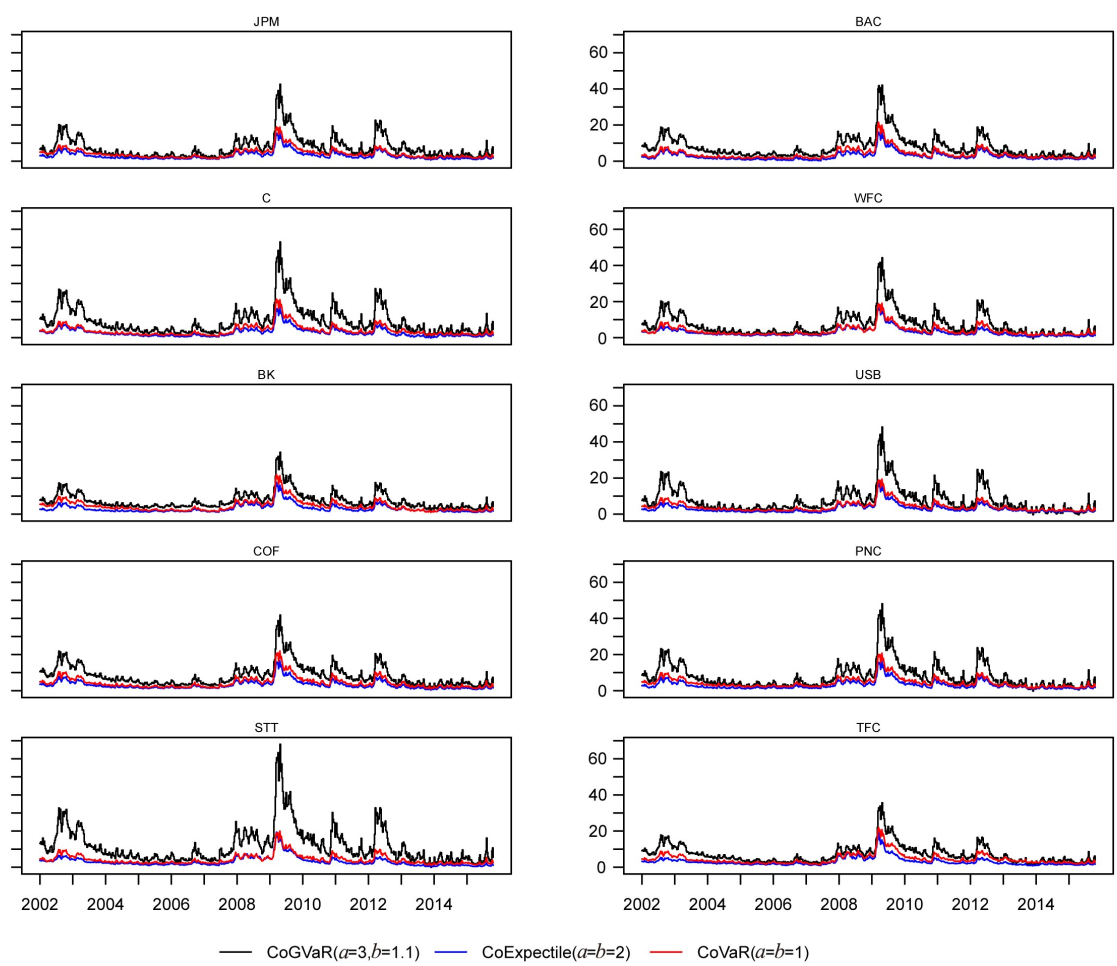


Figure 6. The time series of weekly $\mathrm{CoGVaR}^{i}_{0.9,t}$, $\mathrm{CoExpectile}^{i}_{0.9,t}$ and $\mathrm{CoVaR}^{i}_{0.9,t}$.

Table 4. Comparison of average results with $\alpha=0.9$.

No.	Banks	($a=b=1$) VaR	ΔCoVaR	Rk_1	($a=b=2$) Expectile	ΔCoExpectile	Rk_2	($a=3$, $b=1.1$) GVaR	ΔCoGVaR	Rk_3
1	JPM	4.3457	2.6947	1	3.0304	1.9320	1	10.4483	2.1939	2
2	BAC	5.2580	2.1671	9	4.1157	1.6719	4	14.8647	1.6642	4
3	C	5.7152	2.3818	3	4.9042	1.6119	9	19.2109	0.8023	10
4	WFC	3.1875	2.0404	10	2.4919	1.6536	6	9.1220	0.8954	9
5	BK	3.9847	2.3655	4	2.8225	1.7212	2	8.7764	1.7518	3
6	USB	3.1422	2.2655	7	2.3547	1.5795	10	9.0746	1.2538	6
7	COF	5.2192	2.3276	6	3.6944	1.7102	3	13.0980	1.4009	5
8	PNC	3.5895	2.3414	5	2.7573	1.6622	5	10.9104	1.1773	7
9	STT	4.4037	2.1962	8	3.3292	1.6122	8	26.8743	3.9759	1
10	TFC	3.8575	2.4454	2	2.6413	1.6197	7	7.7480	0.9006	8

Table 5. Comparison of average results with $\alpha=0.95$.

No.	Banks	($a=b=1$) VaR	ΔCoVaR	Rk_1	($a=b=2$) Expectile	ΔCoExpectile	Rk_2	($a=3$, $b=1.1$) GVaR	ΔCoGVaR	Rk_3
1	JPM	5.5773	3.7011	1	4.1182	2.5247	1	10.4978	2.1336	2
2	BAC	7.7618	3.3157	4	5.8852	2.3868	4	14.9459	1.6436	4
3	C	8.9424	3.3070	5	6.7094	2.1928	8	19.0098	0.7317	10
4	WFC	4.9704	3.3409	3	3.7293	2.4661	2	9.6252	1.0562	8
5	BK	5.2754	3.1410	8	3.7568	2.2815	7	9.7177	2.1464	1
6	USB	4.1251	2.4725	10	3.3454	2.1111	9	8.2308	1.0263	9
7	COF	6.9516	3.1869	7	4.9987	2.3056	6	13.8879	1.6543	3
8	PNC	5.0872	3.2844	6	3.8755	2.3119	5	12.3824	1.3469	5
9	STT	6.0806	3.3571	2	4.8760	2.4407	3	20.9327	1.2712	6
10	TFC	4.8587	2.8611	9	3.5061	2.0551	10	8.1673	1.1799	7

Table 6. Comparison of average results with $\alpha=0.8$.

No.	Banks	($a=b=1$) VaR	ΔCoVaR	Rk_1	($a=b=2$) Expectile	ΔCoExpectile	Rk_2	($a=3$, $b=1.1$) GVaR	ΔCoGVaR	Rk_3
1	JPM	2.7551	1.7447	1	1.8921	1.2284	1	8.3743	1.2360	1
2	BAC	2.7833	1.1240	10	2.6205	1.0723	2	12.9719	0.8568	4
3	C	3.4811	1.4166	6	2.9484	0.9121	10	17.6416	0.2464	10
4	WFC	1.8645	1.3382	8	1.4550	1.0033	8	8.2224	0.5530	9
5	BK	2.4628	1.5132	4	1.7629	1.0679	3	7.7141	1.0588	2
6	USB	2.0590	1.5395	3	1.4100	0.9988	9	7.4346	0.5728	8
7	COF	3.4420	1.5797	2	2.2809	1.0301	5	11.7232	0.9423	3
8	PNC	2.1439	1.2822	9	1.6290	1.0085	7	10.6723	0.8445	5
9	STT	2.4784	1.3863	7	2.1181	1.0122	6	19.9938	0.7800	6
10	TFC	2.4439	1.5040	5	1.6083	1.0491	4	6.7481	0.7328	7

while the case with $\alpha=0.8$ yields smaller values of VaR, expectile and GVaR. The systemic risk contribution also yields divergent observations, and the ranks differ greatly. Compared to CoVaR and CoExpectile, CoGVaR ($a=3$, $b=1.1$) behaves more conservatively, especially during the financial crisis period (2008−2010). CoGVaR ($a=3$, $b=1.1$) with the regulator's risk aversion added and more concentration on shortfall risks $(X-x)^+$ (with larger a than CoVaR) may be more suitable in such a context when the upper-tail of a risk is given more concern.

5 Conclusions

In conclusion, we explored the application of generalized quantiles to the systemic risk, inspired by Ref. [1]. It accounts for risk aversion, using two different disutility functions to transform the two risks $(X-x)^+$ and $(X-x)^-$. We proposed the approach to estimate CoGVaR via generalized quantile regression without any distribution assumption and denoted the systemic risk contribution as ΔCoGVaR. We compared CoVaR, CoExpectile and CoGVaR in our Dow Jones U.S. Financials Index case.

In the case study, we found that a high rank in terms of the systemic risk contribution may not coincide with high value of the corresponding VaR, Expectile or GVaR. Controlling an individual risk may not be sufficient to make the whole financial system safe. Our CoGVaR with $\alpha=0.9$, $u_1(x)=x^3$ and $u_2(x)=x^{1.1}$ in the Dow Jones U.S. Financials Index case focuses more on the heavy upper tail of the loss and provides a new perspective on systemic risk contribution.

There is potential for more in-depth investigations. Our regression method has some dependence on subjectively selected state variables, and this dependence does not change over time. That is, on the one hand, the choice of state variables has some potential to improve the regression model for the calculation of our CoGVaR, and on the other hand, the time-invariant dependence may not work as well as time-variant dependence. These are problems to be explored in the future. Compared to CoVaR, the CoGVaR in Dow Jones U.S. Financials Index case has a different application to the systemic risk. For the decision maker who cares much about the upper tail of a risk, a generalized quantile with more convex disutility function u_1 may work better.

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Conflict of interest

The authors declare no conflict of interest.

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应用 CoGVaR 方法度量系统风险贡献

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摘要:基于系统风险度量的角度,提出了一类新的条件风险度量——广义条件风险价值(CoGVaR). 这类新的风险度量是条件分位数的自然广义化,它包括了经典的 CoVaR. 相较于经典的条件风险价值(CoVaR)和条件 expectile(CoExpectile),它在实际中有着更好的应用价值,这一优势来源于它考虑了决策者的风险态度,而这一点目前为止并没有被其他工作关注过. 使用加入状态变量的广义分位数回归方法,在道琼斯美国金融指数实例中给出了具体的计算结果,发现这类风险度量为系统风险贡献的度量提供了一种新的角度. 除此之外,这一结果也显示了该风险度量能够通过使用更凸的负效用函数来更好地捕捉尾部风险.

关键词:广义分位数;条件风险度量;系统风险贡献