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Measure of riskiness based on RDEU model

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Abstract: Motivated by References[3,4], we introduce a new measure of riskiness based on the rank-dependent expected utility (RDEU) model. The new measure of riskiness is a generalized class of risk measures which includes the economic index of riskiness of Reference[3] and the operational measure of riskiness of Reference[4] as special cases. We probe into the basic properties as a measure of riskiness such as monotonicity, positive homogeneity and subadditivity. We study its applications in comparative risk aversion as well. In addition, we present a simulation to illustrate the results.
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1 Introduction

People want to quantify the risk of a decision, far beyond the expectation or variance. Even though many contributions have been made in this area[1,2], no one ever constructed a perfect risk measure both satisfying all the desired properties and applicable in economics. Reference[3] introduced a new measure of riskiness and proved several desired properties. They defined the risk of a specific gamble, which yields both positive and negative outcomes, with the same measurement unit as gambles. However, the risk, according to them, is totally based on the distribution of a gamble about which one may doubt whether others take the gamble as serious as him.

Almost all the measures of riskiness are objective, without the interference of decision-makers[4]. However, a risky asset may be taken as riskless to someone but, as, at the same time, too risky to be accepted by others. In this paper, based on the rank-dependent expected utility (RDEU) model, we propose a measure of riskiness of "gambles" (risky assets) that is subjective: it depends on both the gamble and the one who is considering investing. Even though we have a huge step up, the subjective measure is still ideal to use. Meanwhile, the measure is applicable to all the bounded gambles, making the comparison of different gambles easier.

The RDEU theory is proposed by References[5,6] in which the expectation can be defined as rank-dependent, which permits the analysis of phenomena associated with the distortion of subjective probability and applies better in real than simply weighted expectations, according to References[7,8].

For the discussion of the distortion function, named after the intuition that the expectation is "distorted", it can be concave: this rank-dependent way of modeling pessimism and optimism was suggested before by Reference[5]. It was described in full by Reference [6], which can be convex, and even a mixed pattern of both[8].

In this paper, we apply a new model to the index of riskiness and obtain desired properties. It is natural that some of them are no longer satisfied. However, after assuming the distortion function to be concave, almost all of the properties still hold. Besides, we extend the definition of risks to nearly all gambles, even with no loss, without loss of desired properties.

The rest of the paper is organized as follows. In Section 2, we introduce some preliminaries, such as the RDEU model. In Section 3, we introduce the new risk measure of riskiness based on the RDEU model. Section 4 shows our main results. In Section 5, we present a simulation to illustrate the results.

2 Preliminaries

2.1 The RDEU model

The rank-dependent expected utility (RDEU) model is one of classic models in the economical behavior theory introduced by References[5,6]. A decision decision-maker behaves in accordance with the RDEU model if

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the decision-maker is characterized by an increasing and continuous utility function $u:\mathbb{R}\to\mathbb{R}$ and a probability-perception function h: $[0,1]$ o $[0,1]$, that is, h is increasing with $h(0)=0$ and $h(1)=1$. Such a decision-maker prefers the random variable Y to the random variable X if and only if

$$V_{u,h}(Y) \geqslant V_{u,h}(X),$$

where $V_{u,h}(Y)$, also denoted by $V_{u,h}(G)$, is the RDEU functional or the Quiggin-Yaari functional of Y[5,6] given by

$$V_{u,h}(Y) = \int_{-\infty}^{\infty} u(y),\ \mathrm{d}h(G(y)) \qquad (1)$$

Here G is the cumulative distribution function (CDF) of Y. Any decision maker in the REDU model makes decision according to the RDEU functional $V_{u,h}$ is denoted by a (u,h)-decision-maker.

It is well-known that the RDEU model has the classic expected utility (EU) model and the Yarri′s dual theory as its special case. Specifically, if the probability-perception function $h(s)=s$, $s\in[0,1]$, then the RDEU functional reduces to the classic expected utility functional.

Throughout the paper, the utility function u is the von Neumann-Morgenstern utility function for money. We confine the utility function u in the following set

$$U = \left\{ u \,\middle|\, \begin{array}{l} u(0) = 0,\ u'(0) = 1,\ u \text{ strictly increasing}, \\ \text{concave, twice continuously differentiable} \end{array} \right\}.$$

If the utility function is the linear function (identical function), that is, $u(x)=x$, $x\in\mathbb{R}$, then the RDEU functional reduces to the Yarri's dual utility

$$\mathbb{E}_h(Y) = \int_{-\infty}^{\infty} x\mathrm{d}h(G(y)) = \int_0^1 G^{-1}(s)\mathrm{d}h(s) \quad (2)$$

where $G^{-1}(s)=\inf\{x\in\mathbb{R} : G(x)\geqslant s\}$, $s\in(0,1)$, is the left-continuous inverse function of G. The Choquet integral $\mathbb{E}_h$ is also called the distorted expectation and the probability-perception function is also called the distortion function. The integral follows the standard definition of Lebesgue-Stieljes integral. Throughout the paper, for an increasing R function $g:\mathbb{R}\to\mathbb{R}$ and a function f: $\mathbb{R}\to\mathbb{R}$, the Lebesgue-Stieltjes integral $\int_{\mathbb{R}} f(x)\mathrm{d}g(x)$ is defined (see, for instance, Reference [9] as $\int_{\mathbb{R}} f(x)\mathrm{d}g_+(x)$ or $\int_{\mathbb{R}} f(x)\mu_g(\mathrm{d}x)$, where $g_+(x)=g(x_+)$ and μ_g is a measure defined by $\mu_g([a,b])=g(b_+)-g(a_-)$ for any $a\leqslant b$. In other words, if an increasing function $g:\mathbb{R}\to\mathbb{R}$ is not right-continuous, $g(x)$ in the Lebesgue-Stieltjes integral $\int_{\mathbb{R}} f(x)\mathrm{d}g(x)$ is treated as its right-continuous copy $g(x_+)$. In this way, the integral is well defined. That is, the Yarri's dual utility is expected values calculated based on "distorted" CDFs $h(F_X(x))$. Hence, the formula (2) provides a more clear justification of why the Yarri's dual utility is considered as (distorted) expectations.

One important property of distorted expectation is that $\mathbb{E}_h(aX+b)=a\mathbb{E}_h(X)+b$ holds for real numbers $a\geqslant 0$ and $b\in\mathbb{R}$. However, $\mathbb{E}_h(X+Y)=\mathbb{E}_h(X)+\mathbb{E}_h(Y)$ may not be true for random variables X and Y, and it will hold if X and Y are comonotonic. In the bivariate setting, a random vector $(X;Y)$ is comonotonic if there exists increasing functions f and g such that $X=f(X+Y)$ and $Y=g(X+Y)$ almost surely. This fact leads to problems when dealing with the portfolio investment, a linear combination of different risky assets. Thus, additional presumptions are necessary for the distortion function h. Throughout the paper, the distortion function h is assumed in the following set

$$\mathscr{H} = \{h:[0,1]\to[0,1] \mid h(0)=0, h(1)=1, \\ h \text{ is concave and has no jump at zero}\}.$$

We will also denote h' by the left derivative of distortion function h. There are some examples of distortion functions, and Figure 1 presents a more intuitive picture.

(Ⅰ) Proportional hazard transform function[10], with the distortion function

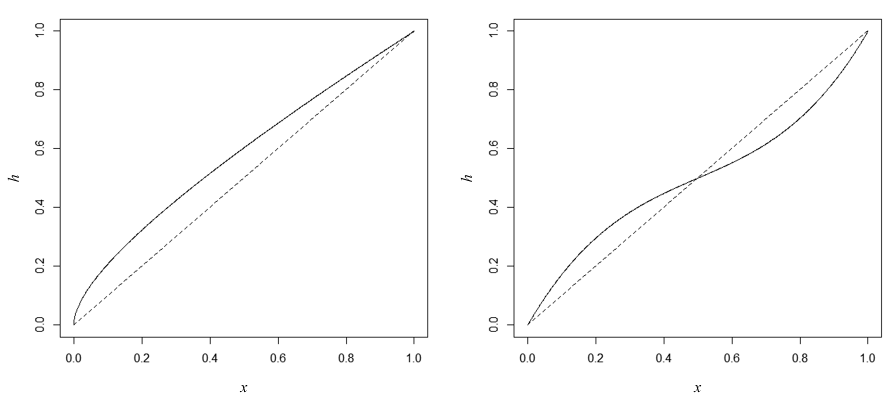

Figure 1. (a) A concave distortion function; (b) A distortion function concave for small probability and convex for moderate and high probabilities.

$$h(x) = x^{\frac{1}{\rho}},\ \rho \geqslant 1.$$

(Ⅱ) Dual-power function[11], with the distortion function

$$h(x) = 1 - (1-x)^{\nu},\ \nu \geqslant 1.$$

(Ⅲ) Wang's transform weighting function, known as the WT weighting function[12], it is applied widely into the pricing of financial derivatives for its fine properties. It is usually represented by

$$h(x) = \Phi(\Phi^{-1}(x) + \alpha),\ \alpha \in \mathbb{R},$$

where $\Phi(x)$ is the cumulative distribution function of a standard normal distribution.

2.2 Comparative risk aversion

Risk aversion is an important concept in the decision theory. We use the notation Reference [3] to describe the comparative risk aversion. Agents i and j are going to decide whether to accept or reject such a gamble.

Definition 2.1 (Ⅰ) A (u,h)-decision maker in the RDEU model accepts gamble X at the wealth level w if

$$V_{u,h}(w+X) > u(w).$$

(Ⅱ) For two agents i and j, we say j is uniformly no less risk-averse than i if whenever j accepts gamble X at wealth level w, i accepts X at any wealth level. It´s denoted by $i \underline{\lhd} j$.

(Ⅲ) For agents i and j, we say j is no less risk-averse than i if whenever j accepts gamble X at wealth level w, i accepts X at some wealth level. It´s denoted by $i \underline{\lhd}_w j$.

As we can see from the definitions, the condition $\underline{\lhd}$ is stronger than that of $\underline{\lhd}_w$, that is, $i \underline{\lhd} \{j\}$ implies $i \underline{\lhd}_w j$.

3 Measure of riskiness based on the RDEU model

In this section, we will introduce a new measure of riskiness based on the RDEU model. To this end, we confine the gambles to some subsets of the family of all gambles.

Definition 3.1 For a given distortion function h we define

$$\mathscr{G} = \{X : X \text{ is bounded and } \mathbb{P}(X=0) < 1\}$$

and

$$\mathscr{G}_h = \{X \mid \mathbb{E}_h(X) > 0,\ \mathbb{P}(X<0) > 0\}.$$

The condition $\mathbb{E}_h(X)>0$ is due to that people will not hesitate to reject a gamble that they think would be nonprofitable, while violating the condition $\mathbb{P}(X<0)>0$ means that the gamble brings no loss at all.

For a utility function u, a distortion function h and a gamble X, we define

$$f_{u,h,X}(\alpha) := f(\alpha) = \mathbb{E}_h(u(\alpha X))$$

on $[0,\infty]$, which is called a scale function throughout the paper. Then, $f \in C^2[0,\infty]$ which means f is second order continuously differentiable. In the following, we state some basic properties for the scale function.

Theorem 3.1 Suppose that the utility function $u \in \mathscr{U}$ has an upper bound and $h \in \mathscr{H}$. For $X \in \mathscr{G}_h$, the scale function $f(\alpha) = \mathbb{E}_h(u(\alpha X))$ is concave on $[0,\infty)$ with $f(0)=0$. Moreover, there exists a real number $\rho_{u,h}(X)>0$ uniquely determined by

$$f(1/\rho_{u,h}(X)) = \mathbb{E}_h u(X/\rho_{u,h}(X)) = 0 \tag{3}$$

Proof It is clear that $f(0) = \mathbb{E}_h[u(0)] = 0$. Note that $f'(0) = \mathbb{E}_h(Xu'(0)) = \mathbb{E}_h(X) > 0$ because of $X \in \mathscr{G}_h$. Hence, there exists an α small enough such that $f(\alpha)>0$. Meanwhile, $f''(\alpha) = \mathbb{E}_h(X^2 u''(\alpha X)) \leqslant 0$ means f is concave on the positive axis.

Assuming now that $p_0 = \mathbb{P}(X < -\epsilon) > 0$ and $\mathbb{P}(|X| \leqslant M) = 1$, let X_0 be a gamble that yields M with probability $1-p_0$ and $-\epsilon$ with p_0. It is obvious that $F_{X_0}(x) \leqslant F_X(x)$ for all x. Then

$$f(\alpha) = \int_0^1 u(\alpha F_X^{-1}(q))\,\mathrm{d}h(q) \leqslant$$

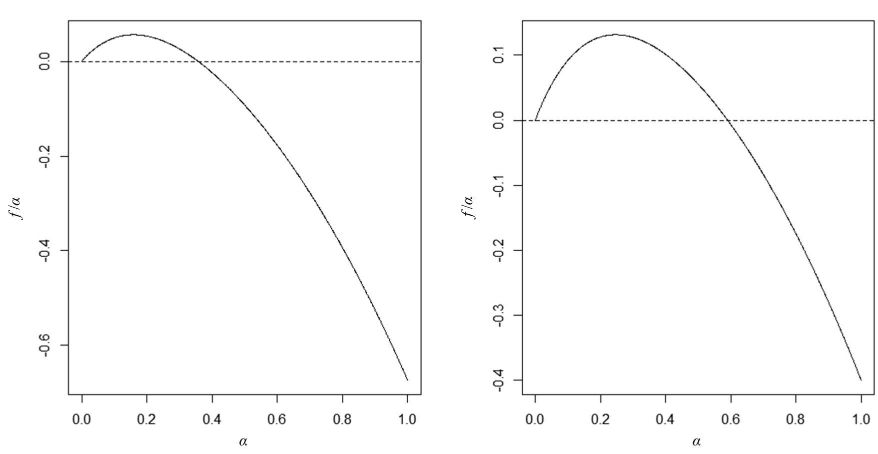

Figure 2. The scale function for (a) with $h(x) = \frac{\sqrt{x}+x}{2}$ and (b) with a linear h. The riskiness is 2.79 for (a) and is 1.69 for (b).

$$\int_0^1 u(\alpha F_{X_0}^{-1}(q))\mathrm{d}h(q) =$$
$$h(p_0)u(-\alpha\epsilon) + (1-h(p_0))u(\alpha M).$$

Thus, the scale function $f(\alpha)$ becomes non-positive for α large enough since u has an upper bound. Up to now, we obtain three observations of the function f:

(Ⅰ) $f(0)=0$ and $f'(0)>0$;

(Ⅱ) f is concave on $[0,\infty)$;

(Ⅲ) There exists an α large enough such that $f(\alpha)<0$.

Hence, there exists a unique $\rho_{u,h}(X)>0$ such that equation (3) holds.

Here, we set $u(x)=1-\mathrm{e}^{-x}$ for all $x\in R$. Let X yield 6 with probability 0.2, 2 with probability 0.3 and −1 with probability 0.5. Checking that it meets all the requirements, for different h, we draw its scale function in Figure 2.

Definition 3.2 For $u\in\mathscr{U}$ and $h\in\mathscr{H}$, the measure of riskiness based on (u,h)-RDEU model is defined by a functional $\rho_{u,h}:\mathscr{G}\to[0,+\infty]$ as the following way

$$\rho_{u,h}(X)=\begin{cases}0, & \mathbb{P}(X<0)=0,\\ +\infty, & \mathbb{E}_h(X)\leqslant 0,\\ \text{the solution of equation (3)}, & X\in\mathscr{G}_h.\end{cases}$$

For the case $\mathbb{E}_h u(X)\leqslant 0$, a decision maker with distortion function h won't take it. Thus, we set its riskiness to be $+\infty$. For another case that $\mathbb{P}(X<0)=0$, people accepts the gamble violating the condition with absolutely no loss. For this one, we set its riskiness to be 0 because nobody would be afraid of it for any reasons.

Example 1 (Ⅰ) If h is the identical function, i.e., $h(p)=p$ for $p\in[0,1]$ and $u(x)=1-\exp(-x)$ for $x\in\mathbb{R}$. The measure of riskiness $\rho_{u,h}$ reduces to the case introduced in Reference[3].

(Ⅱ) If h is the identical function and $u(x)=\log(1+x)$ for $x\in\mathbb{R}$. The measure of riskiness $\rho_{u,h}$ reduces to the case introduced in Reference[4].

(Ⅲ) If $u(x)=x-1$ for $x\in\mathbb{R}$, we have $\rho_{u,h}(X)=\mathbb{E}_h(X)$ for all $X\in\mathscr{G}_h$.

The third one of the above examples illustrates that the constraint of u in Theorem 3.1 is not necessary to guarantee that equation (3) has an unique solution. For some feasible utility function u, $\rho_{u,h}$ can be the Yarri's dual utility.

4 Main results

It follows directly from the definition that two axiomatic characterizations are identical to those of Reference[3]. Hence, the similar results are also obtained for the distorted riskiness.

4.1 Basic properties for the measure of riskiness

Definition 4.1 For any two lotteries with cumulative distribution functions F and G, respectively. We say F first-order stochastic dominates G, denoted by $F\succeq_1 G$, if for any increasing function u

$$\int u(x)\mathrm{d}F \geqslant \int u(x)\mathrm{d}G.$$

Proposition 4.1 For $u\in\mathscr{U}$ with a upper bound and $h\in\mathscr{H}$, the measure of riskiness based on RDEU model has following properties:

(Ⅰ) Monotonicity with respect to the first-order stochastic dominance: For $X,Y\in\mathscr{G}$, if $X\preceq_1 Y$, then $\rho_{u,h}(X)\geqslant\rho_{u,h}(Y)$.

(Ⅱ) Positive Homogeneity: $\rho_{u,h}(\lambda X)=\lambda\{\rho_{u,h}(X)$ for $\lambda>0$ and $X\in\mathscr{G}$;

(Ⅲ) Subdilution: $\rho_{u,h}(X^p)\geqslant\rho_{u,h}(X)$ holds for $p\in(0,1]$ and $X\in\mathscr{G}$, where X^p is a compound gamble that yields X with probability p and 0 with probability $1-p$;

(Ⅳ) Continuity on $\mathscr{G}_h$: $\rho_{u,h}(X_n)\to\rho_{u,h}(X)$, if gamble $X_n\xrightarrow{d}X$, $X\in\mathscr{G}_h$ and $X_n\in\mathscr{G}_h$ are uniformly bounded.

Proof We first consider the properties of $\rho_{u,h}$ on $\mathscr{G}_h$. For $X,Y\in\mathscr{G}_h$ such that $X\preceq_1 Y$, we have $0=\mathbb{E}_h u(X_1/\rho_{u,h}(X))\leqslant\mathbb{E}_h u(Y/\rho_{u,h}(X))$. Recall the properties of scale function in Theorem 3.1, we obtain $1/\rho_{u,h}(Y)\geqslant 1/\rho_{u,h}(X)$, and hence, $\rho_{u,h}(X)\geqslant\rho_{u,h}(Y)$. The positive homogeneity is trivial by the definition of $\rho_{u,h}$.

To prove the subdilution, first note that $X\in\mathscr{G}_h$ implies $X^p\in\mathscr{G}_h$ for $p\in(0,1]$. We use the third form of the rank-dependent expectation for this part. i.e.

$$\mathbb{E}_h(u(X)) = u(M) - \int_{-M}^{M} h(F(x))\mathrm{d}u(x),$$

where M is the bound of X. For the diluted gamble X^p, one writes the CDF $F_{X^p}(x)$ as $pF_X(x)+(1-p)I_{[0,\infty]}(x)$. Thus,

$$\begin{aligned} f_{X^p}(\alpha) &= \mathbb{E}_h u(\alpha X^p) = \\ & u(\alpha M) - \int_{-M}^{M} h(pF_X(x) + \\ &(1-p)I_{[0,\infty]}(x))\mathrm{d}u(\alpha x) \leqslant \\ & u(\alpha M) - \int_{-M}^{M} ph(F_X(x)) + \\ &(1-p)h(I_{[0,\infty]}(x))\mathrm{d}u(\alpha x) = \\ & p\leqslant (u(\alpha M) - \int_{-M}^{M} h(F_X(x))\mathrm{d}u(\alpha x)) + \\ &(1-p)(u(\alpha M) - \int_{-M}^{M} h(I_{[0,\infty]}(x))\mathrm{d}u(\alpha x)) = \\ & pf_X(\alpha). \end{aligned}$$

This implies $f_{X^p}\leqslant\left(\dfrac{1}{\rho_{u,h}(X)}\right)\leqslant 0$, and hence $\rho_{u,h}(X^p)\geqslant\rho_{u,h}(X)$.

For continuity, denote $f_n(\alpha)=\mathbb{E}_h u(\alpha X_n)$ and $f(\alpha)=\mathbb{E}_h u(\alpha X)$. Since $X_n\xrightarrow{d}X$ and X_n is uniformly bounded, it then follows from the dominated convergence theorem that $f_n(\alpha)\to f(\alpha)$ pointwisely. Denote $a_n=\rho_{u,h}(X_n)$ and $a=\rho_{u,h}(X)$. Arguing by contradiction, we assume that there exists $\epsilon_0>0$ such that $|a_n-a|>\epsilon_0$ for all n. If there is a subsequence $\{a_{n_k}\}$ such that $a_{n_k}>a+\epsilon_0$. Then we have $0=f_{n_k}(a_{n_k})\geqslant f_{n_k}(a+\epsilon_0)\to f(a+\epsilon_0)<0$. This yields a contradiction. Similarly, we can also get a contradiction if there is a subsequence $\{a_{n_k}\}$ such that $a_{n_k}<a-\epsilon_0$. Hence, we complete the proof of all properties on $\mathscr{G}_h$.

For gambles on $\mathscr{G}$, one can easily verify the monotonicity, positive homogeneity and subdilution of $\rho_{u,h}$ after classification discussions.

4.2 Measure of riskiness for CARA utility function

The exponential utility function is the only one class of utility functions such that the Arrow-Pratt coefficient is constant, that is, the utility function with constant absolute risk aversion[13]. In this section, we set the utility function $u(x)$ to be $1-\exp(-x)$. For convenience, we denote by R_h the measure of riskiness in this case, i. e.

$$R_h(X):=\rho_{u,h}(X).$$

In the following, more properties of R_h will be found. To present the result, we need the following lemma which is coming from Reference[14].

Lemma 4.1 For any random variables X and Y, denote the inverse of their CDFs by $F_X^{-1}(q)$ and $F_Y^{-1}(q)$. Let $\phi(q)=F_{X+Y}^{-1}(q)-F_X^{-1}(q)-F_Y^{-1}(q)$ and $\Phi(q)=\int_0^q\phi(t)\mathrm{d}t$ on $[0,1]$. Then the following properties hold for $\phi(q)$.

(Ⅰ) $\int_0^1\phi(q)\mathrm{d}q=0$;

(Ⅱ) $\Phi(q)\geqslant 0,\ q\in[0,1]$.

Proposition 4.2 The following two properties hold for gambles $X_1,X_2\in\mathscr{G}$.

(Ⅰ) Subadditivity: $R_h(X_1+X_2)\leqslant R_h(X_1)+R_h(X_2)$, if $X_1+X_2\in\mathscr{G}$;

(Ⅱ) Convexity: $R_h(\lambda X_1+(1-\lambda)X_2)\leqslant\lambda R_h(X_1)+(1-\lambda)R_h(X_2)$ if $\lambda X_1+(1-\lambda)X_2\in\mathscr{G}$.

Proof (Ⅰ) First, assume that $X_1,X_2\in\mathscr{G}_h$ and $X_1+X_2\in\mathscr{G}_h$. The scale function of X is denoted by $f_X(\alpha)=\mathbb{E}_h(1-\exp(-\alpha X))$. By Theorem 3.1, to show the subadditivity $R_h(\cdot)$, we only need to prove that $f_{X_1+X_2}(\frac{1}{r_1+r_2})\geqslant 0$ where $r_k=R_h(X_k)$ for $k=1,2$. i. e.

$$\int_0^1\left(1-\exp\left(-\frac{F_{X_1+X_2}^{-1}(q)}{r_1+r_2}\right)\right)\mathrm{d}h(q)\geqslant 0.$$

To this end, we turn to prove the next two inequalities hold

$$\int_0^1\left(1-\exp\left(-\frac{F_{X_1+X_2}^{-1}(q)}{r_1+r_2}\right)\right)\mathrm{d}h(q)\geqslant \int_0^1\left(1-\exp\left(-\frac{F_{X_1}^{-1}(q)+F_{X_2}^{-1}(q)}{r_1+r_2}\right)\right)\mathrm{d}h(q) \tag{4}$$

$$\int_0^1\left(1-\exp\left(-\frac{F_{X_1}^{-1}(q)+F_{X_2}^{-1}(q)}{r_1+r_2}\right)\right)\mathrm{d}h(q)\geqslant 0 \tag{5}$$

Note that formula (4) is equivalent to

$$\int_0^1\exp\left(-\frac{F_{X_1+X_2}^{-1}(q)}{r_1+r_2}\right)\left(\exp\left(\frac{F_{X_1+X_2}^{-1}(q)-F_{X_1}^{-1}(q)-F_{X_2}^{-1}(q)}{r_1+r_2}\right)-1\right)\mathrm{d}h(q)\geqslant 0.$$

Since $h'(q)$ is non-negative and $e^x-1\geqslant x$ for all $x\in\mathbb{R}$, to prove formula(4), we only need to show that

$$\int_0^1\exp\left(-\frac{F_{X_1+X_2}^{-1}(q)}{r_1+r_2}\right)[F_{X_1+X_2}^{-1}(q)-F_{X_1}^{-1}(q)-F_{X_2}^{-1}(q)]\mathrm{d}h(q)\geqslant 0,$$

or, equivalently,

$$\int_0^1\exp\left(-\frac{F_{X_1+X_2}^{-1}(q)}{r_1+r_2}\right)\phi(q)h'(q)\mathrm{d}q\geqslant 0,$$

where $\phi(q)=F_{X_1+X_2}^{-1}(q)-F_{X_1}^{-1}(q)-F_{X_2}^{-1}(q)$. After substituting $\exp\left(-\frac{F_{X_1+X_2}^{-1}(q)}{r_1+r_2}\right)h'(q)$ by $S(q)$, it is obvious that $S(q)$ is non-increasing. We only need to show that $\int_0^1 S(q)\phi(q)\mathrm{d}q\geqslant 0$. Note that

$$\int_0^1 S(q)\phi(q)\mathrm{d}q=\int_0^1 S(q)\mathrm{d}\Phi(q)=$$

$$S(q)\Phi(q)\mid_0^1-\int_0^1\Phi(q)\mathrm{d}S(q)=-\int_0^1\Phi(q)\mathrm{d}S(q)\geqslant 0,$$

where $\Phi(q)=\int_0^q\phi(s)\mathrm{d}s$ and the last part from the fact that $\Phi(q)\geqslant 0$ by Lemma 4.1. For formula (5),

$$\int_0^1\left(1-\exp\left(-\frac{F_{X_1}^{-1}(q)+F_{X_2}^{-1}(q)}{r_1+r_2}\right)\right)\mathrm{d}h(q)=$$

$$\int_0^1\left(1-\exp\left(\begin{array}{l}-\dfrac{F_{X_1}^{-1}(q)}{r_1}\dfrac{r_1}{r_1+r_2}-\\ \dfrac{F_{X_2}^{-1}(q)}{r_2}\dfrac{r_2}{r_1+r_2}\end{array}\right)\right)\mathrm{d}h(q)\geqslant$$

$$\int_0^1\frac{r_1}{r_1+r_2}\left(1-\exp\left(-\frac{F_{X_1}^{-1}(q)}{r_1}\right)\right)+$$

$$\frac{r_2}{r_1+r_2}\left(1-\exp\left(-\frac{F_{X_2}^{-1}(q)}{r_2}\right)\right)\mathrm{d}h(q)=$$

$$\frac{r_1}{r_1+r_2}\mathbb{E}_h\left(1-\exp\left(-\frac{X_1}{r_1}\right)\right)+$$
$$\frac{r_2}{r_1+r_2}\mathbb{E}_h\left(1-\exp\left(-\frac{X_2}{r_2}\right)\right)=0.$$

Next assume that $X_1, X_2 \in \mathscr{G}$ and $X_1+X_2 \in \mathscr{G}$. There are just two kinds of potential violation of $R_h(X_1+X_2) \leqslant R_h(X_1)+R_h(X_2)$:

(ⅰ) $R_h(X_1+X_2)=\infty$ but $R_h(X_1)$ and $R_h(X_2)$ are positive and finite;

(ⅱ) $R_h(X_1)=0$ and $R_h(X_1+X_2)>R_h(X_2)$.

Here we define $\mathscr{G}^*=\{X: X \text{ is bounded}\}$. For the first one, note that the mapping $\mathbb{E}_h: \mathscr{G}^* \to \mathbb{R}$ satisfies the subadditivity, i.e., $\mathbb{E}_h(X_1+X_2) \geqslant \mathbb{E}_h(X_1)+\mathbb{E}_h(X_2)$ for all $X_1, X_2 \in \mathscr{G}^*$ (see e.g., Theorem 2.2 in Reference [15]. Since $R_h(X_1)$ and $R_h(X_2)$ are positive and the finite it follows that $\mathbb{E}_h(X_1), \mathbb{E}_h(X_2)>0$, we have $\mathbb{E}_h(X_1+X_2)$ is finite. The second case can't happen since R_h is monotonic with respect to first-order stochastic dominance.

(Ⅱ) The convexity follows immediately from that R_h is positively homogeneous.

4.3 The necessary of concavity of distortion function

As someone doubts whether the presumptions of distortion function h can be revised, we claim that concavity is necessary for subadditivity property. We prove it in the following part that there is some violation of subadditivity unless h is concave on $[0,1]$.

Proposition 4.3 Suppose the distortion function has no jump at zero, the validity of subadditivity forces h to be concave.

Proof By reducing absurdity, the violation of concavity of distortion function h implies that there exist p_1 and p_2 such that $p_1<p_2$ and $2h(\bar{p})<h(p_1)+h(p_2)$, where $\bar{p}=\frac{p_1+p_2}{2}$. Note that we can move p_1 and p_2 in a small scale, which keeps the inequality unchanged but makes $p_1>0$ and $p_2<1$. Then we construct a gamble X as follows

$$X=\begin{cases}-x_1, & p_1,\\ -\delta, & \bar{p}-p_1,\\ \delta, & p_2-\bar{p},\\ x_2, & 1-p_2,\end{cases}$$

where $-x_1<-\delta<0<\delta<x_2$. Denote $f(\alpha)=\mathbb{E}_h u(\alpha X)$, then

$$\begin{aligned}\mathbb{E}_h u(\alpha X)= & h(p_1)u(-\alpha x_1)+\\ & (1-h(p_2))u(\alpha x_2)+\\ & (h(\bar{p})-h(p_1))u(-\alpha\delta)+\\ & (h(p_2)-h(\bar{p}))u(\alpha\delta).\end{aligned}$$

Meanwhile, the derivative of f at zero is

$$\begin{aligned}f'(0)= & -x_1 h(p_1)+x_2(1-h(p_2))-\\ & \delta(h(\bar{p})-h(p_1))+\delta(h(p_2)-h(\bar{p})).\end{aligned}$$

Let the summation of the first two terms be zero, that is, $-x_1h(p_1)+x_2(1-h(p_2))=0$. Then $x_2=\frac{h(p_1)}{1-h(p_2)}x_1$.

One can find $f'(0)=\delta((h(p_2)-h(\bar{p}))-(h(\bar{p})-h(p_1)))=\delta(h(p_2)+h(p_1)-2h(\bar{p}))>0$. According to the proof of Theorem 3.1, gamble X has a finite and non-zero riskiness $r>0$. Then we construct a pair gambles (X_1, X_2) with joint distribution

$$(X_1,X_2)=\begin{cases}(-x_1,-x_1), & p_1,\\ (-\delta,\delta), & \bar{p}-p_1,\\ (\delta,-\delta), & p_2-\bar{p},\\ (x_2,x_2), & 1-p_2.\end{cases}$$

As we can see, the marginal distributions of both X_1 and X_2 are the same as that of X, so they have the same riskinesses r. Noting that

$$X_1+X_2=\begin{cases}-2x_1, & p_1,\\ 0, & p_2-p_1,\\ 2x_2, & 1-p_2.\end{cases}$$

One can calculate the riskiness of X_1+X_2 with the similar method. The distorted expectation of X_1+X_2 is $u(-2x_1)\cdot h(p_1)+u(2x_2)(1-h(p_2))$. With a useful fact that $u(x)\leqslant x$ for all x on R, we find $u(-2x_1)h(p_1)+u(2x_2)(1-h(p_2))\leqslant -2x_1h(p_1)+2x_2(1-h(p_2))=0$, which indicates that gamble X_1+X_2 has infinite riskiness. We can conclude that $R_h(X_1+X_2)=\infty>2r=2R_h(X)=R_h(X_1)+R_h(X_2)$, a violation of subadditivity.

4.4 Application in comparative risk aversion

Mentioned by Reference [3], duality implies that less risk-averse agents accept riskier gambles. Once they share the same distortion function, duality holds for the two agents.

Theorem 4.1 (Duality) Given that agents i and j are two decision-maker in the RDEU model, the utility functions of i and j are u_i and u_j, respectively, and they share the same distortion function $h \in \mathscr{H}$. For $X_1, X_2 \in \mathscr{G}$, if $j \trianglelefteq i$, i accepts X_1 at w and $R_h(X_1) \geqslant R_h(X_2)$, then j accepts X_2 at w.

To prove Theorem 4.1, we denote $\rho(w)$ by the Arrow-Pratt coefficient of absolute risk aversion for an agent with the utility function u at wealth level w, i.e., $\rho(w)=-u''(w)/u'(w)$. Besides, we need some extra lemmas, some of which are the direct results in Reference [3].

Lemma 4.2 (Lemma 2 in Reference [3]) For some $\delta>0$, suppose that $\rho_i(w)>\rho_j(w)$ at each w with $|w|<\delta$, then $u_i(w)<u_j(w)$ whenever $|w|<\delta$ and $w\neq 0$.

From Lemma 4.2, we can immediately get the following corollary.

Corollary 4.1 (Corollary 3 in Reference[3]) If $\rho_i(w)\leqslant\rho_j(w)$ for all w, then $u_i(w)\geqslant u_j(w)$ for all w.

Given the changes compared with the definitions by Reference [3], some lemmas also need to be generalized.

Lemma 4.3 If $\rho_i(0)>\rho_j(0)$, there is a gamble X that agent j accepts at 0 but agent i rejects at 0.

Proof By the precondition that utility function is twice continuously differentiable, $\rho(w)$ is continuous. There exists $\delta>0$ such that $\rho_i(w)>\rho_j(w)$ for all $|w|<2\delta$. For $-\delta\leqslant x\leqslant\delta$, let X_X be a gamble yielding $x-\delta$ and $x+\delta$ with probability $p_0\in(0,1)$ and $1-p_0$, respectively, where p_0 satisfies $h(p_0)\in(0,1)$. By lemma 4.2, we can get $u_i(w)\leqslant u_j(w)$ for all $|w|<2\delta$, where the equation is satisfied if and only if $w=0$. Then denote $\mathbb{E}_{h_k}u_k(X_X)-u_k(0)$ by $g_k(x)$ for $k=i,j$. One can compute for $k=i,j$ that

$$g_k(x)=\int_0^1 u_k(F_{X_X}^{-1}(q))\mathrm{d}h(q)=$$
$$h(p_0)u_k(x-\delta)+(1-h(p_0))u_k(x+\delta).$$

Under the condition that $u_i(w)\leqslant u_j(w)$, inequality $g_i(x)<g_j(x)$ holds for $-\delta<x<\delta$. Besides, we find $g_k(\delta)=(1-h(p_0))u_k(2\delta)>0$ and $g_k(-\delta)=h(p_0)u_k(-2\delta)<0$. Thus, it follows from the continuity and monotonic of g_k that there exists some x_0 between $-\delta$ and δ such that $g_i(x_0)\leqslant 0<g_j(x_0)$ holds. Gamble X_{x_0} is the gamble desired.

Lemma 4.4 If $\rho_i(w_i)>\rho_j(w_j)$, then there is a gamble X such that agent j accepts at w_j but agent i rejects at w_i.

Simplification: By standardizing u_i and u_j with $u_i^*(w)=\dfrac{u_i(w)-u_i(w_i)}{u_i'(w_i)}$ and $u_j^*(w)=\dfrac{u_j(w)-u_j(w_j)}{u_j'(w_j)}$, there is no difference in their decision-makings and Arrow-Pratt coefficients after substituting u with u^*, so that $u_k(w_k)=0$ and $u_k'(w_k)=1$ hold at any wealth level for $k=i$; j. u_i and u_j with $u_i^*(w)=\dfrac{u_i(w)-u_i(w_i)}{u_i'(w_i)}$ and $u_j^*(w)=\dfrac{u_j(w)-u_j(w_j)}{u_j'(w_j)}$, we can directly derive the result by Lemma 4.3.

Proposition 4.4 $i\trianglelefteq_w j$ if and only if for all wealth level w, $\rho_i(w)\leqslant\rho_j(w)$.

Proof If $i\trianglelefteq_w j$ and there exists w such that $\rho_i(w)>\rho_j(w)$. Using Lemma 4.4 above, we find there is a gamble X that j accepts at w, but i rejects it at w which contradicts to our assumption that i should accept X at any wealth level. Suppose that $\rho_i(w)\leqslant\rho_j(w)$ holds for all wealth level w, we need to verify that if j accepts at w, then i accepts it at w. Without loss of generality, we assume $w=0$. By Corollary 4.1, we have $u_i(x)\geqslant u_j(x)$ for all $x\in\mathbb{R}$. Thus, we obtain $\mathbb{E}_h u_i(X)\geqslant\mathbb{E}_h u_j(X)$, and this completes the proof.

Proposition 4.5 $i\trianglelefteq j$ if and only if for all wealth level w_i and w_j, $\rho_i(w_i)\leqslant\rho_j(w_j)$.

Proof If $i\trianglelefteq j$ and $\exists$ w_i,w_j such that $\rho_i(w_i)>\rho_j(w_j)$, using Lemma 4 above, we find there is a gamble X such that j accepts at w_j, but i rejects it at w_i which contradicts to our assumption that i should accepts X at any wealth level. Suppose that $\rho_i(w_i)\leqslant\rho_j(w_j)$ holds for all wealth level w_i and w_j. Without loss of generality, we let $w_j=0$, and assume that $\mathbb{E}_h u_j(X)>0$. For $w\in\mathbb{R}$, we define $u_i^{(w)}(x)=u_i(w+x)$. By Proposition 4.4, we have $\mathbb{E}_h(u_i(w+X))=\mathbb{E}_h u_i^{(w)}(X)\geqslant\mathbb{E}_h u_j(X)>0$. This completes the proof.

Proof of Theorem 4.1 Let i, j, X_1, X_2, w be the notations above. With the simplification method, one can set w to be 0 and $u_i,u_j\in\mathscr{U}$, and we first consider the case $X_1,X_2\in\mathscr{G}_h$. Let $f_k(\alpha)=\mathbb{E}_h u(\alpha X_k)$ for $k=1,2$ with $u(x)=1-\exp(-x)$. We set $\alpha_k=\dfrac{1}{R_h(X_k)}$ for $k=1,2$, then $f_1(\alpha_1)=f_2(\alpha_2)=0$. By hypothesis, $\alpha_1\leqslant\alpha_2$ and the condition $j\trianglelefteq i$ implies $\rho_i(w_i)\geqslant\rho_j(w_j)$ for all w_i and w_j. Set $\beta_i=\inf\rho_i(x)$ and $\beta_j=\sup\rho_j(x)$, so that $\beta_i\geqslant\beta_j$. It can be easily shown that $u_i(x)\leqslant u(\beta_i x)/\beta_i$ by Corollary 4.1, since the Arrow-Pratt coefficient of the utility function $u(\beta_i x)/\beta_i$ (to standardize) is constantly β_i, which is no larger than $\rho_i(x)$ for all x. Similarly, one can find $u_j(x)\geqslant u(\beta_j x)/\beta_j$.

Now we assume i accepts the gamble X_1, then we need to show that j accepts the gamble X_2. By definition, $\mathbb{E}_h u_i(X_1)>0$, thus

$$\mathbb{E}_h\frac{u(\alpha_1X_1)}{\alpha_1}=\frac{f_1(\alpha_1)}{\alpha_1}=0<\mathbb{E}_h u_i(X_1)\leqslant\mathbb{E}_h\frac{u(\beta_iX_1)}{\beta_i},$$

resulting in $\beta_i<\alpha_1$. Then one gets $\beta_j\leqslant\beta_i<\alpha_1\leqslant\alpha_2$. Hence, $\beta_j<\alpha_2$. The following inequality holds

$$\mathbb{E}_h u_j(X_2)\geqslant\mathbb{E}_h\frac{u(\beta_jX_2)}{\beta_j}>$$
$$\mathbb{E}_h\frac{u(\alpha_2X_2)}{\alpha_2}=\frac{f_2(\alpha_2)}{\alpha_2}=0.$$

Thus, the duality axiom is satisfied when $X_1,X_2\in\mathscr{G}_h$. Suppose now $R_h(X_2)=0$, so that $X_2\in\mathscr{G}$ and $P(X_2<0)=0$. It is obvious that $E_h u_j(X_2)>u_j(0)=0$. Finally, we will show that i accepts X_1 at 0 and $R_h(X_1)\geqslant R_h(X_2)$ imply $R_h(X_2)<\infty$. Otherwise, we have $R_h(X_1)=\infty$, which implies $X_1\in\mathscr{G}$ with $\mathbb{E}_h(X_1)\leqslant 0$. However, it follows from Jessen's inequality that

$$\mathbb{E}_h u_i(X_1) \leqslant u_i(\mathbb{E}_h(X_1)) \leqslant 0.$$

This means i rejects X_1 at 0, yielding a contradiction.

5 Simulation

For this part, we first show that it is possible to substitute the random variable X by its realizations x_1, x_2,…,x_n to estimate its riskiness by continuity property; then we replace the distortion function f with its estimate $\widehat{f}$ and draw the same conclusion.

With data of a gamble, we can estimate the distribution function from the empirical distribution function. Suppose the data $\{x_k\}_{k=1}^{n}$ is a sequence independent and identically distributed as the gamble X. For simplification, let distortion function be WT weighting function, with parameter $\alpha = 0.25$. i. e., $h(x)=\Phi(\Phi^{-1}(x)+0.25)$, where $\Phi(x)$ stands for the CDF of Gaussian distribution.

Let $\widehat{F}_n(x)$ be the empirical distribution function of n samples. By Glivenk-Cantelli Theorem, the convergence $\widehat{F}_n \to F$ holds, which fulfills the condition for continuity property. By the continuity property, r_n, the risk under x_1, x_2,…,x_n, converges to the risk of the real gamble as $n\to\infty$, even though some r_n can be infinite or zero. Note that r_n is the solution of θ to the equation

$$\int_{-M}^{M} u(x/\theta)\ \mathrm{d}h(\widehat{F}_n(x)) = 0.$$

As for discrete random variables yielding v_i with probability p_i for $i=1,2,\cdots,K$, we can simply summarize the total number of each value by the data. Suppose that value v_i appears n_i times for $i=1,2,\cdots,K$, where K is the total kinds of values appeared. After substituting p_i by $\widehat{p}_i=\dfrac{n_i}{n}$, one can get the estimated scale function

$$\widehat{f}(\theta) = \sum_{i=1}^{K} u(v_i/\theta)\left(h\left(\frac{\sum_{j=1}^{i} n_j}{n}\right) - h\left(\frac{\sum_{j=1}^{i-1} n_j}{n}\right)\right),$$

$$n = \sum_{j=1}^{K} n_j.$$

It is fast for computers to find the unique solution to $\widehat{f}(\theta)=0$ under $0<\theta<\infty$. However, chances are that $\widehat{f}(\theta)$ is always under or above zero. Then let it be in consistent with the extended measure of riskiness. That is to say setting the riskiness to be 0 or ∞, which depends on $\widehat{f}$ if there is no solution. As long as the number of samples is sufficient to reflect the real distribution, the equation has a unique positive solution. Now consider three discrete gambles X_1, X_2, X_3 with probability mass functions given by

$$X_1 = \begin{cases} -1, & 0.5, \\ 2, & 0.3, \\ 6, & 0.15, \\ 10, & 0.05, \end{cases}$$

$$X_2 = \begin{cases} -4, & 0.5, \\ 2, & 0.3, \\ 6, & 0.15, \\ 10, & 0.05, \end{cases}$$

$$X_3 = \begin{cases} 1, & 0.5, \\ 2, & 0.3, \\ 6, & 0.15, \\ 10, & 0.05. \end{cases}$$

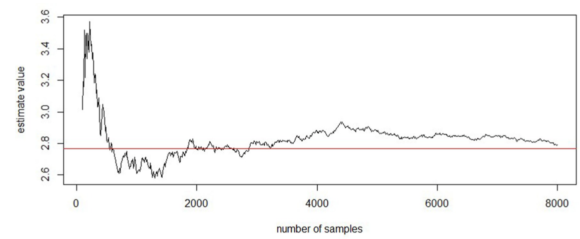

Figure 3. Estimated riskiness of distribution X_1 (wave line) with real riskiness (beeline).

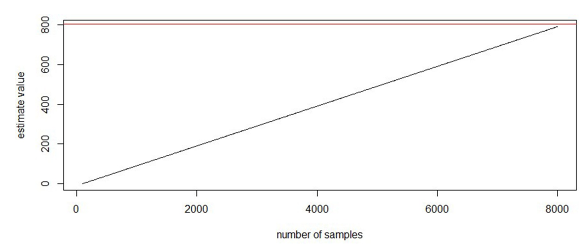

Figure 4. Estimated riskiness of distribution X_2 (wave line) with real riskiness (beeline).

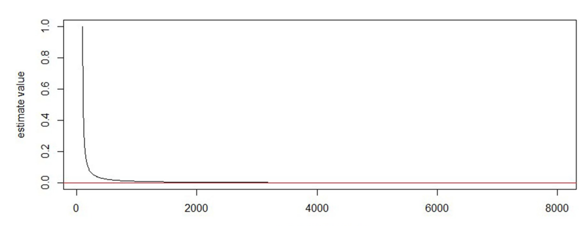

Figure 5. Estimated riskiness of distribution X_3 (wave line) with real riskiness (beeline).

We can check with computer that their risk value are respectively 2.77, ∞ and 0. Figures 3,4 and 5 show how the calculated riskinesses approaches the real riskinesses.

However, it is computational expensive to solve such an equation with a large n in practice for a continuously distributed random variable. By the enlightment of the generalized method of moments, computing the numeric solution can be seen as an optimization problem.

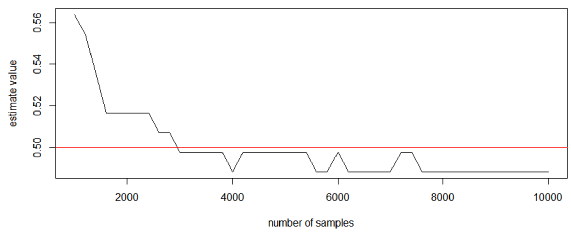

Figure 6. Estimated riskiness of normal distribution with mean 1 and standard deviation 1 under a linear distortion function.

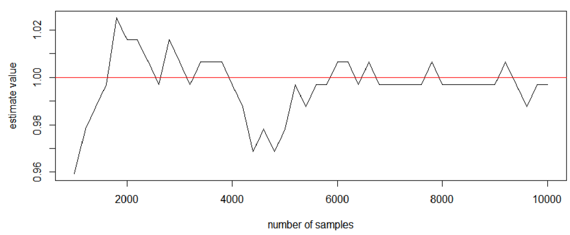

Figure 7. Estimated riskiness of normal distribution with mean 1 and standard deviation 1 under the WT distortion function.

Let $m(\theta)=\mathbb{E}_h u(X/\theta)$. We expect to find θ_0 such that $m(\theta_0)=0$, which is the unique solution by Theorem 3.1. After blending the distortion function into the distribution of X, another distribution, with CDF $G(x)=h(F(x))$, replaces the original one, making the expectation undistorted. That is, $m(\theta)=\mathbb{E}_h u(Y/\theta)$, where $Y\sim G$. But how can we get sample of Y out of x_n? We need to compute the quantile, denoted by q_k of $\widehat{F}$ at each x_k, $k=1,2,\cdots,n$. Then calculate y_k by the left inverse of $h(\widehat{F})$, i. e. $h(\widehat{F}(y_k))=q_k$ for all $k=1,2,\cdots,n$. By the law of large numbers, $\widehat{m}(\theta)=\sum_{i=1}^{n}u(y_i/\theta)$ is a fine estimator for $m(\theta)$ as long as n is large enough, then by the implementation method, we have to find $\widehat{\theta}$

$$\widehat{\theta}=\underset{\theta\in\Theta}{\operatorname{argmin}}\,\widehat{m}(\theta)W\widehat{m}(\theta),$$

where W is the inverse of $\mathrm{var}(Y)$, estimated by

$$\widehat{W}_n(\widehat{\theta})=\left(\frac{1}{n}\sum_{i=1}^{n}U(y_i/\widehat{\theta})^2\right)^{-1}.$$

Note that $\widehat{m}(\theta)W\widehat{m}(\theta)$ ranges from 0 to 1, while $\widehat{m}(\theta)$ can be extraordinary large with the exponential operator.

Suppose a continuous distribution is a normal distribution with mean 1 and standard deviation 1, we can then compute that the riskiness of such an random varible is 0.5 with a linear distortion function and 1 with the WT distortion function. Figures 6 and 7 show how the calculated estimators approach the real riskiness.

Conflict of interest

The authors declare no conflict of interest.

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基于 RDEU 模型的风险度量

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摘要: 风险度量主要应用于人们面对未知风险的情况下,如何准确地量化风险从而做出损失最小化或收益最大化的决策. 准确的风险度量可以极大地帮助投资者调整投资组合进而规避风险,以期实现最大化收益. 为了准确地度量风险,学者根据风险资产服从的客观分布进行量化,但是这种方法的问题在于度量方法是基于人们怀疑的分布,而不同人对待风险资产的态度是不一样的,风险资产可能对某些人来说是无风险的,同时又太过冒险而不被其他人接受. 为了将客观分布和主观感受更好的结合,在秩相依期望效用(rank dependent expected utility) 模型的基础上提出一种主观的"赌博"(风险资产) 风险度量: 它既取决于赌博本身,也取决于决策者的态度. 同时,这种度量方法适用于所有有界赌博,使得不同赌博的比较更加容易.

关键词: 风险度量;RDEU 模型;扭曲函数